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The evolution of inventory dynamics in a post-crisis economy

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Abstract

Inventories are of historical importance when describing business cycles and production volatility. While research in inventories is relatively mature, the growing attention to global value chains and commodity shortages in recent quarters warrants a return and critical reassessment of inventories as a business cycle feature. Herein, we estimate the persistence, volatility, and adjustment rates of inventories over different subsamples, paying particular attention to the period following the Financial Crisis. We find through the estimation of a flexible accelerator model that inventories continue to exhibit strong persistence while the persistence of sales has simultaneously diminished. Using standard volatility metrics within the literature, we illustrate that production volatility relative to sales volatility over long subsamples is highest from 2007 onward. Finally, we uncover evidence from several vector error correction models that the adjustment speed and cointegrating relationship between inventory and sales has deteriorated in the years following the Financial Crisis. This evidence suggests the need for a structural model that can identify the mechanisms underlying these critical changes in inventory dynamics during the post-crisis era.

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1 Introduction

Inventories are by definition unsold units of production. More formally known as inventory investment, the identity $\Delta H_t = Q_t - S_t$ links a firm's inventory growth (ΔH_t) to their production levels (Q_t) less their current period sales (S_t). Much of the attention that inventories have received historically was due to their relationship with the business cycle. Unsold goods held as inventories tend to move procyclically with the business cycle and move with close correspondence to GDP.[1]

It is important to highlight that critical research on macroeconomic inventory investment is at a fairly mature state. Therefore, inventories have not received significant attention in some time. However, given that the Global Financial Crisis and Covid-19 have disrupted global value chains and the production side of the economy altogether, there is an opportunity to reexamine inventory dynamics with new data. We pay particular attention to the persistence of inventories, the relative volatility of inventories, production, and sales as well as the rates of inventory adjustment from the short-run to the long-run.

The organization of this paper begins with an overview of key literature pieces and the setting for the data we utilize in our analysis. We estimate and interpret how inventory persistence has evolved over specific US recessions and longer subsample periods. For the same recessions and subsamples, we then proceed to tabulate all key volatility metrics related to inventories, sales, and production paying particular attention to the period of 2007:1–2022:1. Finally, we construct several vector autoregressive (VAR) and vector error correction (VEC) models over different subsamples and quantify the evolution of the inventory adjustment rate. We have three main results worth highlighting. Firstly, inventories are highly persistent through the present, but sales persistence has weakened considerably. Secondly, the relative volatility of production-to-sales is highest from 2007 onward. Finally, the cointegrating relationship and stationarity of inventory growth have deteriorated from 2007 onward. The decoupling of inventory and sales in recent decades presents a new empirical finding warranting more expansive investigation.

2 Literature & Data Setting

The importance of inventories and their role in describing the business cycle was originally discussed at length in Abramovitz (1950) who noted that changes in inventories correspond very closely to changes in output. Figure 1 summarizes the seminal finding of Abramovitz (1950) quite succinctly.

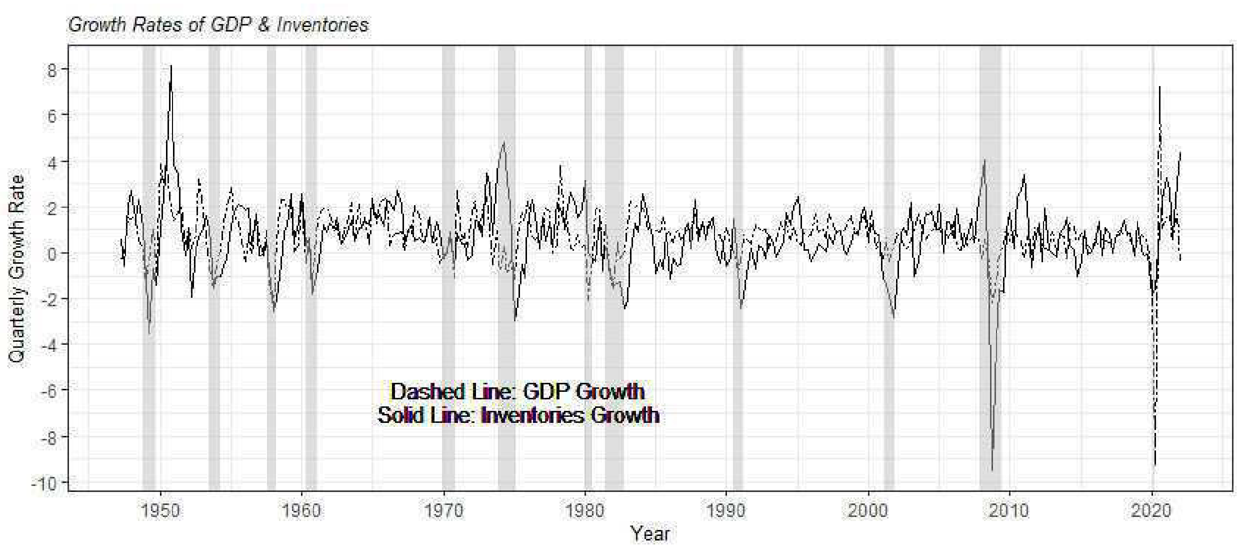

Figure 1: Comparison of Growth Rates

[1] Depending on the environment, inventories can either be seen as a means to smooth production when demand is uncertain or as highly volatile due to the accelerator principle as noted in Ramey & West (1999).

The explanation for the relationship described by Figure 1 was that during times of economic expansion, firms accelerate their inventory positions to meet heightened demand, and liquidate inventories during times of economic contractions to mitigate carrying costs of excess inventories. Ten years later, Holt (1960) introduced the first iteration of the linear-quadratic model (LQ), which has been a staple for empirical work within the inventories literature. While the LQ model is somewhat ad hoc, it conveniently captures two key features of inventories: the first being that at a micro-level, inventories act as a stabilizing force to smooth production and secondly, at a macro-level inventories can be destabilizing due to the accelerator principle. Beyond the benchmark LQ model, several slight variations exist such as M. Lovell (1961) and M. C. Lovell (1962), which introduce a variation of the LQ model known as the buffer-stock model motivated by the notion that some firms will voluntarily hold inventories to dampen the effect of backlogged sales.

The relationship between inventories and sales will be discussed at length throughout our analysis, however, the strong linear relationship between the two motivated a considerable amount of research within this literature. Consider Figure 2 which highlights this relationship.

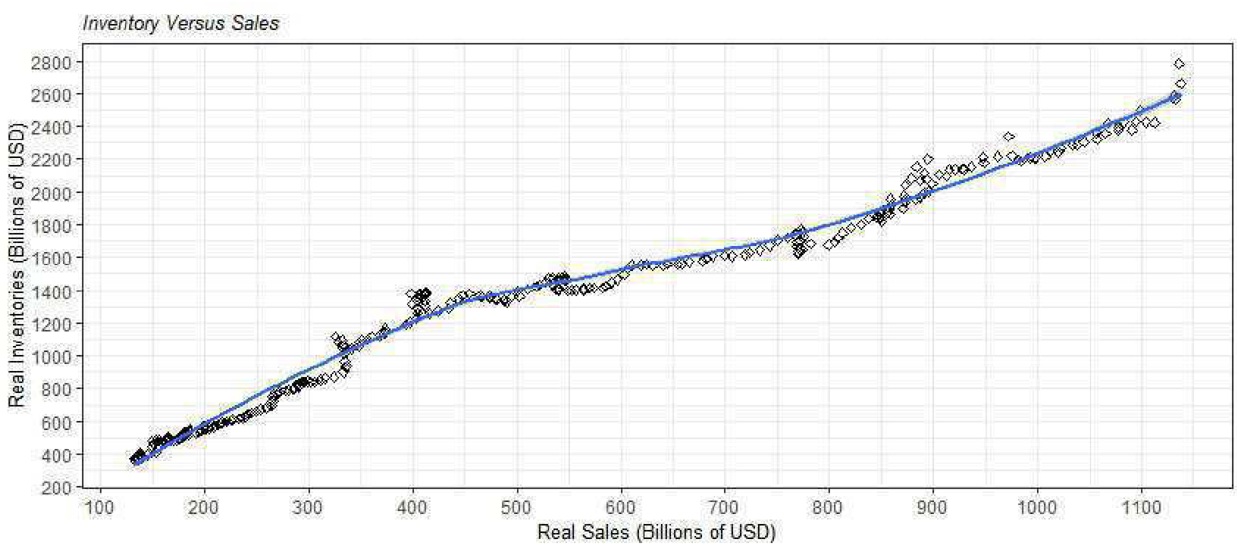

Figure 2: Inventories Versus Sales

Several other key papers have contributed to inventories once being a mainstay of many macroeconomic research agendas. Ramey & West (1999) in the Handbook of Macroeconomics chapter exclusively dedicated to inventories acts as an important primer on the topic alongside Blinder & Maccini (1991), which acts as the best existing survey of the literature to this point. Blinder & Maccini (1991) concisely summarizes the declining interest in inventories despite their macroeconomic relevance to the business cycle.[2]

Beyond the role of inventories in describing the business cycle, other developments in the literature such as Ramey (1989) have considered inventories as a factor of production much like labor or capital. A natural evolution of Ramey (1989) would be papers such as Humphreys et al. (2001) and Iacoviello et al. (2011), which disaggregate inventories down to input and output inventories allowing one to parse out differences in persistence, volatility and adjustment rates in inventories that act as factors of production from inventories that are sold as final goods.

In the same vein as input-output inventories, it is sometimes important to delineate between upstream versus downstream inventories. While most of inventories work has focused on upstream inventories (manufacturers, nonfarm inventories), key works like Arrow et al. (1951) and Khan & Thomas (2007) focus on trade inventories or downstream (retailer) inventories and optimal inventory policies under average demand, inventory carrying costs, and ordering costs. More recent contributions to the structural relationship between dynamics and the business cycle come from works like Alovokpinhou et al. (2022) who utilize a New

[2]Blinder & Maccini (1991) notes in particular that at the beginning of the 1990's, 87% of inventories are held in the manufacturing sector and around 8% are held as farm inventories, thus the application of many empirical approaches in the literature and their subsequent findings are usually within a manufacturing sector setting.

Keynesian model to show how inventories drive persistence across several key macroeconomic aggregates.

In terms of key empirical contributions, Blanchard (1983) is a premier piece testing the assumptions and stylization of inventories using data from the US automobile industry. His findings highlight both the importance of an industry's cost structure and demand process in assessing the degree to which inventories act as a stabilizing or destabilizing force. More recent empirical contributions include works such as Bils & Kahn (2000) who conclude that the sluggishness of inventory adjustments over the business cycle is largely attributable to countercyclical markups. Complementary pieces to Bils & Kahn (2000) are ones such as Crouzet & Oh (2016) and Jones & Tuzel (2013). Additionally, there is a renewed interest in inventories through the lens of total factor productivity (TFP) and "news shocks." Pieces like Görtz et al. (2022) and Görtz & Gunn (2018) in particular examine the interesting and positive comovement between TFP and inventories in response to news about speculative TFP growth.

Finally, across the literature, there are several important inventory puzzles. Maccini et al. (2015) details these quite well including the Wen (2011) puzzle, the variance ratio puzzle, the input cost puzzle and the slow adjustment puzzle. The Wen (2011) puzzle impresses that differences in production volatility relative to sales volatility can be explained by the horizons they are measured across wherein short-horizon production is less volatile than long-horizon production. The variance ratio puzzle articulates the oddity that in an infinite-horizon setting, the variance of production to sales tends to lie precisely at one in spite of production smoothing efforts that firms can pursue. The slow adjustment puzzle shows that, in optimality, firms are exceedingly slow to adjust their current period inventories to their optimum level. Finally, the input cost puzzle illustrates that firms reduce their inventory holdings in light of higher input costs to production.

For our analysis, we focus on four primary data series described in Table I. All data are in quarterly buckets starting in 1947:1 and ending in 2022:1. All units are converted to real terms using the GDP deflator. The inventory aggregate we use is known as *private nonfarm inventories*. This inventory aggregate is common for analysis as its length extends as far back as 1947. The sales aggregate we use is known as *final sales of domestic business*, which is commonly utilized within the literature for its length, which also extends as far back as 1947.

Table I: Data & Descriptive Statistics

Series	Notation	Units	Mean	Std. Dev.	Source
Real Inventories	H_t	Billions of USD	1300.86	624.372	BEA
Real Sales	S_t	Billions of USD	521.554	301.715	BEA
Real Production	Q_t	Billions of USD	530.89	306	$Q_t = S_t + \Delta H_t$
Real GDP	Y_t	Billions of USD	8996.68	5374.95	BEA
Inventory Growth	$\Delta log(H_t)$	Percent	0.67%	1.56%	
Sales Growth	$\Delta log(S_t)$	Percent	0.72%	1.33%	
Production Growth	$\Delta log(Q_t)$	Percent	0.74%	4.16%	
GDP Growth	$\Delta log(Y_t)$	Percent	0.76%	1.16%	

3 Persistence

To illustrate the concept of persistence, we turn to a variation of the LQ model known as the flexible accelerator (FA) model. First penned by M. Lovell (1961), the FA model considers a representative firm who seeks to optimize their inventory stock, H_t, to balance the costs of adjusting inventory levels with the costs of deviations from their optimal target, H_t^*. The explicit inclusion of an inventory target in the objective function is what makes this model slightly different from the traditional LQ models discussed at length in Ramey & West (1999) and West (1993). However, under certain assumptions, the model's solution collapses to a solution identical to the LQ model. The representative firm's objective function is described by equation (3.1).

$$\arg\min_{H_t} \left\{ \frac{1}{2}(H_t - H_t^*)^2 + \frac{1}{2}\mu(H_t - H_{t-1})^2 + \varepsilon_t H_t \right\} \tag{3.1}$$

$\mu > 0$ is the weight of the second cost term relative to the first and ε_t is a disturbance term.[3] The first-order condition (FOC) yields equation (3.2).

$$H_t - H_{t-1} = [1/(1+\mu)](H_t^* - H_{t-1}) - [1/(1+\mu)]\varepsilon_t \tag{3.2}$$

The coefficient $\frac{1}{1+\mu}$ represents the gap between the target and starting inventory levels within a given period. To operationalize this model, H_t^* must be defined. Typically, it is common to assume in an optimal setting with no cost shifters that inventories are some θ proportion of sales or $H_t^* = \theta S_t$ where S_t are sales in the current period. Additionally, S_t must be given an explicit law of motion, the easiest being a simple autoregressive representation such as $S_t = S_{t-1} + e_t$. Plugging these back into the model and simplifying once more, we have:

$$H_t - H_{t-1} = [1/(1+\mu)](\theta S_{t-1} + \theta e_t) - [1/(1+\mu)]H_{t-1} - [1/(1+\mu)]\varepsilon_t$$

Further algebraic manipulation leads to:

$$H_t = [1/(1+\mu)]\theta S_{t-1} + [1/(1+\mu)]\theta e_t - [1/(1+\mu)]H_{t-1} + H_{t-1} - [1/(1+\mu)]\varepsilon_t$$

After collecting similar terms, we arrive at equation (3.3).

$$H_t = [\theta/(1+\mu)]S_{t-1} + [\mu/(1+\mu)]H_{t-1} + [1/(1+\mu)](\theta e_t - \varepsilon_t) \tag{3.3}$$

With these simplifications, we can express (3.3) in a reduced form described by equation (3.4).

$$H_t = \pi_S S_{t-1} + \pi_H H_{t-1} + u_t \tag{3.4}$$

In essence, the reduced form solution for H_t is a simple autoregressive distributed lag model of order one or an ARDL(1,1). Note that we can map our reduced form parameters to the following structural parameters: $\pi_S = \frac{\theta}{1+\mu}$, $\pi_H = \frac{\mu}{1+\mu}$ and with $u_t = \frac{1}{1+\mu}(\theta e_t - \varepsilon_t)$ as the disturbance term. The solution to this model can be thought of as the "traditional" rule to follow for choosing current period inventories; however, if one were to consider a just-in-time (JIT) inventory system wherein $H_t^* = 0$, then equation (3.2) becomes (3.5).

$$H_t - H_{t-1} = [1/(1+\mu)](-H_{t-1}) - [1/(1+\mu)]\varepsilon_t \tag{3.5}$$

This further simplifies to:

$$H_t = [\mu/(1+\mu)]H_{t-1} - [1/(1+\mu)]\varepsilon_t \tag{3.6}$$

In its reduced form, equation (3.6) can be expressed as:

$$H_t = \pi_H H_{t-1} + v_t \tag{3.7}$$

In this instance, π_H still has the same structural mapping as it did in equation (3.4), but as a result of setting $H_t^* = 0$, the model solution collapses to an AR(1) rather than an ARDL(1,1). While our structural model described by equation (3.4) is an ARDL(1,1), in reality, we must be wary of potential autocorrelation in our error term. Alongside this concern, it is likely that the lagged dynamics of both inventories and sales are more complex than singular lags. To remedy this in our estimation procedure, we use the Akaike information criterion (AIC) to estimate a more appropriate ARDL(p, q) model and ensure that our error terms are heteroscedastic and autocorrelation consistent (HAC). Table II presents estimation results in real units for an ARDL(3,2) over select subsamples.[4]

[3] For practical purposes, this is simply a mean zero error term, but in the context of the model, it serves as a scaler or shifter for inventories and is stochastic in nature.

[4] In our appendix, we include estimations of both equations (3.4) and (3.7) in log levels with a constant to match the exact structural model's structure. As noted in Ramey & West (1999) and Blinder & Maccini (1991), it is common to estimate flexible accelerator models in real units or in natural logs.

Table II: Estimated Flexible Accelerator Models: $ADRL(p, q)$

	Dependent Variable: H_t					
Constant	5.12266	4.70152.	45.43967***	-15.1529	134.48767**	123.91908***
	(3.40799)	(2.72837)	(13.00776)	(81.0120)	(37.44923)	(33.92073)
H_{t-1}	1.63583***	1.33214***	1.47075***	1.6050***	1.00160***	1.38291***
	(0.08596)	(0.09882)	(0.08221)	(0.1395)	(0.16789)	(0.10247)
H_{t-2}	-0.81642***	-0.32150*	-0.48313**	-0.9436**	-0.09251	-0.43438**
	(0.24496)	(0.15269)	(0.14936)	(0.3188)	(0.21855)	(0.16148)
H_{t-3}	0.16298	-0.12245	-0.05062	0.2090	-0.10909	-0.11489
	(0.16739)	(0.07806)	(0.08490)	(0.1952)	(0.10469)	(0.09725)
S_{t-1}	0.13202	0.71686*	0.96081**	0.1368	0.36140	1.00433***
	(0.18302)	(0.31400)	(0.29867)	(0.3182)	(0.66440)	(0.28349)
S_{t-2}	-0.09032	-0.41659	-0.88058**	0.1796	-0.03967	-0.79860**
	(0.17682)	(0.32865)	(0.31076)	(0.1228)	(0.65735)	(0.28120)
N	298	99	136	61	40	85
Adj. R^2	0.99	0.99	0.97	0.97	0.98	0.99
Range	1947:4-2022:1	1947:1-1972:2	1973:1-2006:4	2007:1-2022:1	2010:1-2019:4	1981:1-2006:1

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$. We first present our full sample (1947:4-2022:1), the beginning of our sample up until the productivity slowdown (1947:1-1972:2), the productivity slowdown up until the Financial Crisis (1973:1-2006:4), and the Financial Crisis through the end of the sample period (2007:1-2022:1). We present results for a window of time that is the post-Financial Crisis, but pre-Covid 19 pandemic (2010:1-2019:4) in consideration of bias that might arise from both Covid-19 and the Financial Crisis. We also present results for the Great Moderation (1981:1-2006:1) where inventories are of particular importance relative to other subsamples as highlighted in Camacho et al. (2011).

We observe that the autoregressive coefficients of H_{t-1} over all subsamples are highly persistent. We note that the coefficients associated with S_{t-1} carry significant statistical and economic relevance during the productivity slowdown era and the Great Moderation; however, we note that after the Financial Crisis, sales relative economic importance is not statistically different from zero. This is a stark contrast to the traditional importance of sales in the flexible accelerator model.

4 Relative Volatility

Ramey & West (1999) highlights two universally accepted stylized facts about inventories: their procyclical nature and extreme persistence. Given that we have discussed persistence at length, we follow the literature, which points out that relative volatility of inventory, sales and production can be identified from their time series characteristics. In general, inventories tend to trend positively during economic expansions and fall or stagnate during economic contractions. The idea is that as an expansion is occurring, aggregate demand is rising, thus firms are accelerating their inventory positions to meet demand; however, during contractions, firm seek to liquidate their inventory positions to avoid the carrying costs of inventory during periods of low demand. Evidence of recent developments in inventory dynamics can be gleaned from the inventory-to-sales ratio (ISR) described in Figure 3.

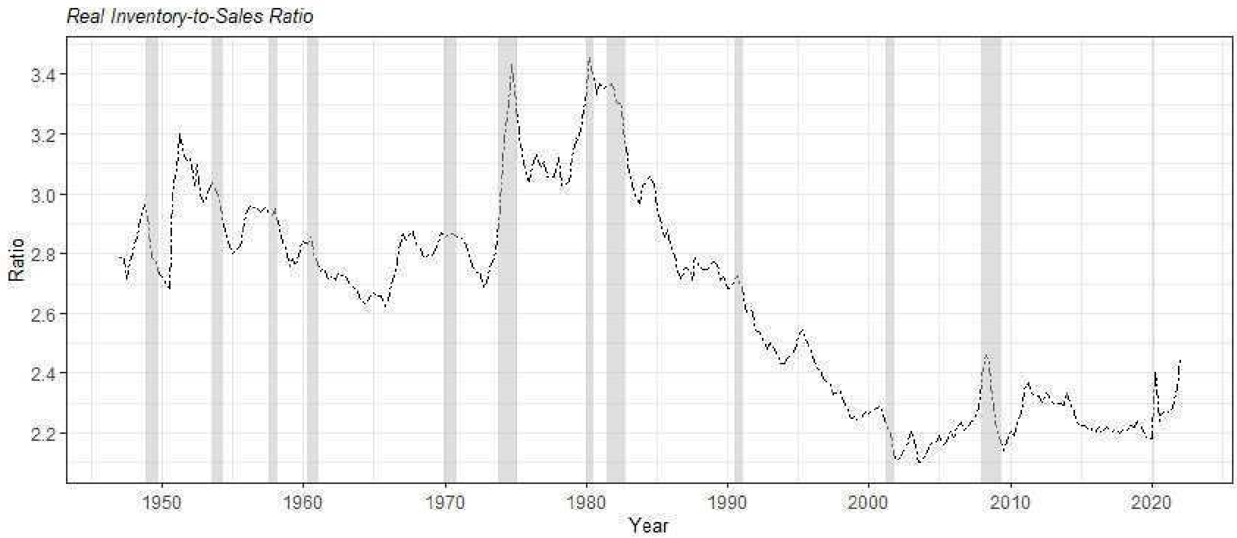

Figure 3: Inventory-to-Sales Ratio

The ISR is typically countercyclical peaking during recessions and falling during expansions.[5] While the level of the ISR has fallen considerably over time, it is still well above unity. During recent recessions like the Financial Crisis and Covid-19, we have seen the ISR reach its highest levels since the early 1990's. This raises several puzzles. The first is the speed of inventory adjustment over recent recessions versus earlier sample periods. The second is the relative volatility of sales during recent economic crises compared to earlier eras. We will address the first puzzle in-depth later. To address the second puzzle, however, there are several standard metrics in the inventories literature used to capture relative inventory volatility. Table III expresses these metrics over various subsamples.

[5]The intuition is straightforward: given the persistence of inventories, upon an economic contraction, demand tends to fall sharply while inventories adjust sluggishly to economic conditions.

Table III: Relative Importance of Inventories (H_t) & Production (Q_t) Volatility

Period	$(\Delta H_t/\Delta Y_t) \times 100$	$cor(S_t, \Delta H_t)$	$var(Q_t)/var(S_t)$	$var(\Delta Q_t)/var(\Delta S_t)$	$mean(H_t/S_t)$
1948:4–1949:4	82.7	-0.76	44.83	70.66	2.84
1954:3–1954:2	35.16	0.65	45.70	36.27	2.99
1957:3–1958:2	31.67	0.90	8.1	7.82	2.93
1960:2–1961:1	298.86	-0.33	413.59	108.65	2.81
1969:4–1970:4	4.66	0.29	39.12	34.18	2.86
1973:4–1975:1	-64.91	0.59	77.9	79.92	3.19
1980:1–1980:3	2.55	0.78	20.81	13.11	3.39
1981:3–1982:4	56.05	0.66	11.91	7.37	3.31
1990:3–1991:1	35.49	0.99	41.04	121.33	2.71
2001:1–2001:4	-158.81	0.13	24.3	6.57	2.20
2007:4–2009:2	32.51	0.71	27.06	120.5	2.33
2020:1–2020:2	2.27	-0.99	0.89	0.22	2.29
1948:1–1972:4	14.09	0.17	1.06	9.85	2.84
1973:1–2006:4	10.76	0.02	1.02	12.15	2.69
2007:1–2022:1	19.17	0.26	1.47	5.49	2.26
1948:1–2022:1	13.57	0.16	1.03	6.24	2.65
1985:1–2006:1	7.18	0.22	1.08	11.44	2.45
2010:1–2019:4	14.71	-0.29	0.92	30.72	2.25
1948:1–1958:2	15.58	-0.11	1.19	10.61	2.93
1958:3–1973:4	16.52	0.35	1.15	14.54	2.77
1974:1–1985:2	15.87	-0.07	1.21	12.69	3.16
1985:3–1996:3	6.62	0.18	1.21	12.39	2.63
1996:4–2022:1	14.36	0.20	1.18	5.74	2.25

Note: This table is divided into three segments. The first are US recessions as identified by the NBER. The second are select subsamples including the pre-productivity slowdown, the post-productivity slowdown up until the Financial Crisis, and the Financial Crisis through the end of our sample. Finally, we utilize a Chow Test to identify multiple structural breaks in the ISR. Our final segment looks across subsamples specific to the break dates generated from our structural break tests.

$(\Delta H_t/\Delta Y_t) \times 100$ describes the endpoint-to-endpoint change in real inventories (H_t) as a fraction of the endpoint-to-endpoint change in real GDP (Y_t) over various subsamples. $cor(S_t, \Delta H_t)$ describes the relationship between real sales (S_t) and real net inventories (ΔH_t). $var(Q_t)/var(S_t)$ describes the volatility of real production (Q_t) relative to the volatility of real sales where Q_t is linked in the data by a standard convention in the literature: $Q_t = S_t + \Delta H_t$. This variance ratio in particular is a proxy for the supply chain phenomena known as the Bullwhip Effect.[6] $var(\Delta Q_t)/var(\Delta S_t)$ is the variance ratio of net real production (ΔQ_t) to net real inventories and $mean(H_t/S_t)$ is the average ISR over each respective subsample.

The relative change in inventories versus GDP varies highly with ΔH_t being almost 300% higher than ΔY_t during the 1960-1961 recession to almost -160% during the dot-com boom. Clearly, the relative magnitude of these measurements are subject to the idiosyncracies of each recessionary period they are measured in; however, if one were to evaluate this same metric over longer time periods, one would see that the era encompassing the Financial Crisis through the present expresses the highest relative proportion of inventory change to GDP change at around 20% compared to roughly 14% for the full sample. If one were to look at the correlation over long periods, one would see that the correlation between inventories and sales is particularly strong between 2007:1–2022:1 relative to the full sample. Whether the strengthening of this linkage is simply spurious or the result of changes in firm behavior is unclear and warrants a deeper empirical treatment.

We note that over recessionary periods, the Bullwhip Effect is highly amplified, however, over long samples, it is relatively close to unity. It would seem the relative volatility of production has increased considerably in recent decades relative to the full sample. A direct consequence of a high Bullwhip Effect is excess inventory investment. Finally, the volatility of the change in production relative to the change in sales, $var(\Delta Q_t)/var(\Delta S_t)$, tends to weakly correspond to the Bullwhip Effect's magnitude over the same

[6] The Bullwhip Effect ordinarily is the ratio of new orders to sales, however, when data on orders is unavailable, it is common to utilize production as noted in Blinder & Maccini (1991). The Bullwhip Effect proxy has a simple interpretation, if $var(Q_t)/var(S_t) < 1$, firms are production smoothing, however, if $var(Q_t)/var(S_t) > 1$, firms are experiencing demand amplification (Bullwhip Effect).

periods. We note, however, that this metric is relatively muted in the 2007:1–2022:1 subsample relative to the full sample and the productivity slowdown subsample. These findings in part echo the inventory puzzles posed in Maccini et al. (2015), particularly the variance ratio puzzle and Wen (2011) puzzle.

5 Adjustment Speeds

The speed of adjustment between inventories and sales was a point of constant analysis in the inventories literature with findings often characterizing inventories as being relatively sluggish in their adjustment to sales. A quick illustration of this can be shown from a simple VAR model. Consider the unrestricted bivariate VAR of $Z_t = \beta_0 + \sum_{i=1}^{8} \beta_i Z_{t-i} + \varepsilon_t$ where $Z_t = [H_t, S_t]^T$ estimated using data from 1947:1–2021:1.[7] The vector of data is in their levels and utilizes eight lags in accordance with the AIC criteria. Post-estimation, we can generate the following impulse-response functions (IRFs) described by Figure 4 and Figure 5.

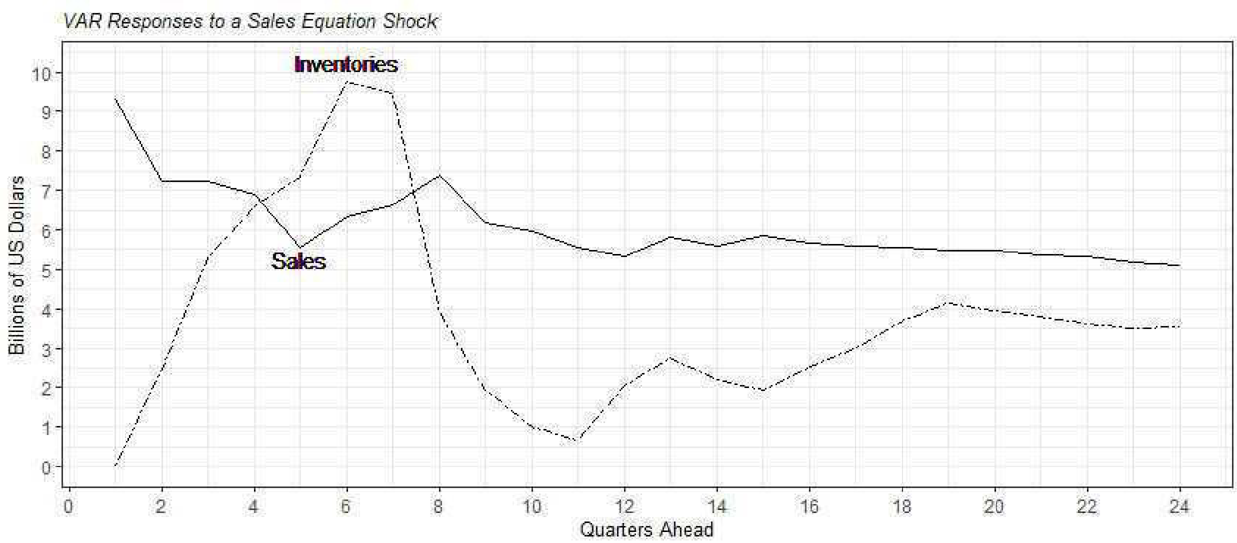

Figure 4: Responses from VAR to a Sales Shock

We observe from Figure 4 that a sales shock has an immediate "jump" impact on sales. We note that it takes time for firms to accumulate necessary inventories for meeting demand, thus, the adjustment of inventories to meet sales is somewhat sluggish. As demand is met, inventories are subsequently depleted settling at a level close to its pre-shock equilibrium.

[7]We refrain from imposing formal restrictions (such as a Choleski decomposition) as our VAR results serve to illustrate a simplistic view of the adjustment rates of inventories and sales to motivate a more conventional vector error correction framework.

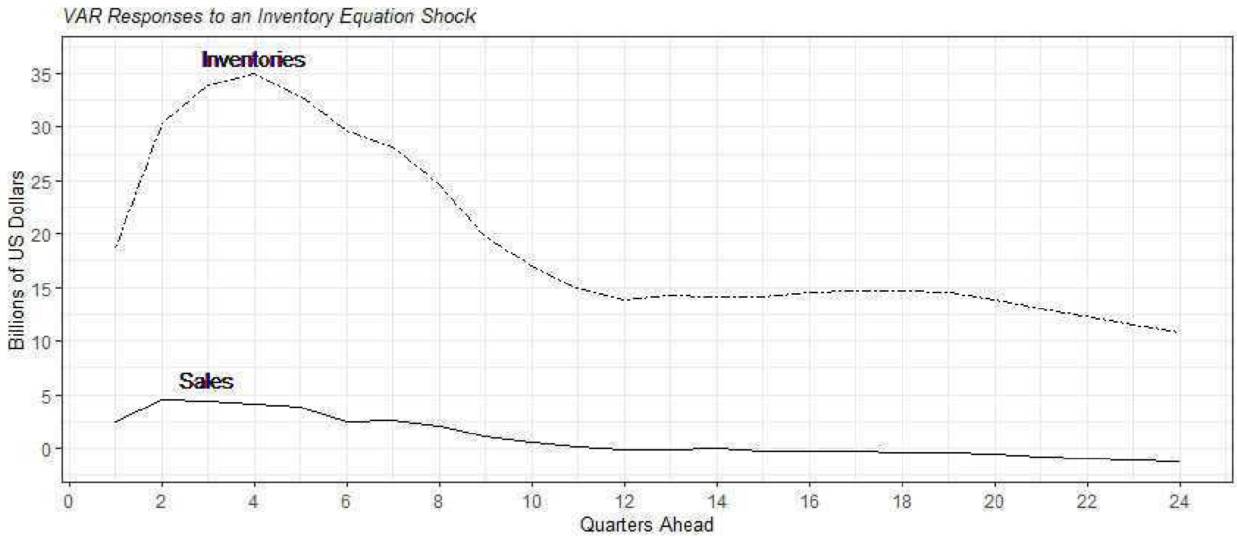

Figure 5: Responses from VAR to an Inventory Shock

From Figure 5, an inventory equation shock dramatically increases the stock of inventories held by firms with a high degree of persistence as well. Sales on the other hand exhibits a weakly positive and transitory response to an inventory equation shock. In reality, demand generally drives production decisions (thus creating inventories), however, the presence of unsold production (inventories) does not necessarily create demand. However, in theory, firms could endogenously price their inventories in such a way to ensure they are always liquidated, but in the aggregate, the evidence suggests this is not the case. This empirical result is somewhat of a contrast to studies like Bils & Kahn (2000), Görtz & Gunn (2018) and Görtz et al. (2022) who assume inventories lead or create sales. One possible explanation for this dichotomy is that in the manufacturing sector, new orders are generated in advance of production. Orders signal the start of the production process whereafter some lead time passes, inventories are created, and then shipped (sold). At the retailer level, however, a prospective customer need only pick an item off the shelf (an inventory) and purchase it. Ultimately, the timing of inventory creation relative to demand signal generation is quite different between retailers and manufacturers. Given that the majority of nonfarm inventories are held in manufacturing, the result described by Figure 5 is not completely surprising.

While this illustrative evidence shows some characteristics of inventory adjustment speeds, it ignores the possibility that inventories and sales are both cointegrated. Papers like Hamilton (2002) and West & Wilcox (1994) discuss the nature of cointegration in the context of the LQ model and its implications extensively. We first opt to evaluate importance of the possible cointegrated relationship in a more straightforward way by examining the inventory-to-sales relationship ($H_t - \theta S_t$) and its response to inventory and sales shocks. To do this, we follow the form of Ramey & West (1999) and first estimate a dynamic OLS model (DOLS) using the methodology described in Stock & Watson (1993). Equation (5.1) describes the structure of this econometric approach.

$$log(H_t) = \beta_0 + \beta_1 log(S_t) + \sum_{i=1}^{8} \gamma_i \Delta log(S_{t+i}) + \sum_{i=1}^{8} \tau_i \Delta log(S_{t-i}) + \varepsilon_t \tag{5.1}$$

Equation (5.1) is estimated with a constant along with eight leads and lags. The Bartlett kernel is used for calculating the long-run variance of the cointegrating relationship. We pay special attention to the β_1 coefficient, which captures the long-run inventory-to-sales relationship. With this in mind, we construct a second bivariate VAR model identical to the one generated earlier, but in log levels rather than real quantities. We construct IRF plots that adjust inventory responses to shocks by netting out its linearly cointegrated relationship with sales.[8] This adjustment produces Figure 6 and Figure 7.

[8]Practically speaking, we adjust a response from H_t such that it is $H_t - \beta_1 S_t$ where S_t is the sales response to the same shock scaled by the cointegrating relationship, $\beta_1 = 1.12$, estimated from equation (5.1).

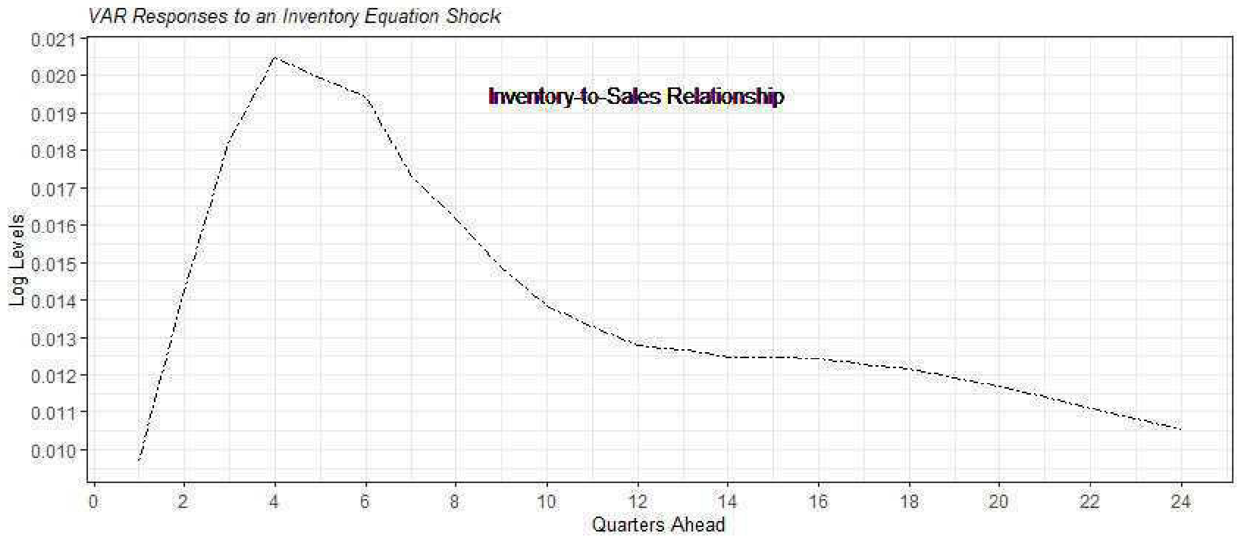

Figure 6: Inventory-to-Sales Relationship: Response to Inventory Shock

Figure 6 provides visual evidence that a surge in inventory levels generally tends to inflate the inventory-to-sales relationship suggesting sales do not respond quickly to such a shock. Additionally, the reversion to the equilibrium is exceedingly slow.

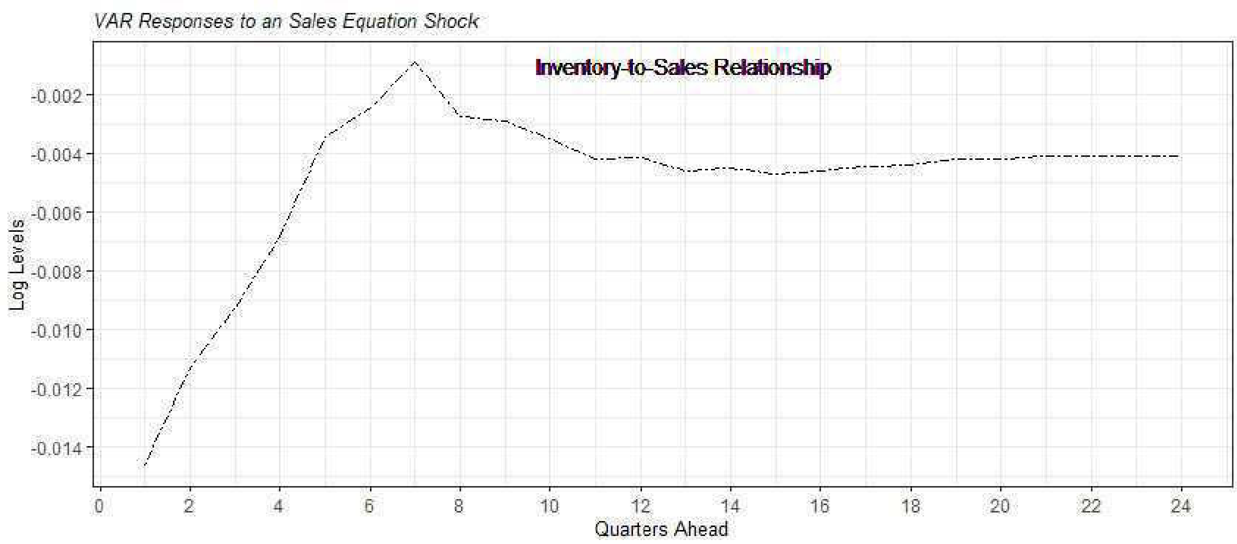

Figure 7: Inventory-to-Sales Relationship: Response to Sales Shock

Figure 7 illustrates the pattern that a sales shock has a disproportionate effect on inventories and sales. The immediate fall in the inventory-to-sales relationship suggests that inventories rise in response to a sales shock, but much more sluggishly than sales. Note in both the cases of Figure 6 and Figure 7 that the inventory-to-sales relationship does not revert completely back to the zero line suggesting that there is still strong persistence in the inventory-to-sales relationship.

A final, more contemporary approach to formally identify and quantify each respective adjustment speed is via a vector error correction model (VECM). We utilize the Johansen (1995) methodology to identify our cointegrating relationships and error correction model (ECM) simultaneously. The basic framework is described by equation (5.2).

$$\Delta Z_t = A_0 + \Pi Z_{t-1} + \sum_{i=1}^{8} \Gamma_i \Delta Z_{t-1} + \varepsilon_t \tag{5.2}$$

From the Π matrix, the reduced form rank condition can be obtained such that $\Pi = \alpha\beta^T$. This allows us to redefine our reduced form VECM as $\Delta Z_t = A_0 + \alpha\beta^T Z_{t-1} + \sum_{i=1}^{8} \Gamma_i \Delta Z_{t-1} + \varepsilon_t$. If we have one cointegrating vector ($r = 1$) in a bivariate model such as equation (5.2), then we can decompose $\alpha\beta^T$ further:

$$\Pi Z_t = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} [1 - \beta] \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} \begin{bmatrix} 1 \\ -\beta \end{bmatrix}^T \begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \alpha\beta^T = \begin{bmatrix} \alpha_{11} \\ \alpha_{21} \end{bmatrix} [z_{1t} - \beta z_{2t}] = \alpha ECT_t$$

ECT_t is our error correction term and identifies the rate of adjustment in the short-run of each endogenous variable to a common shared stochastic long-run trend. To proceed, we must test the stationarity of our variables over each period as well. The ADF test statistics and corresponding p-values are reported in Table IV.

Table IV: Augmented Dickey Fuller Tests

Period/Variable	H_t	ΔH_t	S_t	ΔS_t
Full Sample	-1.608 (0.742)	-6.78 (0.01)	-1.524 (0.777)	-5.865 (0.01)
1948:1–1972:4	-1.44 (0.809)	-4.866 (0.01)	-1.071 (0.923)	-4.003 (0.0117)
1973:1–2006:4	-2.842 (0.226)	-4.192 (0.01)	-1.606 (0.741)	-3.936 (0.0144)
2007:1–2022:1	-2.054 (0.553)	-2.95 (0.189)	-1.831 (0.644)	-4.512 (0.01)
1985:1–2006:1	-2.261 (0.469)	-3.787 (0.0235)	-2.16 (0.51)	-2.586 (0.336)
2010:1–2019:4	-3.287 (0.088)	-4.045 (0.0183)	-3.971 (0.0209)	-3.857 (0.0261)
1948:1–1958:2	-2.463 (0.39)	-4.213 (0.0104)	-1.744 (0.675)	-4.176 (0.0119)
1958:2–1973:4	-3.129 (0.118)	-2.08 (0.542)	-3.001 (0.17)	-3.283 (0.083)
1974:1–1985:2	-1.507 (0.77)	-2.728 (0.285)	-1.986 (0.58)	-2.567 (0.349)
1985:3–1996:3	-2.436 (0.401)	-2.527 (0.365)	-1.723 (0.684)	-2.03 (0.562)
1996:4–2022:1	-3.235 (0.0862)	-4.577 (0.01)	-2.32 (0.444)	-3.062 (0.137)

Note: The ADF test statistic is reported with the corresponding p-value in parentheses. The null hypothesis of the ADF tests is that the data is non-stationary. To rectify this, we first-difference our data and retest for each subsample. If the data is non-stationary in their levels, but stationary in first differences, the data is said to be integrated at order one or I(1).

Our results are mixed. Over the full sample, we find that both H_t and S_t are I(1). The same is true for the subsample stemming from the productivity slowdown up until the Financial Crisis and from the Financial Crisis through the end of the sample. The subsamples based on our estimated structural breaks and the subsamples associated with the Great Moderation and our crisis-free subsample (2010:1–2019:4) are all mixed in some way or another. Despite this, we still proceed to test for cointegration over these same subsamples. We formally test for cointegration using the Johansen (1995) method. Results for these cointegration are listed in Table V.

Table V: Johansen Tests of Cointegration

	Vector: $Z_t = [H_t, S_t]^T$			
	T. Stat	10% CV	5% CV	1% CV
Full Sample	34.56	17.85	19.96	24.60
1948:1–1972:4	19.92	17.85	19.96	24.60
1973:1–2006:4	29.66	17.85	19.96	24.60
2007:1–2022:1	13.66	17.85	19.96	24.60
1985:1–2006:1	17.56	17.85	19.96	24.60
2010:1–2019:4	17.10	17.85	19.96	24.60
1948:1–1958:2	18.95	17.85	19.96	24.60
1958:2–1973:4	15.13	17.85	19.96	24.60
1974:1–1985:2	18.43	17.85	19.96	24.60
1985:3–1996:3	33.23	17.85	19.96	24.60
1996:4–2022:1	9.26	17.85	19.96	24.60

Note: Given that we are testing bivariate vectors, a rejection of the null hypothesis of no cointegrating equations ($r = 0$) allows us to favor an alternative hypothesis that there are $r > 0$ cointegrating vectors.

We observe that over the full sample and the Great Moderation, in particular, we strongly reject the null hypothesis that there is no cointegration. Across other subsamples and structural breaks, we observe a fair degree of heterogeneity in the strength of cointegration between inventories and sales.

With these diagnostics in mind, we still proceed to estimate distinct VEC models with eight lags of the endogenous variables. Each model is estimated over each subsample period described in Tables IV and VI. In Table VI, we report coefficient estimates for each individual equation of ΔZ_t as well as the β long-run coefficient. It should be noted that the mixed results from our ADF and cointegration tests will likely present some results that will be difficult to interpret.

Table VI: VECM Results

Panel A	Full Sample		1948:1–1972:4		1973:1–2006:4		2007:1–2022:1	
	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$
Long-Run β	0.598904		0.915813		0.494673		0.949386	
A_0	0.0218	0.0397*	0.1522*	0.1368*	0.1915**	0.0680	0.3050.	-0.1932
	(0.0146)	(0.0161)	(0.0576)	(0.0652)	(0.0618)	(0.0612)	(0.1605)	(0.2036)
ECT_{t-1}	-0.0059	-0.0092*	-0.1005**	-0.0829.	-0.0464**	-0.0146	-0.2584.	0.1720
	(0.0042)	(0.0046)	(0.0380)	(0.0429)	(0.0148)	(0.0147)	(0.1385)	(0.1756)
$\Delta log(H_{t-1})$	0.5339***	0.1578*	0.4870***	0.2649*	0.5027***	-0.0972	0.7470***	0.3550
	(0.0607)	(0.0670)	(0.1107)	(0.1252)	(0.0876)	(0.0868)	(0.1759)	(0.2230)
$\Delta log(S_{t-1})$	0.2052***	-0.0319	0.2236*	0.0214	0.2308*	0.0468	-0.0409	-0.2005
	(0.0550)	(0.0607)	(0.1114)	(0.1260)	(0.0921)	(0.0913)	(0.1574)	(0.1996)
Panel B	1985:1–2006:1		2010:1–2019:4		1948:1–1958:2		1958:2–1973:4	
	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$
Long-Run β	0.492815		0.753061		0.814435		1.06772	
A_0	0.5536.	0.2865	0.3276	-0.4028	0.7097**	0.4056	0.1367.	0.1669.
	(0.3100)	(0.2552)	(0.4386)	(0.2475)	(0.2231)	(0.3466)	(0.0803)	(0.0976)
ECT_{t-1}	-0.1335.	-0.0679	-0.1255	0.1714	-0.3517**	-0.1990	-0.2042.	-0.2358
	(0.0744)	(0.0612)	(0.1759)	(0.0993)	(0.1125)	(0.1747)	(0.1175)	(0.1427)
$\Delta log(H_{t-1})$	0.5192***	-0.0417	-0.0988	0.0076	0.4140.	0.3559	0.1667	-0.1833
	(0.1320)	(0.1086)	(0.2615)	(0.1475)	(0.1977)	(0.3071)	(0.1786)	(0.2169)
$\Delta log(S_{t-1})$	0.3965*	-0.0884	-0.7521.	-0.0330	0.1251	-0.0949	-0.1134	0.0044
	(0.1711)	(0.1409)	(0.3874)	(0.2186)	(0.1890)	(0.2935)	(0.2101)	(0.2552)
Panel C	1974:1–1985:2		1985:3–1996:3		1996:4–2022:1			
	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$	$\Delta log(H_t)$	$\Delta log(S_t)$		
Long-Run β	0.950798		0.313754		1.09388			
A_0	0.3644	1.2727***	1.5129.	1.6679*	0.0214*	-0.0029		
	(0.2356)	(0.2928)	(0.8581)	(0.7088)	(0.0105)	(0.0123)		
ECT_{t-1}	-0.2466	-0.8485***	-0.2860.	-0.3128*	-0.1112.	0.0417		
	(0.1578)	(0.1962)	(0.1616)	(0.1334)	(0.0622)	(0.0726)		
$\Delta log(H_{t-1})$	0.6964*	0.7683*	0.6121*	0.0019	0.5689***	0.3178*		
	(0.2583)	(0.3211)	(0.2258)	(0.1865)	(0.1278)	(0.1491)		
$\Delta log(S_{t-1})$	-0.2269	-1.2091**	0.2834	-0.4809.	0.0530	-0.2776*		
	(0.2677)	(0.3328)	(0.3039)	(0.2510)	(0.1169)	(0.1365)		

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$; $^{\cdot}p < 0.10$. Despite estimating our model with eight lags, we opt to conserve on space and present results for the constant, first lag and our lagged ECT. After one lag, economic and statistical significance diminish greatly.

The results for the lagged endogenous variables are largely as expected with the first lagged differences of both H_t and S_t carrying the highest degree of significance and persistence compared to all other lags. Secondly, we note that the long-run coefficients vary quite significantly over each model. For the full sample, the β coefficient is around 0.60 and we see that inventories do not seem to adjust at a rate different from zero, however, sales exhibits a very slow and marginally significant adjustment speed to the shared long-run trend. Over the various subsamples, despite estimation risk, we see that the signs of each ECT_t are either zero or negative, which is appropriate. Finally, we note from Panel A of Table VI that our crisis subsample exhibits a statistically weak adjustment rate for inventories and an adjustment rate of zero for sales. Both pose a sharp contrast to other longer subsamples including the full sample, the productivity slowdown era, and the era before the productivity slowdown. The results herein also the echo a combination of findings from Wen (2011) and Maccini et al. (2015) wherein shorter subsamples tend to exhibit faster adjustment speeds (albeit at a weaker level of significance) while other longer subsamples all pose exceedingly slow adjustment speeds. Given the precariousness of S_t as an I(1) variable along with varying evidence of cointegration depending on the subsample, these results are not too surprising.

6 Conclusion

The aim of this paper was to revisit the fairly mature literature of inventories and assess the degree to which inventory dynamics have evolved with particular attention given to the period of time after the Financial Crisis through the beginning of 2022. We find that inventory levels estimated from a conventional flexible accelerator model are still persistent in recent decades, but the reverse is true for sales. This underscores the present deterioration of the link between inventories and sales.

Secondly, we find that the relative change in inventories as a percent of the change in GDP is considerably higher from the Financial Crisis onward when compared to the full sample period. This suggests changes in inventory levels over this period are on magnitude considerably higher than changes in GDP. We further uncover that the proxy Bullwhip Effect over long subsamples is strongest from the Financial Crisis onward when compared to the full sample. This suggests that domestic value chains have become less efficient, rather than more efficient in recent decades. We also find that the correlation between the level of sales and inventory growth is the highest during the era following the Financial Crisis, however, this correlation could very well be spurious, particularly when evaluating the results from our VEC models.

Finally, our analysis of the results from our cointegrated and VEC models illustrate that adjustment speeds are quite heterogeneous over different subsamples. Furthermore, we discover that the cointegrating relationship and stability of inventory growth deteriorate considerably after the Financial Crisis. Further research beyond this paper would do well to focus on the structural relationship between inventories, sales and output after the Financial Crisis and through the present day.

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A Appendix

Additional Flexible Accelerator Model Results

Table VII: Estimated Flexible Accelerator Models

	Dependent Variable: $log(H_t)$							
	Eq. (3.4)	Eq. (3.7)	Eq. (3.4)	Eq. (3.7)	Eq. (3.4)	Eq. (3.7)	Eq. (3.4)	Eq. (3.7)
Constant	0.067***	0.027**	0.2101***	0.03245	0.4364***	0.15114**	-0.0803	-0.1546
	(0.021)	(0.011)	0.0562	0.03942	0.0791	0.04944	0.2279	0.2223
$log(H_{t-1})$	0.974***	0.997***	0.8538***	0.99618***	0.8973***	0.97998***	0.9055***	1.0209***
	(0.010)	(0.002)	0.0347	0.00621	0.0197	0.00682	0.0915	0.0289
$log(S_{t-1})$	0.020**		0.1368***		0.0501***		0.1182	
	(0.009)		0.0329		0.0113		0.0890	
N	300	300	101	101	136	136	61	61
Adj. R^2	0.99	0.99	0.997	0.996	0.994	0.994	0.955	0.954
Period	1947:1–2022:1	1947:1–2022:1	1947:1–1972:4	1947:1–1972:4	1973:1–2006:4	1973:1–2006:4	2007:1–2022:1	2007:1–2022:1

Note: $^{***}p < 0.001$; $^{**}p < 0.01$; $^{*}p < 0.05$

Structural Break Tests

Table VIII: Chow-Test Results

Break Dates Via Chow Tests				
Break Date	1958:2	1973:4	1985:2	1996:3
F-Statistic	40.3565	554.15	728.182	128.272