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Bayesian GARCH modeling for return and range

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Abstract

This paper introduces a new generalized autoregressive conditional heteroskedasticity (GARCH) structure, which models the daily return of an asset and the price range simultaneously to describe the time-varying volatility of an asset return. New equations that link the price range to volatility are added to the GARCH and related models based on the density of the range. An algorithm for the Bayesian estimation of the parameters and one-step-ahead forecasting is provided by using the adaptive Markov chain Monte Carlo. The approach is applied to stock index data in Japan and the United Kingdom. The estimation results reveal that the proposed models capture the stylized features of an asset return, such as volatility clustering and asymmetry of the volatility to the return (leverage effect). The downward bias of the range, due to non-trading hours and the market microstructure, is suggested in the estimation. Model comparisons are conducted based on the predictive ability for the volatility, which shows that the new GARCH-type models perform equal to or better than the competing models for the return and the corresponding realized measure of the volatility.

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1 Introduction

In recent decades, researchers in financial econometrics have investigated the unobserved variance (volatility) of the return of an asset because it is of critical significance to option pricing or risk management. A vast amount of empirical literature has suggested the occurrence of periods of high and low volatility for each asset; that is, volatility clusters emerge (e.g., Bauwens, Hafner, and Laurent (2012)).

The realized generalized autoregressive conditional heteroskedasticity (GARCH) model introduced by Hansen, Huang, and Shek (2012) proposes the joint modeling of the return of an asset and the corresponding realized measure of the volatility. The realized measure is a proxy for a latent variable that is computed using high-frequency financial data (e.g., Andersen and Bollerslev (1998)). Although such proxies may be biased due to non-trading hours and market microstructure noise (e.g., Hansen and Lunde (2006)) in the real market, they are far more informative and attractive to the researchers who investigate assets' latent volatility. The realized GARCH $(1, 1)$ model with a log-linear specification is given by the following equations:

$$y_t = \sigma_t z_t, \;\; z_t \sim \mathrm{N}(0, 1), \;\; t = 1, \dots, n, \tag{1}$$

$$\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \alpha x_{t-1}, \;\; t = 2, \dots, n, \tag{2}$$

$$x_t = \xi + \varphi \log \sigma_t^2 + \tau(z_t) + u_t, \; u_t \sim \mathrm{N}(0, \sigma_u^2), \;\; t = 1, \dots, n, \tag{3}$$

where x_t is the logarithm of a realized measure for the volatility and τ is a function with the expectation of 0. $\mathrm{N}(m, s^2)$ denotes the normal distribution with mean m and variance s^2. Hansen and Huang (2016), Gerlach and Wang (2016), Wang, Chen, and Gerlach (2019), and Chen, Watanabe, and Linc (2021) investigated the further extension of the model. Takahashi, Omori, and Watanabe (2009) and Dobrev and Szerszen (2010) proposed the simultaneous modeling of the return of an asset and the corresponding realized measure of the volatility in line with the stochastic volatility model .

Although the realized GARCH and related models succeed in the estimation of the unobserved volatility of financial time series (e.g., Watanabe (2012)), the models' accuracy relies heavily on the realized measure used for the estimation. Degiannakis and Livada (2013) contended that the realized volatility on a day is less accurate than an estimator computed only from the transaction price range (i.e., the difference of the logarithms of the highest and the lowest transaction prices) on the day if the sampling frequency of the intraday returns is not sufficiently high. Volatility estimations based on the price range have been discussed by, e.g., Parkinson (1980), Garman and Klass (1980), Rogers and Satchell (1991), Kunitomo (1992), and Yang and Zhang (2000) because the dataset can be easily obtained and computed.

In this paper, I propose new GARCH-type simultaneous modeling of the return and the associated transaction price range. I add a new equation to integrate the observed price range and the unobserved volatility into the GARCH model with a log-linear specification represented in Equations (1) and (2). Referencing Feller (1951), Kurose (2021), and Hansen, Huang, and Shek (2012), the equation is designed to be exactly consistent with the probability distribution of the range. Chou (2005), Brandt and Jones (2006), Chen, Gerlach, and Lin (2008), and Molnár (2016) investigated GARCH-type models using the approximated density of the range. Asymmetry (leverage effect) is also incorporated in the proposed model, and is a stylized feature in the stock market, i.e., a negative shock is followed by larger conditional volatility than a positive one. I construct a simple Bayesian estimation and forecasting algorithm to implement empirical studies using real-world data.

The remainder of this article is organized into three sections. Section 2 introduces the proposed new GARCH-type modeling for an asset return and the range-based volatility proxy. I adopt a Bayesian approach to investigate an estimation method for the proposed model to obtain the posterior density of the model parameters. Section 3 applies the proposed model to daily returns and the price range data for a stock index in Japan and the United Kingdom. Section 4 concludes.

2 GARCH-type models and volatility proxy based on range

2.1 The models

Assume that $p(s)$, the log-price of an asset at time s, is subject to a continuous stochastic process, i.e.,

$$dp(s) = \sigma(s)dB(s), \tag{4}$$

where $B(s)$ denotes a standard Brownian motion. Define $R_t = \log H_t - \log L_t$ as the price range in day t, where H_t is the highest price and L_t is the lowest price of the day t, respectively.

The probability density function of R_t is given in two forms as

$$f_{\text{range}}(R_t|\sigma_t^2) = \begin{cases} 8\sum_{n_d=1}^{\infty}(-1)^{n_d-1}\frac{n_d^2}{\sqrt{2\pi\sigma_t^2}}\exp\left(-\frac{n_d^2R_t^2}{2\sigma_t^2}\right), \\ 8\sum_{n_d=1}^{\infty}\left\{\frac{(2n_d-1)^2\pi^2\sigma_t^4}{R_t^5}-\frac{\sigma_t^2}{R_t^3}\right\}\exp\left\{-\frac{(2n_d-1)^2\pi^2\sigma_t^2}{2R_t^2}\right\}. \end{cases} \tag{5}$$

(See Feller (1951) and Kurose (2021).)

Parkinson (1980) revealed the daily returns' p-th moment and the second moment as

$$\mathrm{E}(R_t^2) = (4\log 2)\sigma_t^2. \tag{6}$$

Parkinson's estimator $R_t^2/(4\log 2)$ has been used as the proxy for the variance of the asset return on day t. While Parkinson's estimator is consistent with the theory of stochastic process and is easy to compute, many researchers have suggested that it tends to be (negatively) biased due to discretization errors of the price or non-trading hours.

In this paper, I introduce a GARCH model for return and range (GARCH-RG), written as follows:

$$y_t = \sigma_t z_t, \quad z_t \sim \mathrm{N}(0,1), \quad t = 1,\dots,n, \tag{7}$$

$$\log\sigma_t^2 = \omega + \beta\log\sigma_{t-1}^2 + \alpha\log\{R_{t-1}^2/(4\log 2)\}, \quad t = 2,\dots,n, \tag{8}$$

$$R_t \sim f_{\text{range}}(R_t|\widetilde{\sigma}_t^2), \quad \log\widetilde{\sigma}_t^2 = \xi + \varphi\log\sigma_t^2 + \tau(z_t), \quad t = 1,\dots,n. \tag{9}$$

$\tau(z_t)$ is the *leverage function*, which describes the leverage effect, the (negative) correlation between the return and the one-step-ahead volatility. Based on Hermite polynomials, $\tau(z_t)$ is set as $\delta_1 z_t + \delta_2(z_t^2-1)$ (Hansen, Huang, and Shek (2012)). Notice that the expectation of $\tau(z_t)$ takes the value of 0.

ξ is a bias correction term for the range. If ξ takes the negative value, R_t's have downward bias because $\mathrm{E}(\tau(z_t)) = 0$ as stated above. Garman and Klass (1980) and Rogers and Satchell (1991) suggested that the daily price range of an asset may have a negative bias due to non-trading hours or market microstructure noise, e.g., price discretization error.

Note that the one-step-ahead forecast of the volatility is obtained as

$$\sigma_{n+1}^2 = \exp[\omega + \beta\log\sigma_n^2 + \alpha\log\{R_n^2/(4\log 2)\}]. \tag{10}$$

I extend the GARCH-RG model described above to the exponential GARCH model for return and range (EGARCH-RG) written as follows:

$$y_t = \sigma_t z_t, \quad z_t \sim \mathrm{N}(0,1), \quad t = 1, \ldots, n, \tag{11}$$

$$\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \rho_1 z_{t-1} + \rho_2 (z_{t-1}^2 - 1) + \gamma \log(R_{t-1}^2 / \widetilde{\sigma}_{t-1}^2), \quad t = 2, \ldots, n, \tag{12}$$

$$R_t \sim f_{\mathrm{range}}(R_t | \widetilde{\sigma}_t^2), \quad \log \widetilde{\sigma}_t^2 = \xi + \varphi \log \sigma_t^2 + \tau(z_t), \quad t = 1, \ldots, n. \tag{13}$$

$\tau(z_t)$ is the same as the GARCH-RG model. This is an extension of the exponential GARCH (EGARCH) model introduced by Nelson (1991)[1].

Note that the one-step-ahead forecast of the volatility is obtained as

$$\sigma_{n+1}^2 = \exp[\omega + \beta \log \sigma_n^2 + \rho_1 z_n + \rho_2 (z_n^2 - 1) + \gamma \log(R_n^2 / \widetilde{\sigma}_n^2)]. \tag{17}$$

2.2 Estimation and forecasting

Priors. Let denote $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}$ and $\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}$ as $(\omega, \beta, \alpha, \xi, \varphi, \delta_1, \delta_2, \log \sigma_1^2)'$ in the GARCH-RG model and $(\omega, \beta, \rho_1, \rho_2, \xi, \varphi, \delta_1, \delta_2, \log \sigma_1^2)'$ in the EGARCH-RG model, respectively. I assume that the priors $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}} \sim \mathrm{N}(\boldsymbol{m}_{\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}0}, S_{\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}0})$, $\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}} \sim \mathrm{N}(\boldsymbol{m}_{\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}0}, S_{\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}0})$.

Bayesian estimation and forecasting. I propose an adaptive Markov chain Monte Carlo (MCMC) algorithm for the GARCH-RG (EGARCH-RG) model described as follows:

1. Initialize $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}$ ($\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}$).
2. Generate $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}$ ($\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}$) using the adaptive random walk Metropolis algorithm (Haario, Saksman, and Tamminen (2001), Andrieu and Thoms (2008)).
3. Compute $\log \sigma_{n+1}^2$.
4. Go to 2.

Using the algorithm described above, I obtain adaptive MCMC samples of $\{\sigma_t^2\}_{t=1}^n$, $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}$ ($\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}$), and σ_{n+1}^2. In the following, I use the estimated posterior mean $\overline{\sigma^2}_{n+1}$ as the one-day volatility forecast.

Evaluation of volatility forecasts. To evaluate the volatility forecasts obtained from the proposed models and competing models, I use the following loss functions,

$$L_{\mathrm{MSE}}(\widehat{\sigma}_t^2, h_t) = \frac{(h_t - \widehat{\sigma}_t^2)^2}{2}, \tag{18}$$

[1] Hansen and Huang (2016) proposed the realized EGARCH framework that uses an asset return and the realized measure of the volatility of the return. The simplest model among the realized GARCH class is given by the following equations:

$$y_t = \sigma_t z_t, \quad z_t \sim \mathrm{N}(0,1), \quad t = 1, \ldots, n, \tag{14}$$

$$\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \rho_1 z_{t-1} + \rho_2 (z_{t-1}^2 - 1) + \gamma u_{t-1}, \quad t = 2, \ldots, n, \tag{15}$$

$$x_t = \xi + \varphi \log \sigma_t^2 + \tau(z_t) + u_t, \ u_t \sim \mathrm{N}(0, \sigma_u^2), \quad t = 1, \ldots, n, \tag{16}$$

where x_t is the realized measure of the volatility.

$$L_{\mathrm{QLIKE}}(\widehat{\sigma}_t^2, h_t) = \frac{\widehat{\sigma}_t^2}{h_t} - \log \frac{\widehat{\sigma}_t^2}{h_t} - 1, \tag{19}$$

where $L_{\mathrm{MSE}}(\cdot,\cdot)$ denotes the minimum squared error (MSE) loss, $L_{\mathrm{QLIKE}}(\cdot,\cdot)$ denotes the quasi-likelihood (QLIKE) loss, h_t is a forecast of the volatility on day t, and $\widehat{\sigma}_t^2$ is a volatility proxy. Since no researchers in the financial market can observe the true volatility of an asset return, volatility proxies are inevitably used in the loss functions. Volatility proxies may be contaminated with noise and the ranking between the two volatility forecasts based on those loss functions seems to be inconsistent with the one using true volatility. Patton (2011) demonstrated that a certain class of loss functions, including the two stated above, preserves the ranking between the two forecasts of the volatility if the unbiased volatility proxy is substituted by the true volatility.

3 Empirical study

3.1 Data

This section presents an empirical analysis of the time-varying volatility of asset returns in the stock market. I use the daily asset return series of Nikkei 225 index and FTSE 100 index from January 3, 2012, to December 31, 2020. The data for Nikkei 225 and for FTSE 100 include 2,194 and 2,274 returns, respectively. The daily return of an asset is calculated as $y_t = (\log p_t - \log p_{t-1}) \times 100$, where p_t is the closing price of the asset on day t. The daily price range data of the asset is used in the proposed model and is calculated as $R_t = (\log H_t - \log L_t) \times 100$, where H_t and L_t are the highest and the lowest prices of the asset on day t.

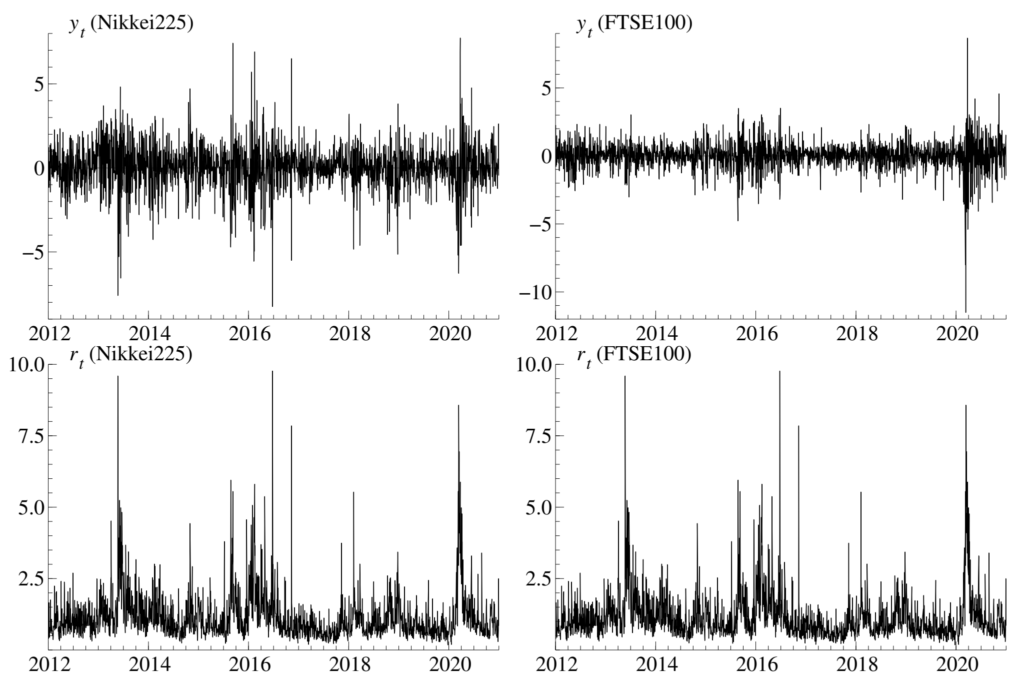

Figure 1: Transition of y_t and R_t.

The transitions of y_t and R_t for Nikkei 225 and FTSE 100 data are presented in the four panels of Figure 1.

To illustrate the validity of the models considered, I conduct a model comparison using the realized volatility (five-minute sub-sampled) associated with the returns, which is retrieved from the "realized library" provided by Heber, Lunde, Shephard, and Sheppard (2009).

3.2 Estimation results

The priors in the models are assumed as $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}} \sim \mathrm{N}(\mathbf{0}_8, 100I_8)$, and $\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}} \sim \mathrm{N}(\mathbf{0}_9, 100I_9)$, where $\mathbf{0}_p$ denotes a p-dimensional zero vector and I_p denotes a p-dimensional unit matrix.

I use 20,000 adaptive MCMC samples after discarding 10,000 samples as burn-in.

Table 1 presents the estimated posterior means, 0.95 probability credible intervals and inefficiency factors (IF) for $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}$ ($\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}$). The inefficiency factor is calculated as $1 + 2\sum_{j=1}^{\infty} \rho(j)$, where $\rho(j)$ is the sample autocorrelation at lag j. It is used to perform convergence diagnostics (see Chib (2001)). For Nikkei 225 and FTSE 100 data, the acceptance rates of $\boldsymbol{\vartheta}^{\mathrm{GARCH-RG}}$ ($\boldsymbol{\vartheta}^{\mathrm{EGARCH-RG}}$) of the GARCH-RG (EGARCH-RG) model in the adaptive Metropolis algorithm are 0.231 (0.215) and 0.259 (0.253), respectively. It indicates that the proposed estimation algorithm for the model functions effectively and adaptive MCMC samples are obtained efficiently.

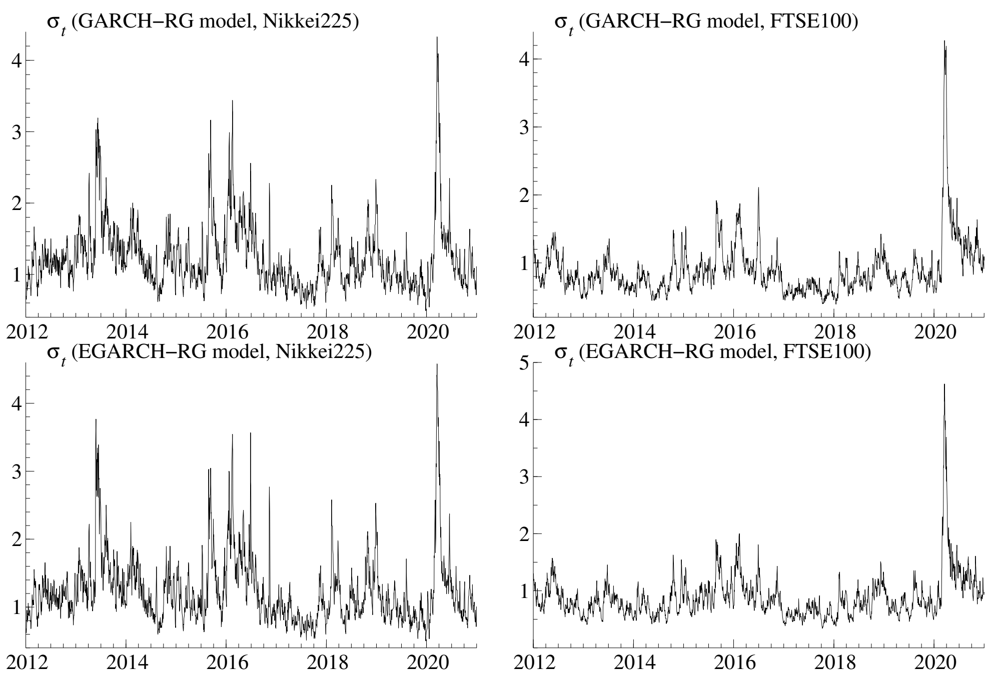

Figure 2: Evolution of the estimated posterior means of σ_t's for GARCH-RG and EGARH-RG models.

Figure 2 depicts the evolution of the estimated posterior means of σ_t, indicating a high persistence of volatility and volatility clustering, as demonstrated in past empirical studies (e.g., Bauwens, Hafner, and Laurent (2012)). In fact, for Nikke 225 data,

Table 1: Bayesian estimation results of the models.

Parameter	Mean	95% interval	IF
Nikkei225, GARCH-RG model			
ω	0.365	(0.323, 0.408)	19.4
β	0.682	(0.648, 0.714)	25.1
α	0.244	(0.217, 0.271)	14.3
ξ	-1.290	(-1.354, -1.230)	21.8
φ	1.086	(1.023, 1.151)	30.9
δ_1	-0.101	(-0.124, -0.077)	25.7
δ_2	0.198	(0.183, 0.215)	18.8
$\log \sigma_1^2$	-0.751	(-1.483, -0.051)	24.4
Nikkei225, EGARCH-RG model			
ω	0.020	(0.013, 0.028)	26.1
β	0.968	(0.961, 0.975)	27.2
ρ_1	-0.028	(-0.036, -0.022)	36.4
ρ_2	0.068	(0.059, 0.078)	43.4
γ	0.011	(0.007, 0.015)	52.0
ξ	-1.319	(-1.388, -1.251)	39.2
φ	1.116	(1.044, 1.193)	25.1
δ_1	-0.087	(-0.112, -0.063)	34.8
δ_2	0.193	(0.178, 0.210)	27.2
$\log \sigma_1^2$	-0.569	(-1.168, -0.063)	46.2
FSTE100, GARCH-RG model			
ω	0.118	(0.100, 0.136)	16.9
β	0.766	(0.742, 0.789)	24.5
α	0.207	(0.189, 0.228)	20.0
ξ	-0.491	(-0.542, -0.442)	15.2
φ	0.962	(0.923, 1.003)	48.4
δ_1	0.007	(-0.018, 0.033)	27.3
δ_2	0.271	(0.252, 0.291)	24.0
$\log \sigma_1^2$	0.41	(-0.423, 1.227)	19.9
FSTE100, EGARCH-RG model			
ω	-0.173	(-0.202, -0.146)	16.1
β	0.965	(0.956, 0.975)	46.3
ρ_1	-0.136	(-0.154, -0.121)	21.6
ρ_2	0.027	(0.019, 0.036)	35.0
γ	0.183	(0.153, 0.213)	13.9
ξ	-0.439	(-0.490, -0.385)	38.9
φ	0.987	(0.942, 1.031)	26.6
δ_1	-0.007	(-0.034, 0.020)	33.7
δ_2	0.281	(0.262, 0.301)	42.4
$\log \sigma_1^2$	0.493	(-0.269, 1.305)	40.5

the sample autocorrelations of $\log \sigma_t^2$ for the GARCH-RG (EGARCH-RG) model at lags 1 and 2 are 0.942 (0.967) and 0.884 (0.935), respectively. For FTSE 100 data, they are 0.969 (0.961) and 0.938 (0.921). As expected from Figure 1, the estimated volatility increased in March 2020 due to the market turmoil induced by the COVID-19 pandemic. An increase in August 2015 and in 2016 resulting from the turbulence in the global stock market is also apparent. The estimated volatilities of Nikkei 225 are high in 2013, corresponding to the massive rise of the index.

The posterior probabilities of negative bias ξ for the GARCH-RG (EGARCH-RG) model are over 0.975, which implies that the measured range tends to have a downward bias owing to non-trading hours or market microstructure noise as highlighted by previous literature (e.g., Garman and Klass (1980)).

Note that the 95% credible intervals of φ for the GARCH-RG and EGARCH-RG models do not necessarily include the value of 1, while the posterior means are close to 1; thus, there is a possibility of overfitting. I adopt the models with $\varphi = 1$ as well as those without this restriction for φ in the model comparison presented in the next subsection.

News impact curve. To investigate the asymmetric effect of daily price shock on the volatility, I use the news impact function defined as $\nu(z) = \mathrm{E}(\log \sigma_{t+1}^2 | z_t = z) - \mathrm{E}(\log \sigma_{t+1}^2)$ (Engle and Ng (1993)), which is interpreted as an unexpected shock for the one-day-ahead forecast of the logarithm of the volatility. Note that $\nu(z) = \alpha\tau(z)$ for the GARCH-RG model and $\nu(z) = \tau(z)$ for the EGARCH-RG model.

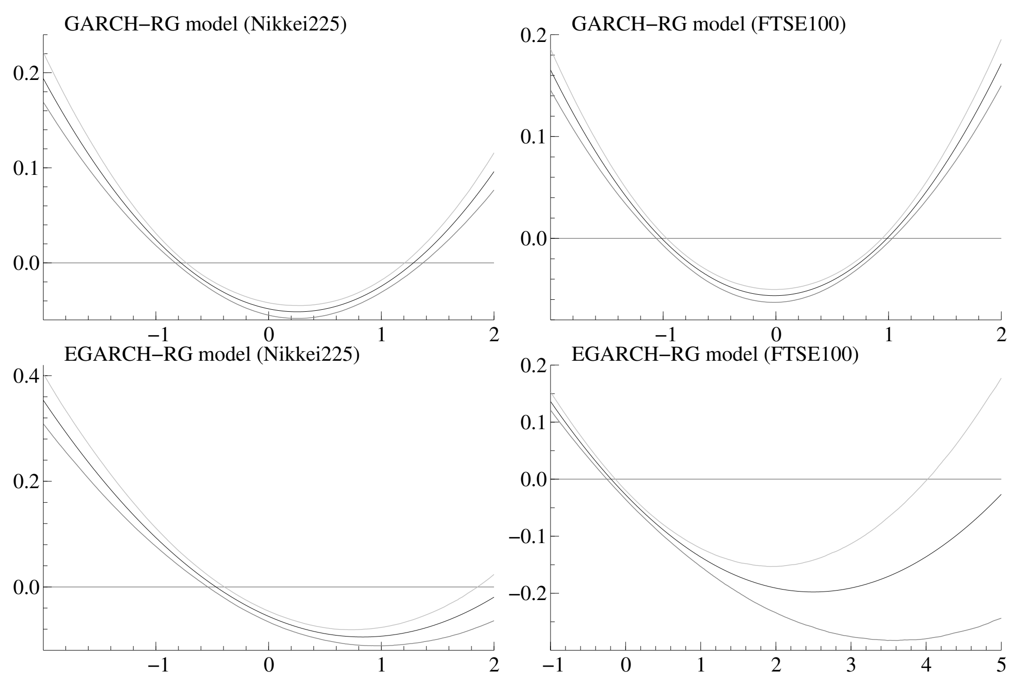

Figure 3: News impact curves for GARCH-RG and EGARH-RG models; the posterior means (black lines) and the upper and lower bounds of the 95% interval (gray lines).

Figure 3 illustrates the news impact curves associated with the news impact function for the GARCH-RG (EGARCH-RG) model, revealing that the estimated news impact functions are not symmetric about 0, and indicating the existence of asymmetry (except for the case of the GARCH-RG model and FTSE 100 data).

3.3 Model comparison based on forecasting

In this subsection, I focus on the predictive ability of the proposed GARCH-type models and conduct a model comparison examining forecasting performance.

I divide the full sample period into two parts: Period (I), from January 3, 2012, to December 31, 2018, and Period (II), from January 3, 2019, to December 31, 2020. For Nikkei 225 (FTSE 100), the former and the latter contain 1,717 (1,769) and 477 (505) business days, respectively. I regard Period (II) as a prediction period, adopt a rolling window approach with a fixed size window, and conduct a forecast of one-day-ahead volatility as stated in Section 2.2.

In this study, I use Parkinson's range-based estimator (RG) and the realized volatility (RV) as unbiased volatility proxies for evaluating the forecasts' performance. Because some authors contended that Parkinson's estimator and the realized measure for the volatility of an asset return tend to have a downward bias due to the overnight effect or market microstructure, I scale these proxies as follows:

$$Proxy_t^{\text{scale}} = c_{\text{scale}} Proxy_t, \quad c_{\text{scale}} = \frac{\sum_{s=1}^{n} (y_t - n^{-1} \sum_{s'=1}^{n} y_{s'})^2}{\sum_{s=1}^{n} Proxy_t}. \tag{20}$$

(See Hansen and Lunde (2006).)

The realized GARCH (1,1) model given by Equations (1)–(3) and the realized EGARCH model given by Equations (14)–(16) are used for the comparison. Additionally, I consider the two realized GARCH-type models with the realized measure replaced by the Parkinson's volatility estimator $R_t^2/(4\log 2)$ (Realized GARCH-RG and Realized EGARCH-RG). I use the sample mean of Parkinson's estimator for the last 25 days as a one-step-ahead forecast.

Table 2 presents the averages of the loss function in the MSE and QLIKE forms using the RV and RG as unbiased volatility proxies. The EGARCH-RG models outperform the other competing models except for the case of FTSE 100 and the RG volatility proxy. The restriction $\varphi = 1$ does not necessarily improve the forecasting performance of the GARCH-RG and EGARCH-RG models.

I also conduct a test of the predictive ability provided by Giacomini and White (2006). Although the ranking obtained from the loss functions in Equations (18) and (19) are consistent and reliable, it is unclear whether the difference of the loss between the two forecasts is statistically significant or not. I choose a constant and the lagged difference of the losses as the test function, and measure the significance of the loss difference between the two volatility forecasts using the Giacomini-White test. The MSE loss differences between the models are not necessarily significant at the 0.01 level. The QLIKE loss differences between the EGARCH-RG models and the other competing models are not necessarily significant for the case of the Nikkei 225 data, whereas they are significant for the case of FTSE 100 data and the RV proxy. The QLIKE loss differences between the realized GARCH models and the other models are not necessarily significant when FTSE 100 data and RG proxy are used. Overall, the proposed models perform equivalently (or better) than the other competing models.

Table 2: Average loss of forecasts.

	RV		RG	
Model	MSE	QLIKE	MSE	QLIKE
Nikkei225				
GARCH-RG	18.059	2.406	11.969	4.259
GARCH-RG($\varphi = 1$)	17.399	2.388	11.748	4.255
EGARCH-RG	16.286	**2.260**	10.634	**4.047**
EGARCH-RG($\varphi = 1$)	**15.914**	2.261	**10.522**	4.054
Realized GARCH	17.602	2.494	11.766	4.299
Realized EGARCH	21.983	3.272	14.528	5.061
Realized GARCH-RG	18.163	2.414	12.051	4.269
Realized EGARCH-RG	21.528	3.291	14.154	5.123
Parkinson's Est.	27.418	10.459	19.342	11.926
FTSE100				
GARCH-RG	15.830	2.623	14.705	4.068
GARCH-RG($\varphi = 1$)	15.985	2.605	14.863	4.063
EGARCH-RG	**15.013**	2.449	14.304	3.930
EGARCH-RG($\varphi = 1$)	15.120	**2.434**	14.395	3.925
Realized GARCH	15.383	2.617	**14.126**	**3.897**
Realized EGARCH	18.645	3.284	17.410	4.536
Realized GARCH-RG	15.501	2.563	14.406	4.021
Realized EGARCH-RG	18.994	3.484	17.965	4.790
Parkinson's Est.	20.621	4.136	19.674	5.657

Bold figures indicate the minimum of the column. Losses for QLIKE are multiplied by 10.

4 Conclusion

This paper extends the univariate GARCH-type models for a daily asset return to models incorporating the daily return and the associated price range. The simple and efficient Bayesian estimation and forecasting algorithm using the adaptive MCMC method is proposed to accurately obtain the posterior density of the model parameters and the predictive density of the one-step-ahead latent volatility. Moreover, I present an application of the GARCH-type models with return and price range to Nikkei 225 index and FTSE 100 index data. My analysis indicates that the volatility is subject to a persistent process, and estimated news impact curves support the existence of asymmetry or leverage effect. The model is demonstrated to function effectively regarding prediction performance in comparison to the competing models known as the realized GARCH-type models for the daily return and the associated realized measure of volatility, which is much more informative than the range.

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