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A dynamical explanation about price formation in illegal markets

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Abstract

This paper introduces a dynamical extension to the static theoretical framework proposed by Slim (2009), where the author analyzes price formation in illegal markets. We develop a simple model with two regimes that can show a rich variety of dynamic behaviors, such as cyclical or even chaotic fluctuations without needing changes in exogenous variables. The analysis does not use random shocks to introduce irregular fluctuations, these arise because of the presence of intrinsic forces associated with nonlinear relations. This model can provide intuitions to explain price changes in illegal markets, particularly in drug ones, where price dynamics can become unpredictable given both the dangerous environment and the unequal degree of trust between the participants in these markets.

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1. Introduction

While it is true that illegal markets are markets in every sense of the word, they differ from conventional markets in many ways, particularly in how prices are determined (Caulkins, 2007) and also in the existence of large price variations (Gallais-Hamonno, Hoang and Oosterlinck, 2019), (Caulkins and Reuter, 2004). One theoretical proposal that has tried to explain both particularities is the work developed by Slim (2009). In his approach, the author generalizes the mark-up pricing based on the work of Kalecki (1971) through the introduction of a function of social discrimination, which is justified by the author in terms of both sociological and anthropological considerations. Slim's theoretical framework analyzes the process of pricing in static terms. Thus, our aim in this paper is to introduce a dynamic extension to this model, which will allow us to characterize a wide variety of dynamic behaviors.

In line with different contributions of Puu (Puu, 2000; Panchuk & Puu, 2018) this paper attempts to understand the illegal markets from the point of view of increasing complexity. The existence of different attractors provides the possibility of telling different stories and even pondering different scenarios. In our dynamical approach, the model can show a rich variety of dynamic behaviors, such as cyclical or even chaotic fluctuations associated with nonlinear relations. This fact reminds us, as pointed out by Puu (2000: 76), that determinism and predictability are two completely different concepts.

The starting point of the Slim model is the empirical observation that in illegal markets the exchange of homogeneous products occurs despite the fact that there is no unique price (Caulkins & Reuter, 1998; 2004). According to this approach, the diversity of situations observed is due to the absence of a legal framework as well as the absence of an authority to manage possible conflicts. Due to the above, the exchange takes place in an environment characterized by incomplete information and uncertainty.

Thus, Slim's model proposes to explain price variations, for the same good, in an illegal context. For this, the author formalizes the price following a mark-up equation with oligopolistic competition, augmented by a function of social discrimination. Based on sociological and anthropological approaches, the social discrimination function considers the degree of closeness that exists between the seller and the buyer. In this way, in illegal markets, prices transmit, in addition to traditional information, a component of affection or closeness that indicates the level of social integration that characterizes market participants. According to Slim, the existence of a function of social discrimination makes it possible to explain the diversity of prices generated by the illegal nature of exchanges. In other words, the degree of closeness between the seller and the buyer influences the formation of prices, which makes it possible to explain the existence of abnormally high prices.

The remainder of this paper is organized as follow. Section 2 presents the theoretical framework developed by Slim (2009). In section 3 we transform the static version into a dynamic one and identify the kind of behaviors produced by the model. Finally, section 4 provides some conclusions.

2. Price formation in illegal markets

Slim (2009) proposes a theoretical framework to analyze price formation in illegal markets. His work generalizes the equation of mark-up pricing in oligopolistic markets by introducing a function of social discrimination, which is justified by the author in terms of both sociologic and anthropologic approaches.

According to Slim, illegal markets are characterized by a multiplicity of situations that produce a wide variety of prices. According to the author, there is a continuum of advertised prices for the same good and there may be situations in which the seller rejects the exchange or even decides to donate the property.

For this reason, price formation is based on a mark-up equation, first formulated by Kalecki (1971). The model is based on two postulates. First, the environment is subject to endogenous uncertainty, inherent in the illegal nature of the activities considered (Caulkins et al, 2006). This principle is observed, according to Slim, in the fact that one of the main characteristics of illegal markets is the opacity of information. Consequently, the illegality of the exchange implies uncertainty associated, on the one hand, with the possibility of being detained by the competent authorities and, on the other, with the absence of legal or institutional frameworks in the event of non-compliance with the terms of exchange.

A second source of uncertainty is associated with the type of market structure. Slim's model assumes the existence of oligopolistic competition, in which sellers are on the long side, as price creators, and demanders, on the short side, as price takers. According to the margin price equation, suppliers decide the selling price, based on production costs, or average variable costs, to which a margin is applied, which includes risk taking.

Following a sociological and anthropological approach, Slim considers that illegal prices transmit a degree of social recognition, or level of affection, between the seller and the buyer (Offer, 1997). This implies that the standard hypothesis of undifferentiated individuals is abandoned and admits, on the other hand, that individuals are inserted in a society structured in social networks. Thus, the seller distinguishes individuals through the level of affection or closeness, which implies a social discrimination based on the level of integration in the network. The price discrimination pattern, thus defined, is no longer based on the characteristics of the product, but rather based on the social characteristics of the agents. Therefore, to the traditional margin equation a social discrimination function is added that reflects the level of affect, whose generic form is written as follows:

$$p = \pi(\varepsilon)c(1+\mu) \qquad (1)$$

Where p is the price, c the cost, μ the mark-up and $\pi(\varepsilon)$ is the function of social discrimination, which is defined as follows

$$\pi(\varepsilon) = \frac{1}{\varepsilon}e^{1-\varepsilon} \qquad (2)$$

where $\varepsilon > 0$. Observe that for $\varepsilon = 1$, equation (1) becomes $p = c(1+\mu)$, which is the conventional equation of mark-up pricing in oligopolistic markets. For $\varepsilon > 1$, the function of social discrimination is less than 1, which implies that the price is below the mark-up pricing.

This situation might arise when agents are members of particular networks, for instance a network of familiar ties. Thus, the levels of both social integration and trust between participants interacting into personal exchanges have an impact on the price determination. On the other hand, when $0 < \varepsilon < 1$, the function of social discrimination is greater than 1 and, consequently, the price is higher than the one get under conventional mark-up pricing. This situation reflects riskier exchanges, as those observed in drug markets, where agents set higher prices in order to compensate for the lack of trust in their counterparts.

3. Dynamical analysis

In what follows equation (1) is transformed into a dynamical expression. In the mark-up pricing formulated by Kalecki (1971), the mark-up μ can be considered as the degree of monopoly. Following this idea, in this work we assume that μ, the mark-up, is given by the ratio between the price in the previous period, p_{t-1}, and the price set by a monopoly, p^*. Thus, equation (1) becomes

$$p_t = \pi(\varepsilon)c\left(1 + \frac{p_{t-1}}{P^*}\right) \qquad (3)$$

This analysis assumes that equation (3) presents a discontinuous change when a critical value of the price is reached. This critical value, labeled as p^s, separates the evolution of the price in two regimes: regime 1 (labeled as *R1*) corresponds to the situation when the price is below the critical value and regime 2 (labeled as *R2*) when it is above. Thus, equation (3) has been transformed into a two-regime model. Intuitively, a regime is understood as a qualitative behavior that can be distinguished from another qualitative behavior. In formal terms, given a dynamical system (D, f) and a partition $\{D_1, D_2, \dots, D_n\}$ of the domain D, a regime is a pair (D_i, f_i) $(i = 1,2, \dots, n)$ where $f_i = f|D_i$ is the restriction of the function f to the setD_i(Brida, 2006). The idea of using dynamics of regimes in the economic analysis has been widely explored in (Brida 2006), (Brida, 2006a), (Brida, 2008), (Brida, Puchet and Punzo, 2011), (Brida and Punzo, 2003), (Brida and Punzo, 2008). Equation (3) is now is defined by:

$$f(p) = \begin{cases} \pi(\varepsilon_1)c\left(1 + \frac{p}{P^*}\right) & \text{if } p \le p^s \\ \pi(\varepsilon_2)c\left(1 + \frac{p}{P^*}\right) & \text{if } p > p^s \end{cases} \qquad (4)$$

Thus, the model is represented by the first-order differential equation $p_{t+1} = f(p_t)$. Observe that in regime 1 the value of the function of social discrimination is given by $\pi(\varepsilon_1)$, whilst in regime 2 it corresponds to $\pi(\varepsilon_2)$. Consequently, the dynamics of regimes are associated to changes in parameter ε. The map$f(p)$ is linear in each regime. However, given the discontinuity at $p^s$the overall system is nonlinear. There are a great variety of cases, which depend both on the slopes of$f$(p) , $\frac{\pi(\varepsilon_1)c}{P^*}$and $\frac{\pi(\varepsilon_2)c}{P^*}$, and on the number of fixed points. We restrict the analysis to the interval $[z_2, z_1]$, where z_1 and z_2 are defined as follows: $z_1 = \pi(\varepsilon_1)c\left(1+\frac{p^s}{P^*}\right)$ and $z_2 = \pi(\varepsilon_2)c\left(1+\frac{p^s}{P^*}\right)$. Therefore, the restricted domain $[z_2, z_1]$ is a trapping set: all trajectories must enter this interval and remain there. If the starting point is outside, all the trajectories after a finite number of iterations go toward $[z_2, z_1]$ and remain trapped there, or diverge to infinity. It is important to note that the invariant interval $[z_2, z_1]$ is a restricted domain that excludes the parameter values for which the price diverge to infinity.

In what follows we consider only two extreme cases to show that the model is capable of generating very complex regime dynamics. Given a partition *P*, these two cases verify the covering property on *P*: an element of *P* is mapped by *f* onto a union of elements of the partition. If a partition verifies the covering property, the regime dynamics is represented by a shift of finite type and then by a transition graph *G* and its associate matrix. Transition graph *G* is constructed using a covering rule: an edge from vertex *i* to vertex *j* is drawn if and only if the image *f(Ri)* of regime *Ri* contains the regime *Rj* . Coded dynamics is then described by all possible walks through the labeled directed graph *G*. Thus, if we have been able to construct a covering partition *P* of the domain of the corresponding dynamical system, its associated coded dynamics can be effectively represented by a graph.

The first case is showed in Graph 1. This is a very simple case. If the process starts inside R_2, the price decreases and moves below p^s , the next period the price enters in R_1 and increases and moves above p^s, so p moves to R2 and the process starts again. We have two possible symbolic sequences: $\{R_1,R_2,R_1,R_2\ldots\}$ or $\{R_2,R_1,R_2,R_1\ldots\}$. All the punctual trajectories in this simple case are cycles of period 2. Figure 1 illustrates the transition graph and transition matrix representing this particular case.

Graph 2 displays the most complex case, where $f(R_2) = f(R_1) = R_2 \cup R_1$. The covering rule implies that the regime dynamics can be represented by the full shift in two symbols, and every symbolic sequence is possible. Under these conditions the price becomes unpredictable. Figure 1 shows the transition graph and transition matrix representing for the four cases. Case (a) is very simple: the only possibility for a trajectory is to alternate regime and then the possible sequences of regimes are 121212…. = $(12)\infty$ and 212121… = $(21)\infty$ only. All of the punctual trajectories in this simple case are cycles of period 2. Cases (b)-(d) illustrate some possible complex dynamics in the model. Here, regime dynamics can be viewed as a Markov chain in two states and unstable non-periodic fluctuations occur for almost all of the initial levels. The complexity of each case is not the same: e.g. cases (b) and (c) have lower entropy than (d). Surely (d) is the most complex case, in which $f(R_2) = f(R_1) = R_2 \cup R_1$ and then the covering rule implies that any sequence of regime changes can be produced by the model. Thus, regime dynamics is represented by the full shift in two symbols, and every symbolic sequence is possible. In this case the price becomes unpredictable unless we know the initial condition with infinite accuracy.

Note that in a generic case, regime dynamics cannot be represented by a two-vertex directed graph, unlike for the extreme cases. Accordingly, the possibilities for regime switching range from the simplest case (a) where every regime sequence is periodic with switches between Regime1 and 2 to the more complex case (d) represented by the full shift where every regime sequence is possible, including cycles of any order and non-periodic sequences. Despite this, it is important to remark that in a generic case, regime dynamics can be represented in a symbolic way with a shift of finite type (see Brida (2006a, 2006b); Brida *et al* (2008, 2011)). In particular, it is always possible to verify whether a certain sequence in the symbols 1 and 2 is a possible path representing the regimes traversed by a trajectory of our model.

At this point, we are able to discuss the complexity of our model. The natural way to measure it is the symbolic entropy. From the dynamic viewpoint, entropy measures the complexity and randomness of a dynamical system. From the information theory viewpoint, it is a measure of its "information capacity" or ability to transmit messages. The entropy of a space of symbolic sequences X measures the exponential growth rate of n blocks in X. The number $\#B(X)$ of n blocks appearing on points of X gives us some idea of the complexity of X: the larger the number of n blocks, the more complicated the space. Instead of using the individual numbers *#Bn(X)* for n= 1; 2; : : : , we can summarize their behavior by computing their growth rate. As can be seen, $\#B(X)$ grows approximately like $2cn$, and the value of the constant c is the growth rate. Then this value should be roughly $(1/n)$log2($\#Bn(X)$) for large n. This motivates the definition of entropy: if X is a shift space, then the (topological) entropy of X is defined by:

$$h(x) = \lim_{n \to +\infty} \frac{1}{n} log_2(\#Bn(X))$$

Note that if X is a space defined in a set of R symbols, then $\#B(X) \leq Rn$ for all n and this implies that $h(X) \leq log_2(R)$. If X is the full shift of R symbols, we obtain that $\#Bn(X) = \#Rn$, so it is $h(X) = log_2(R)$. With a two-regime model, we can obtain a maximal entropy level $h = log_2(2) = 1$, case (d), and a minimal entropy $h = 0$, case (a). For the particular cases (a) – (d) with regime dynamics represented by a direct vertex graph and its transition matrix, it is simple to calculate the entropy. Here, we derive that $h = log_2(\alpha A)$, where αA is the largest positive eigenvalue (called the Perron-Frobenius eigenvalue) of the transition matrix A. Then, it is clear that in case (a) is $h = 0$, in case (d) is $h = 1$, and in cases (b) and (c) is $h = log_2\left(\frac{1+\sqrt{5}}{2}\right)$. For other parameter values, the computation is not so easy. It can be shown that if m and n are the natural numbers such that z! leaves the regime R_2 at the nth iterate and Z_2 leaves regime R_1 at the *mth* iterate, then increases monotonically with n and m from 0 to 1.

4. Conclusions

In line with the contributions of Puu (2000), in this paper we analyzed markets as an example of a non-linear phenomenon. Following Slim (2009), where the author analyzes price formation in illegal markets, we introduced a dynamical extension to this theoretical framework. We presented a simple model with two regimes that can show aggregate fluctuations in absence of stochastic components.

The most important result of this study is that, unless the initial conditions of the system are exactly known, the deterministic system can become unpredictable. This is accentuated because p^s is private information of the company (it is part of its pricing strategy), so it is not possible to anticipate changes in the price regime. On the other hand, as previously mentioned, p^s is an indirect indicator of the market power of the company. The restriction imposed on this model implies that if we are in $[z_2, z_1]$ then this company has a minimum market share threshold among type 2 individuals and a maximum market share threshold among type 1 individuals.

This means that if the price is in [z_2, z_1], with $z_2<z_1$ then the company will participate in both sub-markets.

The lack of predictability and the high margins that the selling part obtains, make government intervention through active policies desirable. The answer à la Becker (2006) consists of an internalization of the activity, through its legalization; in this way the power of the oligopolies decreases as competition increases. On the other hand, the problem of trust between buyers and sellers can be attacked; by improving communication and transparency in transactions, it is possible to access better information on market conditions and the payment of cost overruns is avoided. These cost overruns take the form of higher interest rates or risk premiums.

Finally, it is even beneficial from a tax point of view. By taxing activities, the market loses efficiency but it is very possible that the situation is better than the initial condition and it is also the Government that captures part of the income from the activity, which can be used for active policies.

It is worthwhile to mention that, in spite of its simplicity, the model is capable of generating a wide variety of dynamics, such as cyclical or even chaotic fluctuations without needing changes in exogenous variables. This dynamical model can provide intuitions to explain price changes in illegal markets, particularly in drug markets, where price dynamics can become unpredictable given both the dangerous environment and also the unequal degree of trust between the participants. High price volatility may not reflect a successful repressive policy but rather respond to its own dynamics.

In addition to the innovations proposed in the present work, some extensions are proposed. This model can be considered in a framework of monopolistic competition, with consumers seeking to maximize consumption in an intertemporal way. Another possible extension of this work looks at the conditions of competition and distribution of the market with firms that endogenously choose $\pi(\varepsilon)$. A question that arises is whether in a model with these characteristics there will be market segmentation and the impact of this segmentation on volatility and relative prices.

References

Becker, G. S., Murphy, K. M., & Grossman, M. (2006). The market for illegal goods: the case of drugs. *Journal of Political Economy*, 114(1), 38-60.

Brida, J.G. (2006a). A model of inflation and unemployment with multiple regimes. *International Mathematical Forum* 1(23): 1125 - 1144.

Brida, J. G. (2006b). Multiple Regimes Model Reconstruction Using Symbolic Time Series Methods. *International Journal of Applied Mathematics & Statistics*, 5(S06), 19-40.

Brida, J.G. (2008). The dynamic regime concept in Economics. *International Journal of Economic Research* 5(1): 55-76.

Brida, J. G., Anyul, M. P., & Punzo, L. F. (2003a). Coding economic dynamics to represent regime dynamics. A teach-yourself exercise. *Structural Change and Economic Dynamics*, 14(2), 133-157.

Brida, J. G., & Punzo, L. F. (2003b). Symbolic time series analysis and dynamic regimes. *Structural Change and Economic Dynamics*, 14(2), 159-183.

Brida, J. G., & Punzo, L. F. (2008). Multiregime dynamics: modeling and statistical tools. *Brazilian Journal of Business Economics*-BJBE, 8(1), 15-28.

Caulkins, J. (2007). Price and Purity Analysis for Illicit Drug: Data and Conceptual Issues'. *Drug and Alcohol Dependence*, 90, 61–68.

Caulkins, J., Reuter, P., (2004). Illegal "lemons": price dispersion in cocaine and heroin markets. *Bulletin on Narcotics*, 54 (1,2),141-165.

Kalecki M., (1971). *Selected Essays on the Dynamics of the Capitalist Economy*, Cambridge University Press, Cambridge.

Gallais-Hamonno, G., Hoang, T. H. V., & Oosterlinck, K. (2019). Price formation on clandestine markets: the case of the Paris gold market during the Second World War. The *Economic History Review*, 72(3), 1048-1072.

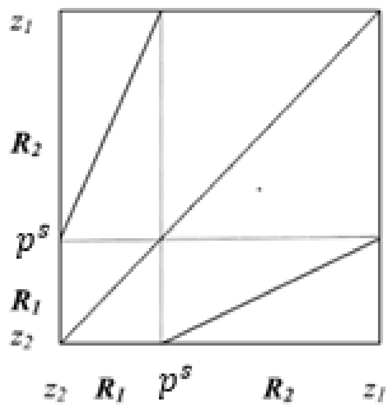
Offer, A. (1997).Between the gift and the market: the economy of regard. *Economic History Review*. 50(3): 450-476.

Panchuk, A., & Puu, T. (2018). Dynamics of a durable commodity market involving trade at disequilibrium. *Communications in Nonlinear Science and Numerical Simulation* 58: 2-14.

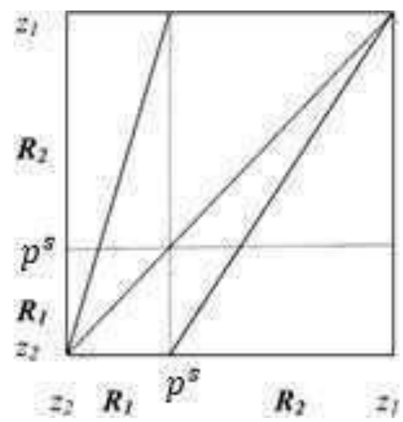
Puu, T., (2000), *Attractors, Bifurcations, & Chaos: Nonlinear Phenomena in Economics*, Berlin New York, Springer.

Slim, S. (2009). Du refus de vente au don: Une explication de la formation du prix par l´affect. MPRA paper no. 15317.

Graphs and figures



Graph 1



Graph 2

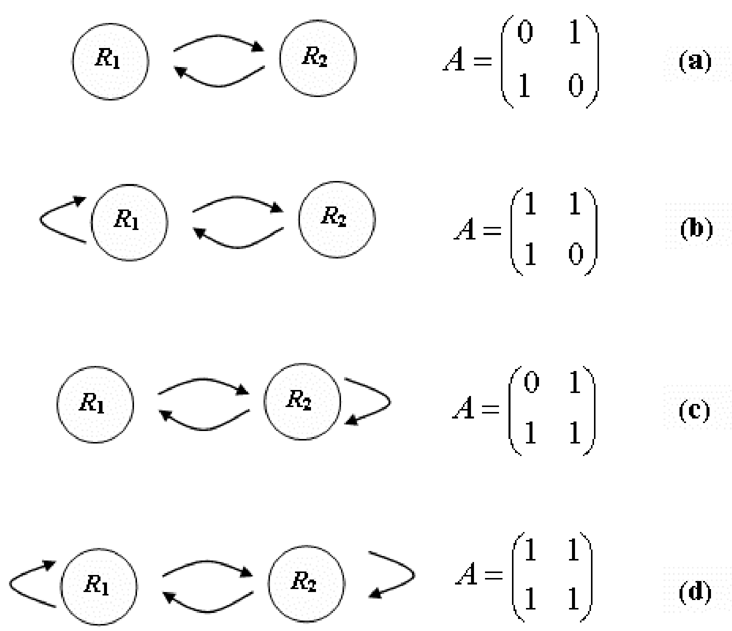


Figure 1. A zero in the ij entry means that there is no arrow from i to j and a one means that there is an arrow from i to j, where i; j = R1;R2.