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Consumption function: nondurable and durable goods

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Abstract

Modeling the consumption of nondurable and durable goods separately is equivalent to assuming that consumers` resources exclusively finance one category of goods. To overcome this problem, I modelled these decisions jointly. Not by chance, the new consumption functions for nondurable and durable goods depend not only on consumers` resources, but also on the lagged stock of durable goods.

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1 Introduction

In a seminal work, Hall (1978) derived the well-known nondurable consumption random walk hypothesis, which, along with the intertemporal budget constraint, implies that expenditure on nondurable goods is determined by the lifetime expected resources. Mankiw (1982) later applied Hall's approach to durable goods, concluding that the stock of durable goods follows an autoregressive process. Although I have extended Mankiw's analysis by deriving the respective consumption function for durable goods, both approaches share the same drawback: they assume that consumers' resources are used exclusively to buy one type of good. To overcome such undesirable assumptions, I combine both analyses in a single framework.

In this paper, I solve the consumer intertemporal problem by assuming a nonseparable utility in nondurable and durable goods. After all, the new consumption functions for nondurable and durable goods depend not only on consumers' resources, but also on the lagged stock of durable goods. As discussed in Section 3, the omission of this term may lead to a mistaken conclusion that consumer preferences reflect habit formation.

The rest of the paper is organized as follows. Section 2 presents the consumption functions related to the approaches of Hall (1978) and Mankiw (1982). Section 3 derives the new consumption functions. Finally, Section 4 summarizes the main contributions.

2 Consumption function: previous approaches

2.1 Nondurable Goods

Hall (1978) considers an economy lived by a representative agent who maximizes their lifetime well-being, given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t)$, where C_t is the nondurable consumption, $u(\cdot)$ is the instantaneous utility function, $\beta \in (0,1)$ is the intertemporal discount factor, and $\mathbb{E}_t[\cdot]$ is the conditional expectation. In each period, the consumer faces the following budget constraint:

$$A_{t+1} = R\,(A_t + Y_t - C_t) \tag{1}$$

where A_t and Y_t are, respectively, wealth and income, and $R = 1 + r$ is the gross interest rate, with $r > 0$. In addition, Hall (1978) made Assumptions 1 and 2:

Assumption 1 *The interest rate is the reciprocal of the intertemporal discount factor:* $\beta R = 1$.

Assumption 2 *The instantaneous utility function is quadratic:*

$$u(C) = C - \frac{b_c}{2} C^2, \;\; b_c > 0 \tag{2}$$

Under Assumptions 1 and 2, the consumer's Euler equation is such that $\mathbb{E}[C_{t+1}] = C_t$. Thus, the nondurable consumption follows a random walk process, and the intertemporal budget constraint yields the well-known consumption function:[1]

$$C_t = \frac{r}{1+r} \left[A_t + \sum_{i=0}^{\infty} R^{-i} \mathbb{E}_t[Y_{t+i}] \right] \tag{3}$$

where the right-hand side is the perpetuity of the consumer's lifetime expected resources which, therefore, exclusively finance expenditures on nondurable goods.

[1]To be precise, I also assume that $\mathbb{E}_t \left[\lim_{j\to\infty} R^{-j} A_{t+j} \right] = 0$. This hypothesis is used to derive all consumption functions in this paper.

2.2 Durable Goods

Mankiw (1982) followed Hall's approach to model the consumer's decision on durable goods. Consequently, the lifetime utility becomes $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t v(K_t)$, where K_t is the stock of durable goods, and $v(\cdot)$ is the instantaneous utility function. As became the standard in the literature, Mankiw (1982) assumes that the service flow from durable goods is proportional to their stock[2], which evolves as follows:

$$K_t = (1 - \delta)\, K_{t-1} + D_t \tag{4}$$

where D_t is the expenditure on durable goods and $\delta \in (0, 1)$ is the depreciation rate. Furthermore, the consumer faces the following budget constraint:

$$A_{t+1} = R\,(A_t + Y_t - D_t) \tag{5}$$

Following Hall (1978), Mankiw (1982) assumed a quadratic utility, as described in Assumption 3:

Assumption 3 *The instantaneous utility function is quadratic:*

$$v(K) = K - \frac{b_k}{2} K^2, \;\; b_k > 0 \tag{6}$$

Given Assumptions 1 and 3, the consumer Euler equation is such that $\mathbb{E}_t[K_{t+1}] = K_t$. Hence, the stock of durable goods follows a random walk process.[3] Mankiw (1982) did not put forward the consumption function for durable goods, a gap filled by Proposition 1. As it can be seen, the consumption of durable goods depends positively on consumers' resources and negatively on the lagged stock of durable goods. In this case, the consumers' resources finance only the expenditures on durable goods.

Proposition 1 *Under Assumptions 1 and 3, the consumption function for durable goods is as follows:*

$$D_t = \frac{r}{r+\delta}\left[A_t + \sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[Y_{t+i}]\right] - \frac{\delta(1-\delta)}{r+\delta} K_{t-1} \tag{7}$$

Proof 1 *See Appendix A.*

3 Consumption function: new approach

In practice, consumers' resources are not used to buy exclusively one type of good, thus it is important to jointly derive the consumption functions on durable and nondurable expenditures. To do so, I employ the following lifetime utility and budget constraint, respectively:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u(C_t, K_t)\right] \tag{8}$$

$$A_{t+1} = R(A_t + Y_t - C_t - P_t D_t) \tag{9}$$

[2]See also Ogaki and Reinhart (1998a) and Ogaki and Reinhart (1998b).

[3]Furthermore, the revision of the expenditure on durable goods, ΔD_t, follows an MA(1) process.

where $u(C_t, K_t)$ is the instantaneous utility and P_t is the relative price of durable goods. Consequently, the Bellman equation for the consumer intertemporal problem becomes

$$V(A_t, K_{t-1}) = \max_{C_t, D_t} \{u(C_t, K_t) + \beta \mathbb{E}_t [V(A_{t+1}, K_t)]\} \tag{10}$$

which is subject to equations (4) and (9), and the initial values A_0 and K_{-1}. The first order conditions for C_t and D_t, and the envelope theorem for A_t and K_{t-1}, yield the Euler equations:

$$u_1(C_t, K_t) = \beta R \mathbb{E}_t [u_1(C_{t+1}, K_{t+1})] \tag{11}$$

$$u_2(C_t, K_t) = \beta \mathbb{E}_t [u_1(C_{t+1}, K_{t+1}) (P_t R - (1-\delta) P_{t+1})] \tag{12}$$

Regarding the Euler equation (12), $P_t R$ measures the current opportunity cost of one unit of a durable good, while $(1-\delta)P_{t+1}$ measures how much of said cost could be recovered in the next period. Despite that, to derive analytical solutions for the consumption functions, Assumption 4 is needed. Furthermore, Assumption 5 describes the instantaneous utility.

Assumption 4 *The relative price is constant:* $P_t = P$ *for all* t*.*

Assumption 5 *The instantaneous utility is quadratic:*

$$u(C, K) = b_0 + b_1 C - \frac{b_2}{2} C^2 + b_3 K - \frac{b_4}{2} K^2 + b_5 CK \tag{13}$$

As previously, I assume that $b_i > 0$ for $i = 1, ..., 4$. Given that, the utility function (5) is concave when $b_2 b_4 > b_5^2$.[4] Although these conditions do not prevent b_5 from being negative, I assume that $0 < b_5 < \sqrt{b_2 b_4}$, which means that the marginal utility of one good increases with the other. Finally, Proposition 2 presents the Euler equations and the consumption functions for this setup.

Proposition 2 *Under Assumptions 1, 4 and 5, the Euler equations (11) and (12) imply that*

$$\mathbb{E}_t[C_{t+1}] = C_t, \tag{14}$$

$$\mathbb{E}_t[K_{t+1}] = K_t, \tag{15}$$

$$K_t = \lambda_1 + \lambda_2 C_t \tag{16}$$

where

$$\lambda_1 \equiv \frac{b_3 - b_1 P[1-\beta(1-\delta)]}{b_4 + b_5 P[1-\beta(1-\delta)]}, \quad \lambda_2 \equiv \frac{b_5 + b_2 P[1-\beta(1-\delta)]}{b_4 + b_5 P[1-\beta(1-\delta)]} > 0, \tag{17}$$

and the consumption functions for nondurable and durable goods are, respectively:

$$C_t = \theta_1 \left[A_t + \sum_{i=0}^{\infty} R^{-i} \mathbb{E}_t[Y_{t+i}] \right] + \theta_2 K_{t-1} + \theta_3 \tag{18}$$

$$D_t = \lambda_1 + \lambda_2 \theta_3 + \lambda_2 \theta_1 \left[A_t + \sum_{i=0}^{\infty} R^{-i} \mathbb{E}_t[Y_{t+i}] \right] + [\lambda_2 \theta_2 - (1-\delta)] K_{t-1} \tag{19}$$

where

$$\theta_1 \equiv \frac{r}{1 + r + (r+\delta) P \lambda_2} > 0, \quad \theta_2 \equiv \theta_1 P (1-\delta) > 0, \quad \theta_3 \equiv -\theta_1 \frac{(r+\delta) P \lambda_1}{r} \tag{20}$$

[4]Therefore, these conditions are sufficient for quasi-concavity of the utility function (13). Furthermore, for $b_5 = 0$, the condition $b_2 b_4 > b_5^2$ is automatically satisfied because b_2 and b_4 are strictly positive.

Proof 2 *See Appendix B.*

As before, the nondurable consumption expenditures and the stock of durable goods are described by random walk processes. Furthermore, an intratemporal condition links the two types of goods – equation (16). Regarding the consumption functions, expenditures on nondurable and durable goods depend positively on consumers' resources. While expenditures on nondurable goods increase with the lagged stock of durable goods, the consumption of durable goods decreases ($\lambda_2\theta_2 < (1-\delta)$). Therefore, when the stock of durable goods is relatively high (low), the expenditure on nondurable (durable) goods grows faster.

Cantor (1985) and Caballero (1990) adopted a CARA utility to derive the consumption function for nondurable goods. Alessie and Lusardi (1997) extended their analysis by adding habit formation and, as a result, the consumption of nondurable goods depends on its lagged level.[5] In consumption function (18), the lagged stock of durable consumption affects the consumption of nondurable goods. However, the intratemporal condition (16) allows the substitution of K_{t-1} by C_{t-1}, which leads to

$$C_t = \theta_1 \left[A_t + \sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[Y_{t+i}] \right] + \Psi_1 + \Psi_2 C_{t-1} \tag{21}$$

where $\Psi_1 \equiv \theta_2\lambda_1 + \theta_3$ and $\Psi_2 \equiv \theta_2\lambda_2$. The term in square brackets is the standard consumption rule from Hall (1978) – see equation (3) – and the last term will appear as the coefficient of autoregression. In this sense, apparently there is habit formation, and the usual strategy to regress the consumption growth rate on its own lag to detect habit formation – see, for instance, Dynan (2000) and Sommer (2007) – would be compromised by ignoring the decision on durable goods.

4 Conclusions

Modeling the consumption of nondurable and durable goods separately is equivalent to assuming that consumers' resources exclusively finance one category of goods. To overcome this problem, I show that if these decisions are modelled jointly, expenditures on nondurable and durable goods depend on both consumers' resources and the lagged stock of durable goods. Because this stock is related to the lagged consumption of nondurables goods, its omission may lead to the mistaken conclusion that consumer behaviour is motivated by habit formation.

A Proof of Proposition 1

Solving forward the budget constraint (5) and taking the conditional expectation yields

$$\sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[D_{t+i}] = A_t + \sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[Y_{t+i}], \tag{22}$$

as long as $\mathbb{E}_t\left[\lim_{j\to\infty} R^{-j}A_{t+j}\right] = 0$. Equation (4) and the random walk process for K_t imply that

$$\mathbb{E}_t[D_{t+i}] = \mathbb{E}_t[K_{t+i}] - (1-\delta)\mathbb{E}_t[K_{t+i-1}] = K_t - (1-\delta)K_t = \delta K_t, \tag{23}$$

[5] See also Gomes (2020).

for $i > 0$. Consequently, equation (22) becomes

$$D_t + \sum_{i=1}^{\infty} R^{-i}\delta K_t \;=\; A_t + \sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[Y_{t+i}] \tag{24}$$

The substitution of K_t using equation (4) leads to the consumption function (7).

B Proof of Proposition 2

Under Assumptions 1, 4 and 5, the Euler equations (11) and (12) become

$$b_5 K_t - b_2 C_t = \mathbb{E}_t[b_5 K_{t+1} - b_2 C_{t+1}] \tag{25}$$

$$b_3 - b_4 K_t + b_5 C_t = \beta b_1[PR - (1-\delta)P] + \mathbb{E}_t[b_5 K_{t+1} - b_2 C_{t+1}]\beta[PR - (1-\delta)P] \tag{26}$$

Substituting (25) in (26) yields the condition (16). This condition and equation (25) imply that $\mathbb{E}_t[C_{t+1}] = C_t$. This result and condition (16) imply that $\mathbb{E}_t[K_{t+1}] = K_t$.

Solving forward the budget constraint (9) and taking the conditional expectation yields

$$\sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[C_{t+i}] + P\sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[D_{t+i}] = A_t + \sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[Y_{t+i}], \tag{27}$$

as long as $\mathbb{E}_t\left[\lim_{j\to\infty} R^{-j}A_{t+j}\right] = 0$. Equation (4) and the random walk process for K_t imply that

$$\mathbb{E}_t[D_{t+i}] = \mathbb{E}_t[K_{t+i}] - (1-\delta)\mathbb{E}_t[K_{t+i-1}] = K_t - (1-\delta)K_t = \delta K_t, \tag{28}$$

for $i > 0$. Furthermore, $\mathbb{E}_t[C_{t+i}] = C_t$ because C_t follows a random walk process. These results lead equation (27) to become the following:

$$\sum_{i=0}^{\infty} R^{-i} C_t + PD_t + P\sum_{i=0}^{\infty} R^{-i}\delta K_t = A_t + \sum_{i=0}^{\infty} R^{-i}\mathbb{E}_t[Y_{t+i}] \tag{29}$$

Substituting $D_t = K_t - (1-\delta)K_{t-1}$ and $K_t = \lambda_1 + \lambda_2 C_t$, equation (29) becomes consumption function (18). Finally, notice that

$$D_t = K_t - (1-\delta)K_{t-1} = \lambda_1 + \lambda_2 C_t - (1-\delta)K_{t-1} \tag{30}$$

Substituting C_t by consumption function (18) leads to consumption function (19).

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