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Optimization of water use in agriculture

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Abstract

The present work uses a theoretical modeling approach to model the integrated management of water resources in a context of resource scarcity. It aims at finding an optimal balance between the use of different water sources while taking into account the constraints associated with their use.

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1. Introduction

Climate change, water scarcity and projected population growth pose a great threat to water supplies for agriculture, compelling societies to seek alternative sources of water (De Fraiture et al. 2010) Wastewater is a viable irrigation water alternative in countries facing increasing water shortages (Cirelli et al, 2012 ; Hanjra et al, 2012). In fact Agriculture remains the largest consumer of water with around 70% of the world's fresh water currently used for irrigation, (Alexandratos et al, 2012). Because of the competition for the use of fresh water that already exists between the different sectors, particularly in areas where water is scarce, agriculture has ended up gradually ceding its share to non-agricultural uses (Rosegrant et al, 2000; Qadir et al, 2004; Qadir et al, 2010).

The use of fresh water for non-agricultural activities has generated wastewater, the volume of which has increased in proportion to rapid population growth, urbanization and economic development (Lazarova and Bahri, 2005; Asano et al, 2007). The vulnerability of the resource during the long periods of drought in recent years, overexploitation and the regular increase in needs have led to an imbalance between supply and demand, in particular chronic deficits. In most regions, irrigation is confronted with the drying up of riverbeds, the salinization and contamination of water resources, the insufficient filling rate of dams, conflicts of use and the drop in groundwater levels. Classical supply management no longer manages water properly. To deal with these problems, a new approach has been adopted taking into account the management of demand and the use of non-conventional water resources.

The main objective of wastewater reuse is not only to provide additional quantities of good quality water by accelerating the natural water purification cycle, but also to ensure the balance of this cycle and the protection of the surrounding environment. By definition, this reuse is a voluntary and planned action that aims to produce additional quantities of water for different uses in order to make up for water deficits. Depending on the quality requirements of consumers, two main classes of reuse can be defined: potable uses which can be direct, after extensive treatment, or indirect, after passing through the natural environment, and non-potable uses in agricultural sectors (irrigation), industrial and urban. The majority of wastewater reuse projects are for agricultural uses. For this sector, water reuse improves crop yields and brings financial benefits.

Domestic water use has a significant impact on the economics of indirect wastewater reuse. Greenhouse cultivation influences Stage II irrigation demand, primarily by reducing water availability. Additionally, it could further affect demand if communities evolve to be more concerned with the use of groundwater resources. The space of possibilities shows that the reuse of wastewater has a strong influence on groundwater and could alleviate agricultural water deficits through the diversification of irrigation sources (Jeong, 2020). Land and water are the most basic resources of food production systems. However, impending water scarcity threatens the sustainability of food production systems and poses food security challenges. Agricultural production on marginal and degraded land using unconventional water resources can help ensure food security for future generations.

Legislation should be adopted to encourage farmers to use Non-conventional water (NCW), water treatment technologies should be advocated and implemented for the use of NCW (Hussain et al,2019). and growing plant species (safflower, sorghum, millet, carrots, radish, cucumber, tomatoes, eggplant, lettuce) that accumulate relatively very low amounts of metals in their edible parts, especially in pre-urban areas. Reusing wastewater in agriculture reduces the water footprint of food production on the environment; it also involves activities such as higher crop yields and changes in cropping patterns, which also reduce the carbon footprint. However, there is a need to better integrate water reuse into key water governance frameworks to effectively address the challenges and harness the potential of this vital resource for protecting environmental health. The document also presents a blueprint for future water governance and public policies to protect environmental health (Hanjra et al,2012).

In order to increasingly use treated wastewater in agriculture, we have sought the optimal conditions for the combined use of conventional and unconventional water. this research aims to determine the optimal conditions for mobilizing conventional and unconventional water resources in agriculture and provide planning and management mechanisms for the different sources of irrigation water and help decision-makers choose their hydro investment agricultural policy to increase water availability in space and time. Our approach is based on the optimal control technique in order to model the integrated management of water resources in a context of resource scarcity. The hypothetical situation modeled is that of a watershed where agricultural activity uses three sources of water: groundwater from groundwater pumping, surface water from a dam and unconventional water from a wastewater treatment unit.

To model the management of water resources, we made three basic assumptions:. The first hypothesis considers that the quality of treated wastewater does not pose a problem for agricultural recovery. The second hypothesis concerns the possibility of access by farmers to the various conventional and unconventional water resources and the third hypothesis concerns the existence of an agency responsible for the planning and management of surface and groundwater resources.

2. Function Specification

Let $Q_{wt,t}$ the quantity of water mobilized from the water table, $Q_{d,t}$ the quantity of surface water from the dam, δ_t the quantity of unconventional water, πt the piezometry level. $\overline{ct}$ the cost of treated wastewater, $\overline{cb}$ the cost of surface water from the dam and CM The average cost function. the total instantaneous cost of using the different water sources is written:

$$CM(\pi_t)Q_{wt,t} + \overline{ct}\delta_t + \overline{cb}Q_{d,t}$$[1]

Let $\delta_t^{-1}(x)$ be the inverse function of the global water demand with (x) the total quantity of water and for an infinite time horizon and a discount rate $r > 0$. The expression of the social surplus based on the work of is written:

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[1] The average cost function *(CM)* is assumed to be positive $CM(\pi_t) \geq 0$,, decreasing $dCM'/d\pi_t < 0$, and convex $dCM''/d\pi_t \geq 0$.

$$S = \int_0^{\infty}\left[\int_0^{Q_{wt,t}+Q_{d,t}+\delta_t} \delta_t^{-1}(x)dx - CM(\pi_t)Q_{wt,t} + \overline{ct}\delta_t + \overline{cb}Q_{d,t}\right]e^{-rt}dt \quad (1)$$

This specification is an extension of the "objective" function used by (Hogner, 1982; Guthrie and Parker,1989; Patten, 1991 ; Patten, 1992). Groundwater resource dynamics describes the movement of water underground, which can be influenced by factors such as precipitation, water withdrawals, human activities and climate change. the dynamics of groundwater resources are introduced as a constraint in the model:

$$\pi_t = \frac{d\pi}{dt} = \rho - F(\pi_t) - Q_{wt,t} \quad (2)$$

Equation (2) represents the evolution of the groundwater balance. In the absence of exploitation of the water table ($Q_{wt,t} = 0$), the level of the piezometry depends only on the difference between the Groundwater recharge ρ and the Water leaks $F(\pi_t)$. the piezometric level may increase when ($\rho > F(\pi_t)$), remain constant when ($\rho = F(\pi_t)$), and decrease when ($\rho < F(\pi_t)$). In a groundwater exploitation situation, $Q_{wt,t} > 0$, the evolution of the groundwater piezometry depends on the difference between ρ and $F(\pi_t) + Q_{wt,t}$. The situation of overexploitation of the aquifer can thus be represented by the situation where $\rho < F(\pi_t) + Q_{wt,t}$.

3. Model

Our problem is the following: what quantities $Q_{wt,t}$ (quantity of water mobilized from the water table) and δ_t (quantity of unconventional water) should be used to maximize the net social surplus and have a better allocation of water resources in agriculture, taking into account environmental considerations related to the overexploitation of the water table and given exogenous $(Q_{d,t})$?. The mathematical modeling of the problem is as follows:

$$Max\, S = Max \int_0^{\infty}\left[\int_0^{Q_{wt,t}+Q_{d,t}+\delta_t} \delta_t^{-1}(x)dx - CM(\pi_t)Q_{wt,t} + \overline{cb}Q_{d,t} + \overline{ct}\delta_t\right]e^{-rt}dt \quad (3)$$

With the constraints:

$$\pi_t = \frac{d\pi}{dt} = \rho - F(\pi_t) - Q_{wt,t} \quad (4)$$

$$Q_{wt,t} \geq 0 \quad (5)$$

$$Q_{d,t} \geq 0 \quad (6)$$

$$\delta_t \geq 0 \quad (7)$$

This is an optimal control problem with two control variables $Q_{wt,t}$ and δ_t and a state variable π_t. No constraint was imposed on the variable $Q_{d,t}$ because, in the model, it is considered as an exogenous variable. the necessary condition for the control variables to maximize the objective function, given the constraints considered, is that there exists a variable φ_t called the adjoint variable associated with the state equation, (Hamilton, 1866) associated with the problem, expressed in current value and defined by:

$$H = \int_0^{Q_{wt,t}+Q_{d,t}+\delta_t} \delta_t^{-1}(x)dx - CM(\pi_t)Q_{wt,t} + \overline{cb}Q_{d,t} + \overline{ct}\delta_t + \lambda_t(\rho - F(\pi_t) - Q_{wt,t}) \tag{8}$$

It reaches its maximum for the sought optimal values $Q_{wt,t}^*$ and δ^* and their corresponding solution π^*. Thus, the following conditions must be verified:

$$\frac{\partial H}{\partial \lambda_t} = \dot{\pi}_t = \rho - F(\pi_t) - Q_{wt,t} \tag{9}$$

$$\varphi_t - r\varphi_t = -\frac{\partial H}{\partial \pi_t} = CM'(\pi_t)Q_{wt,t} + F'(\pi_t)\varphi_t \tag{10}$$

$$\frac{\partial H}{\partial Q_{wt,t}} = \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t) - CM(\varphi_t) \leq 0 \qquad \rightarrow \ (Q_{wt,t} > 0) \tag{11a}$$

$$\frac{\partial H}{\partial Q_{wt,t}} = \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t) - CM(\varphi_t) < 0 \qquad \rightarrow (Q_{wt,t} = 0) \tag{11b}$$

$$\frac{\partial H}{\partial Q_{d,t}} = \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t) - CM(\varphi) \leq 0 \qquad \rightarrow \ (Q_{d,t} > 0) \tag{12a}$$

$$\frac{\partial H}{\partial Q_{d,t}} = \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t) - CM(\varphi) < 0 \qquad \rightarrow \ (Q_{d,t} = 0) \tag{12b}$$

$$\frac{\partial H}{\partial \delta_t} = \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t) - \overline{cb} \leq 0 \qquad \rightarrow \ (\delta_t > 0) \tag{13a}$$

$$\frac{\partial H}{\partial \delta_t} = \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t) - \overline{cb} < 0 \qquad \rightarrow \ (\delta_t = 0) \tag{13b}$$

Considering that the global demand for water is very high, the system of equations (9) to (13) can be rewritten in the following form (replacing $p_t \equiv \delta_t^{-1}(Q_{wt,t} + Q_{d,t} + \delta_t)$):

$$\dot{\pi}_t = \rho - F(\pi_t) - Q_{wt,t} \tag{14}$$

$$\varphi_t - r\varphi_t = CM'^{(\pi_t)}\, Q_{wt,t} + F'(\pi_t)\varphi_t \ = \ r\varphi_t + F'(\pi_t)\varphi_t = \dot{\varphi}_t - CM'(\pi_t)Q_{wt,t} \tag{15}$$

$$\varphi_t = p_t - CM(\pi_t) \tag{16}$$

$$p_t \leq \overline{ct} \qquad \rightarrow (\delta_t > 0) \tag{17a}$$

$$p_t < \overline{ct} \qquad \rightarrow \ (\delta_t = 0) \tag{17b}$$

according to the equation (15), $r\varphi_t + F'(\pi_t)\varphi_t$ represents the part of the interest-related economic benefits associated with the net price of the resource ($r\varphi_t$) and the value of the future reduction in leakage ($F'(\pi_t)\varphi_t$). $\dot{\varphi}_t - CM'(\pi_t)Q_{wt,t}$ presents the cost component and includes the marginal increase in the net price ($\dot{\varphi}_t$) and the marginal increase in the future cost of pumping ($C'(\pi_t)Q_{wt,t}$). Equation (17) represents the condition of whether or not to use non-conventional water according to its cost and the net price of the resource. When the price of water is strictly lower than the cost of reusing treated wastewater ($p_t < \overline{ct}$), treated wastewater is not used. On the other hand, their use begins as soon as the two prices are equal ($p_t = \overline{ct}$) .

4. Results

To determine the steady state of equilibrium, it is considered that to fill the deficit due to the lack of water from the dam, the basin agency will use groundwater and treated wastewater, i.e. say ($\varphi_t = p_t - CM(\pi_t)$) et ($p_t = \overline{ct}$) .

Thereby :

$$\dot{\pi}_t = \rho - F(\pi) - Q_t \tag{18}$$

$$\overline{ct} - CM(\pi_t) = -\frac{CM'(\pi_t)(\rho - F(\pi))}{r + F'(\pi_t)} \tag{19}$$

$$if\ \left((\rho - F(\pi)) = 0\right)\ then\ \left(\overline{ct} = CM(\pi_t)\right) \tag{20}$$

when the recharge is equal to the leaks $\left((\rho - F(\pi)) = 0\right)$, equation (19) becomes $\left(\overline{ct} = CM(\pi_t)\right)$, which supposes that at any time the trade-off must be made by comparing the price of treated wastewater to the average cost of pumping. Then, when $\left((\rho - F(\pi)) = 0\right)$ and also the marginal increase in leakage is zero ($F'(\pi_t) = 0$), (Nahorski et al 2000; Jørgensen et al, 2001; Haji Rahimi and Ghaderzadeh, 2008 ; Harold, 1929).

The optimal condition becomes ($\dot{p} = r\varphi_t$) , which corresponds to the optimal condition for managing exhaustible resources. This rule stipulates that the marginal increase in the net price of the resource ($\dot{p}_t$) must at all times equalize the present value of the marginal cost of use ($r\varphi$) for the exploitation of the resource to be optimal. The stationary solutions of the problem are obtained by solving the system of equations $\dot{\pi}_t = 0\ et\ (\dot{p}_t = 0)$;

$$\dot{\pi}_t = R - F(\pi_t) - Q_{wt,t} \tag{21}$$

$$\dot{p}_t = \left(\overline{ct} - CM(\pi_t)\right)\left(r + F'^{(\pi_t)}\right) + CM(\pi_t))(r + F(\pi_t)) \tag{22}$$

Determining the singular control (π^*) and the corresponding solutions ($Q^*_{wt,t}\ and\ \delta^*_t$) requires knowing the explicit forms of the leakage functions ($F(\pi_t)$) , the average pumping cost ($CM(\pi_t)$) and the estimate econometrics of the coefficients. Once the values of the stationary solutions are known, they can be used as tools to help plan and manage the waters of the catchment area. Starting from a known initial situation ($\pi_t(0) = \pi_0$) , if the price of water is strictly lower than the cost of reusing treated wastewater ($p_t < \overline{ct}$) , non-conventional water is not used.

Therefore, the irrigation water deficit must be filled exclusively by pumping from the aquifer. During this period, the optimal trajectory of the price of the resource is governed by equation (22) and that of the quantities to be pumped by equation (21). The rate of pumping from the water table depends in this case on the quantity received from the dam, ie $\left(Q_{wt,t} = \delta_t(p_t) - Q_{d,t}\right)$. On the other hand, when the price of water reaches that of non-conventional water ($p_t = \overline{ct}$), the reuse of treated wastewater is an interesting option to meet the deficit in overall demand. During this period, the groundwater level is maintained at its equilibrium level (π^*) and the price of water is constant ($p_t = 0$). If conventional and unconventional water are used together, i.e. if the price of water is equal to the cost of reusing wastewater ($p_t = \overline{ct}$) and if the groundwater level is at its equilibrium level ($\pi_t = \pi^*$), given that Q_{bt} is known, the global demand is satisfied by the reuse of treated wastewater by providing $\left(s_t = \delta_t(p_t) - Q_{d,t} - Q_{wt,t}\right)$.

5. Concluding Discussion

the use of unconventional water, particularly the reuse of treated wastewater, as a solution for irrigation is a complex issue that takes into account both economic and environmental factors. The level of piezometry plays a crucial role in determining whether the use of treated wastewater is a viable option, as low levels of piezometry can indicate a shortage of water for irrigation. Additionally, the quality of the treated wastewater, the demand for irrigation water, and the potential risks to human health and the environment must also be considered. As the model presented suggests, arbitration can be made between different water sources to balance the economic goal of maximizing the net social surplus with the environmental objective of protecting groundwater from overexploitation. This may involve assessing the cost-benefit of using treated wastewater for irrigation compared to other sources of water, such as groundwater or surface water, and considering the potential impact on the groundwater levels and the environment. The results of our model show how arbitration can be made between the different water sources to reconcile the economic objective of maximizing the net social surplus with the environmental objective relating to the protection of the groundwater against overexploitation.

the results obtained reflect a certain simplism in the various assumptions made at the outset. The most important of these hypotheses is that relating to the control of the exploitation of the aquifer, which would certainly not fail to pose certain problems at the level of practical feasibility (institutional, organizational and economic aspects). Then, the hypothesis related to the use of treated wastewater would imply accessibility to the technology, the availability of financial means and the organization of the beneficiaries. However, until now, only the public authorities that encourage the investment in this field and the request for a financial participation to the producers would perhaps pose problems of acceptance. The model presented can be developed in several ways, both theoretically and empirically. Theoretically, it would be interesting to endogenize the quantity of water coming from the dam, by linking it to the quantity of water pumped from the groundwater and to the stock of the dam, which would imply introducing a new constraint for the dynamics of water coming from the dam. This would allow for integrating water resources into a single management plan with various control possibilities. Additionally, it would be more realistic to consider the average pumping cost not only as a function of the piezometric level, but also as a function of the quantity of water pumped, which might make unconventional water more competitive for use in agriculture. Empirically, collaboration with specialists working on the physical aspects of the resource should be intensified to apply the model and validate it in several regions.

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