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Why and how a well-intended (local) government can hide information from citizens for their own good: The case of public goods provision in less developed areas

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Abstract

In developing countries, particularly in African cities or rural areas, despite the existence of the free-riding phenomenon, it is common to see ordinary citizens being engaged in private initiative to produce a public good (roads repairing, makeshift bridges building, traffic regulation, etc.). This everyday observed fact reveals that preferences of agents are such that there is a minimal level of the public good under which the free-riding problem seems to vanish. In this paper, we develop a theoretical model that matches with this everyday reality in order to derive a development policy. Specifically, we revisit agents' preferences by considering a utility function that involves a satiation level. We find that, in a non-cooperative game, agents make their choice around the satiation level, and this outcome is socially optimal. As a policy implication for local authorities, before raising taxes, a better strategy could be to make citizens believe that the public good production is below the satiation level. We present the particular cases of public light provision and the management of inadequate occupation of public spaces.

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1 - Introduction

In developing countries, despite the scarcity of resources, an effort has to be made to build public infrastructure as it is essential for economic development. Because of the free-riding problem (i.e., ordinary citizens would not properly contribute), public goods production is usually put in the hands of government (authorities). But, raising taxes makes the authorities unpopular, and voluntary contribution is hard to implement. However, in some cities or villages in developing countries (particularly in Africa), a fact is observed almost everyday. Indeed, for some public goods, when the amount of the public good comes down to a given level, *ordinary* people decide to produce the good, instead of waiting for local authorities. Many makeshift roads, schools and bridges are built or repaired by voluntary citizens' initiatives. Likewise, when traffic is busy and policemen are absent, traffic is usually regulated by ordinary people who receive verbal or financial encouragement from fellow citizens. Another most recent example (in countries like Burkina Faso) is the ordinary citizens' commitment to fight against terrorism because official army is snowed under with the crisis. Intuitively, this means that preferences of agents are such that there is a minimal level of the public good under which the free-riding is mitigated.[1] But, in such a situation, the outcome is that people produce the public good for an amount that is just needed to enjoy it. For instance, if the road is impassable because of potholes, people's initiative would consist in just filling those potholes, but not really refurbishing the road.

In this paper, we base on a theoretical model that conforms with this everyday reality to derive a development policy regarding public goods production. Specifically, we first consider agents' preferences that account for congestion, that is, at a certain level of use, the public good becomes rival. Precisely, we consider preferences that involve a *satiation level* above which the marginal benefit of the private good is positive, and below which the marginal benefit is non-positive. In addition to the setup (see details in the next section), this is the first paper that clearly expresses this stylized fact about infrastructure provision in the world's poorest countries.

We find that, in a non-cooperative game, agents make their choice around the satiation level. We then compare the social planner's outcomes with that of a non-cooperative game (a symmetric Nash equilibrium), and the result is that the two types of equilibrium coincide. In other words, these ordinary citizens' initiatives we described above is socially optimal. Indeed, given the characteristics of preferences, in a non-cooperative game, agents will make their choice around the satiation level. Around that satiation level, the public good is relatively scarce, its production is of great importance to agents, and no agent has an interest in a decrease of the public good amount.

As revealed by our model and observed in real world, ordinary citizens' initiatives (Nash equilibrium) produce just the needed amount of the public good (around the satiation level). However, just *filling potholes on a road* cannot put a community on the development pathway. So, a well-intended government may be willing to produce the public good more than the amount of the satiation level, by the means of internal

[1] We may think that agents are rather altruist. However, two facts reveal that there is no such an altruism we may imagine. First, in relatively big African cities there is perceptible inequality in standards of living between citizens. For example, in the same area, some inhabitants may be living in high-level chic houses while, a stone's throw from there, some others are living in slums. Altruism should make richer people to forgone a part of their life conditions and help the poorer ones. Second, the existence of corruption, misappropriation of public funds and reluctance to pay taxes are not a signal of altruistic citizens.

resources. On the other hand, another reality can be observed in several urban and suburban areas in Africa: there is a discrepancy between the use of private goods and the availability of public goods.[2] So, a well-intended government can try to make citizens to forgo some units of private goods to participate in the production of public goods. But, since the Nash equilibrium is optimal, raising taxes may need some arrangements in order to avoid government unpopularity, tax fraud, etc. Therefore, before collecting taxes, the strategy could consist in making people believe that the current amount of the public good is below the satiation level.

The remainder of the paper is organized as follows. We provide a brief review of the literature, introduce the model and derive results respectively in Sections 2, 3 and 4. The findings give rise to some development policy implications presented in Section 5. We end with a discussion on the satiation level and some concluding remarks respectively in Sections 6 and 7.

2 - Short review of the literature

Voluntary contributions or private provisions are needed to produce public goods in many situations (Allouch, 2015; Honga and Lim, 2016), and particularly in a context of high mistrust for policymakers, reluctance to contribute, or lack of enforcement of tax collection. Samuelson (1954) argued that public goods should be provided by planners, despite the existence of a Lindahl equilibrium of personalized taxes charged according to the agent's marginal benefit (Conley and Smith, 2003), because none would like to voluntarily contribute to the production of public goods. However, the existing literature has extensively examined the conditions under which non-altruistic citizens might freely interact to provide the optimal amount of the public good. This literature can be divided into two branches. A majority of the papers in the first branch show that efficiency can be achieved through voluntary contributions if an incentive mechanism is properly designed (see, for example, Birulin (2006), Shinohara (2009), Konishi and Shinohara (2014), Slavov (2014), and Honga and Lim (2016)).

The second branch suggests that the form of agents' preferences, the nature of the public good and the structure of the economy may lead to an efficient (or near-optimal) voluntary provision. Most of the studies of this branch consider public goods that are subject to congestion (see, for instance, Tiebout (1956), Wooders (1980), Allouch and Wooders (2008), and Lindsay and Dougan (2013)). Tiebout (1956) considered an economy with several jurisdictions and local public goods and conjectured that, if there is mobility between jurisdictions, each consumer will move to the jurisdiction where his preference is satisfied the most. Agents' choices therefore, nearly decentralize the social planner's decision. Wooders (1980) provided a setup to give a formal proof of Tiebout's conjecture. Allouch and Wooders (2008) considered Tiebout's economy but with more general characteristics. In their model, agents may belong to several jurisdictions and there are communication costs. It is shown that the price-taking equilibrium is optimal.

Our work belongs to the second branch, but departs from Tiebout (1956), Wooders (1980) and Allouch and Wooders (2008) in several ways. We do not employ Tiebout's economy of multiple jurisdictions. Rather, we consider just one jurisdiction. Our public

[2] We could give the following two examples: 1) everyday traffic jams because of the use of cars and scanty existence of roads (of bad quality), 2) ordinary citizens who build nice and costly houses on a site which is not prepared for construction (no really passable road access, no electricity from the national power grid, no water conveyance, etc.).

good is local only in the sense that agents have identical preferences. Second, we do not define congestion with respect to the population size only. In our model, the public good becomes more congested as the population size or the use of the private good increases. We argue that this setup is more realistic. A ten-kilometer-long road in a city with a hundred inhabitants and where people use cars is more congested than a ten-kilometer-long road in a city of the same size where people use bicycles.

To the best of our knowledge, this is the first paper on the efficiency of the private provision of public goods in which congestion depends on both population size and on the amount of private good consumed. Oakland (1972) used a similar congestion specification, but his analysis did not look at the efficiency of voluntary contribution. Lindsay and Dougan (2013) employ a utility function with decreasing utility of the public good, but not of the private good. Decreasing utility of the public good does not necessarily produce congestion.

Moreover, in contrast with previous literature, agents in our model contribute to the public provision using a common resource, not private resources or endowments. In other words, our work is similar to a single period Great Fish War problem with a public good.[3] The last two characteristics of our model have a fundamental implication. Congestion is exacerbated here in comparison with Wooders (1980), Allouch and Wooders (2008) and Lindsay and Dougan (2013). This setup is meant to reflect the situation in economies suffering a severe lack of public infrastructure.

3 - Theoretical model

We consider n agents $i = 1, \cdots, n$, living in the same community and with utility function $U_i(.) \equiv U(.)$. The economy consists of two goods x and y, where x is a private good and y is a public good, as in Oakland (1972), Conley and Smith (2003) and Gravel and Poitevin (2019). In other words, for a given agent i, $U(.) = U(x_i, y)$, where x_i is the amount of private good consumed by agent i. We denote by $U_1(.)$ and $U_2(.)$ the first derivatives with respect to the first and second arguments respectively. We assume that $U_2(.) > 0$, namely that the marginal utility of the consumption of public goods is positive. But, as we will see, this is not the case for the private good because of congestion. We also assume that $U_{12} \geq 0$, where U_{12} stands for a second derivative. In other words, there is complementarity between the private good and the public good, as in Fraser (1996). We need the public good to enjoy the private good, and most of the time we cannot enjoy the public good if we do not have private good. This complementarity between the two goods has an implication for the model: it is no longer interesting to keep consuming the private good if the amount of the public good is not sufficient enough.

As one could see, we consider agents with identical preferences (Wilson, 1983; Fraser, 1996; Parry, 2002; Keenan et al., 2006; Gravel and Poitevin, 2019). This assumption contrasts with that of Bai et al. (2009), Chen et al. (2013), Andreoni (1990) and Kotchen (2009). Some authors have argued that the framework of heterogeneous preferences is the most reasonable assumption for public goods provision (Boardman and Lave, 1977; Hellwig, 2007). However, within the context of local public goods in small jurisdictions, homogeneous preferences fit realities as people with similar tastes tend to live together. Indeed, as mentioned by Tiebout (1956), if there is mobility, agents will move to the jurisdictions where choices better meet their preferences.

[3] See Levhari and Mirman (1980) for a seminal paper on the Great Fish War problem.

To highlight the existence of satiation level on the public good side (as suspected from observed empirical facts), we assume that the utility function is such that there exists $\bar{y}$ such that $\forall i$,

$$\begin{array}{lll} \frac{\partial U}{\partial x_i}(x_i, y) > 0 & \text{if} & y - \sum_{j=1}^{n} x_j \geq \bar{y} \\ \frac{\partial U}{\partial x_i}(x_i, y) \leq 0 & \text{if} & y - \sum_{j=1}^{n} x_j < \bar{y}. \end{array} \tag{1}$$

Property (1) tells that it is no longer profitable to keep using the private good if the public good is overused, that is, the gap between the total consumption of the private good and the amount of the public good is high. Because of congestion, there exists a threshold that could be called a *satiation level*, as in Lindsay and Dougan (2013). At the satiation level or above, agents still obtain utility from consumption of an additional unit of the private good. Below this satiation level, any additional unit does not increase utility. Jackson and Nicolo (2004) discussed the existence of $\bar{y}$ by considering a generalized single-peaked utility function, while assuming $\bar{y}$ to be independent of population size. If congestion is present, there should be a threshold above which the consumption of the private good becomes non-profitable or even harmful.

The *satiation level* identified by Lindsay and Dougan (2013) is not exactly the one we consider here. Lindsay and Dougan (2013) specified a utility function with a decreasing utility in the public good (above a given amount), but not in the private good. At a certain level of public good provision, it is no longer profitable to produce the public good. In other words, our concept of *satiation level* expresses the idea of congestion while that of Lindsay and Dougan (2013) does not necessarily.

In much of the literature, public good congestion depends on the size of the population using that public good (Craig, 1987; Oates, 1989; Edwards, 1990; Reiter and Weichenrieder, 1999; Guengant et al., 2002; Brinkman, 2016). In our model, congestion depends on both the population size and how the public good is used (in proportion to the quantity or quality available) through the use of the private good, as considered in Oakland (1972).[4] Indeed, on a road, car users or people who commute frequently contribute more to traffic jams than motorcycle users or those who are less mobile.

We could specify a more general setup for Property (1) by considering a monotonic continuous function $g(.)$ such that

$$\begin{array}{lll} \frac{\partial U}{\partial x_i}(x_i, y) > 0 & \text{if} & g\left(y - \sum_{j=1}^{n} x_j\right) \geq \bar{y} \\ \frac{\partial U}{\partial x_i}(x_i, y) \leq 0 & \text{if} & g\left(y - \sum_{j=1}^{n} x_j\right) < \bar{y}. \end{array} \tag{2}$$

But here, for simplicity and without loss of generality, we consider the setup of (1).

The public good y is produced according to the following constant-returns-to-scale technology, that is, a linear production function.[5]

$$y = K - \sum_{j=1}^{n} x_j. \tag{3}$$

[4] Malik and Babiker (2010) avoided considering congestion depending on population size. Studying road congestion, the authors related traffic to the number of cars moving per unit of time.

[5] The use of a linear production function of the public good is common in the literature on public goods. See, for example, Fraser (1996), Keenan et al. (2006), Brett and Weymark (2008), Slavov (2014), Allouch (2015), Gravel and Poitevin (2019) and Banerjee and Gravel (2020). Here, since the utility function is general, we adopt a linear production technology to simplify the model and its solving.

K is the stock of a public resource that is shared between private consumption and the production of a public good, and we assume that $K > \bar{y}$. Here, instead of considering the most common setup of endowed agents who make a voluntary or mandatory contribution to the public good (Andreoni, 1990), we adopt Hellwig (2007) and Kotchen (2009). Indeed, there is an exogenous stock of resource that is shared between private good consumption and public good production. Agents contribute to the public good by forgoing some amount of the private good.

4 - Agents' decision and outcomes

Agents interact strategically and maximize their utility. They decide on the amount of the private good (and then the contribution to the public good) in order to maximize their utility. We first derive the social optimum and then find the Nash equilibrium (non-cooperative decision). The social planner maximizes the sum of all agents' utilities. Technically, for each type of solution, the approach consists in solving the problem for $y - \sum_{j=1}^{n} x_j \geq \bar{y}$ and for $y - \sum_{j=1}^{n} x_j < \bar{y}$.

4.1 The social planner decision

The social planner decides on the amount of the public good to produce and the amount of the private good for each agent in order to maximize the social welfare, that is, the sum of utilities. Specifically, the problem of the social planner is

$$\begin{array}{l} \max\limits_{x_l,\, l=1,\cdots,n} \ \sum_{i=1}^{n} U\left(x_i, y\right) \\ s.t. \quad y - \sum_{j=1}^{n} x_j \geq \bar{y} \quad \text{or} \quad y - \sum_{j=1}^{n} x_j < \bar{y} \\ y + \sum_{j=1}^{n} x_j = K. \end{array} \tag{4}$$

We consider the symmetric equilibrium of the social planner. Indeed, for the social planner's solution, symmetric equilibrium is natural. If agents with identical characteristics interact cooperatively, it is less reasonable to assume that some of them will consume the private good more than the others. We derive the following lemma.

Lemma . *Let x_i^s be the social planner consumption amount of the private good by agent i, and y^s the social planner production of the public good. Then,*

$$x_i^s = \frac{K - \bar{y}}{2n} \quad \forall i, \quad \text{and} \quad y^s = \frac{K + \bar{y}}{2}. \tag{5}$$

Moreover, we have

$$\frac{\partial U}{\partial x_i}\left(x_i, y\right)|_{\left(x_i^s, y^s\right)} > \frac{\partial U}{\partial y}\left(x_i, y\right)|_{\left(x_i^s, y^s\right)} \ \forall i. \tag{6}$$

Proof. See Appendix A. □

The social planner's decision suggests consuming the private good and producing the public good at the satiation level. Indeed, at the satiation level it is not possible to increase the satisfaction of an agent without decreasing that of another one.

4.2 The non-cooperative decision

In this section, we derive individuals' private good consumption and the public good amount within the context of selfish agents who maximize their own utility. The problem of agent i is

$$\begin{array}{l} \max\limits_{x_i} U(x_i, y) \\ s.t. \quad y - \sum_{j=1}^{n} x_j \geq \bar{y} \quad \text{or} \quad y - \sum_{j=1}^{n} x_j < \bar{y} \\ \qquad y + \sum_{j=1}^{n} x_j = K. \end{array} \tag{7}$$

Proposition . *The non-cooperative decision generates an allocation that is socially optimal.*

Proof. See Appendix B. □

The proposition gives three key results. First, agents make their decision exactly at the satiation level, i.e., at equilibrium, the difference between the public good level and the total private consumption is equal to the threshold $\bar{y}$. This finding is quite intuitive. As long as it remains possible to increase utility with an additional unit of the private good, people will do so. More precisely, let us suppose that a consumer decides to deviate unilaterally from the Nash equilibrium implied by the proposition. If he increases his private good consumption, the economy falls below the satiation level and (because of the sign of the marginal utility) the deviation is not profitable. If he decreases it, he bears a loss in terms of utility since at the satiation level the marginal utility is positive.

Second, at equilibrium, the marginal benefit of the private good is higher than that of the public good (Eq (6)). Agents have more interest in consuming the private good than in contributing to the public good provision. This result seems to reveal the intention of agents to achieve a twofold goal. While pursuing the selfish goal of private good consumption, people make the required effort to preserve a sufficient level of the public good that allows them to keep using the private good. This finding may foretell the near optimality of the non-cooperative outcome.

Third, the non-cooperative and the cooperative decisions coincide. In other words, laisser-faire leads to efficiency. The intuition behind this finding is quite simple. Stiglitz (1982) raised the matter of a possible privateness (or quasi-privateness) of local public goods and asked the question of whether the Fundamental Theorem of Welfare Economics holds, and under what conditions. Of course, this privateness is not supported by a part of the literature (McMillan, 1989; Oates, 1989), and the context of Stiglitz (1982) is different from ours. For Stiglitz (1982), inside the jurisdiction the local public good is a pure one, but between communities the local public good has a private good characteristics, because the public good produced in profit of the citizens of a given jurisdiction provides no benefit to people in other jurisdictions. In our setup, the efficiency of laisser-faire lies in the existence of the satiation level $\bar{y}$. Around that satiation level, the public good is relatively scarce. Its production is of great importance to agents. None has an interest in a decrease of the amount of the public good. Hence, the local public good subject to congestion acts like a private one.

It is worth mentioning that our results seem to support some of the intuitions of the previous literature. Tiebout (1956) imagined an optimal size of the population above which the jurisdiction will attract new residents, and below which the opposite happens. Within another context, Lindsay and Dougan (2013) argued that if satiability holds and the marginal cost of provision is low, then voluntary contribution leads to a provision that is close to efficiency (not exactly the efficient level).

Finally, for policy purposes, it may be necessary to discuss the uniqueness and the stability of the Nash equilibrium. Bramoullé et al. (2014) showed that any n-players game can be analyzed as a network game, and if the Nash equilibrium is unique then it is stable. As we show in Appendix B, on Page 24, the Nash equilibrium is unique and symmetric, thus stable.

5 - Development policy implications

In developing areas, it is still necessary to build public infrastructure, even though resources are scarce. Of course, foreign aids may help achieve the goal, but we should also rely on internal resources. This is possible since, in several urban and suburban areas in Africa, there is a discrepancy between the use of private goods and the availability of public goods. So, to produce public goods today, we could find a way to make citizens accept to forgo some units in today private goods consumption. For instance, citizen can accept to reduce the use of cars in order to pay for roads construction. However, at the statu quo raising taxes without any caution may be delicate. Our findings above have major policy implications in that way. Indeed, we have learned that people voluntarily contribute if the amount of the public good is below the satiation level. So, to make citizens contribute to the local public good or pay taxes (*more than the amount recommended by the social planner*), an effective strategy could be to make them believe that the public good is below its satiation level. It does not consist in letting the public good reach its threshold $\bar{y}$. As we said above, no community will get developed if infrastructure is at the satiation level. But the approach is to make people believe that the economy is below the threshold and there is a need of provision. It is to insinuate an apparent worrying reality. For example, government could stop the functioning of traffic and public lights, close roads because they are allegedly in poor repair, slow down the internet, reduce the number of policemen, etc. In other words, we propose to destroy (or mimic destruction of) the public good to induce people contribution and avoid local authorities unpopularity. In Sections 5.1 and 5.2, we provide details for some examples of public goods.

Our results suggest that, optimally, the marginal benefit of the private good is greater than the marginal benefit of the public good. Currently, some cities in the poorest countries are pursuing a policy of forcibly evicting people from public spaces. Many houses and firms have been destroyed by such operations. Indeed, to reduce congestion, any strategy should consist in increasing the quantity $y - \sum_{j=1}^{n} x_j$, by either decreasing $\sum_{j=1}^{n} x_j$ or increasing y. With such an *eviction* policy, governments consider the first approach. However, such a policy without any consideration of an apparent worrying reality would increase the marginal benefit of the private good. So, any *eviction* will be followed by new establishments. This is what we can observe in cities in West Africa in 2020.

5.1 Local provision of public lights

Let us consider the example of a community that is reachable during both day- and night-time by a congested road. We suppose that the number of street lights that are currently working along the road is twenty, but the amount desired by authorities is thirty. We also assume that the satiation level suggests fifteen street lights. In the current situation, there is more street lights than what is recommended by the social planner. No citizen will voluntarily contribute to provide street lights because the satiation level is lower than the *status quo*. The strategy could consist in making people believe that only (for example) ten street lights are working. This could be achieved by intentionally stopping ten lights from functioning. This strategy would foster voluntary contributions (tax payment) because the members of the community would agree to contribute to the provision of five more lights. At this stage, people believe that fifteen street lights are working but the community actually has twenty-five at its disposal. Taking a step further, the local government could stop, let's say, two lights. Citizens will then accept to contribute to provide two more lights. The community has now twenty-seven street lights.

A strategy of deliberately disabling lights might continue until funding has been obtained for all of the thirty lights desired by the government. Let us specify that, at each step, authorities have to make citizens believe that the number of lights is below the satiation level, by disabling some lights. By simulating the breakdown of existing lights, local governments are able to produce more public goods than the optimal amount. This is achieved through voluntary contributions. The approach is effective if the mechanism is conducted ingeniously and discreetly in order to conceal the strategy from the members of the community.

5.2 Management of inadequate occupation of public spaces

Getting rid of anarchic occupation of public spaces is a serious challenge for some governments. Sometimes, machines, police and army are used to drive people away from public spaces. Here, instead of forcing people to leave public spaces, we propose to increase the marginal utility of the good that attracts them to public spaces: the public good. Indeed, if people love establishing their businesses or houses by the roadside, it is because of the benefits offered by the road in terms of visibility and security (public lights, easy access to police, etc.). So, instead of destroying the private goods, we could fake the destruction of the road and its benefits. When a road is closed for repair for a long time, businesses along that road may suffer and some owners may agree to relocate voluntarily. In other words, the strategy may consist in seemingly bringing the public goods located on the public space below the satiation level.

It might be useful to mention that our policy implications are not that agents will deviate from the optimum or will produce the public good more than the optimal level if they know the current state of the economy. Of course, if the amount of the public good is below the optimal level, agents will contribute to bring it to the optimum. If the amount is optimal and agents have this information, they will not contribute and the public good production will stay at the optimum. But, if the amount is optimal and the local government successfully hides this information, agents will contribute and, in the end, the available amount will be higher than the amount recommended by the social planner.

Let us clarify that the policy strategy we are deriving from our results is not necessary to promote a pure voluntary contribution (that is with no public intervention), but local government should devise a way to coordinate this voluntary contribution.

6 - Revealing the satiation level $\bar{y}$

As argued, there is evidence for the existence of $\bar{y}$. It is in the interest of any public decision-maker to know this threshold. We propose a "tâtonnement" process to identify $\bar{y}$, as in Drèze and de la Vallée Poussin (1971). Based upon our policy implications, the approach consists in *destroying* little by little and step by step the public good until its provision becomes a serious concern for citizens. Suppose that local authorities would like to know the satiation level for the speed of internet connectivity. The first step is to announce the will to improve internet quality, and to pretend to foresee a decline in the service because of supposed congestion due to high demand. Authorities will invite citizens to contribute voluntarily. We propose a gradual decrease in the quality of internet over time, until the majority (or a certain percentage of the population) of agents accept to increase contributions. The approach we propose does not change the satiation level, but just reveals it.

Remark . *Our model aims to fit stylized facts about infrastructure provisions in areas suffering a severe lack of public infrastructure. However, we could argue that the satiation level exists for any community whether developed or not. This satiation level varies with the level of development of the jurisdiction. As the jurisdiction becomes wealthier, preferences change, and agents are more demanding in terms of public goods (i.e., $\bar{y}$ increases). This reality could have a re-enforcing effect on the effectiveness of the strategy described in Section 5.1. However, finding $\bar{y}$ should be harder for more developed communities.*

7 - Conclusion

We base on an everyday fact to build a theory of public good that gives rise to a public good policy. The contribution of the paper is twofold. First, we provide a theoretical model that describes a common behaviour of agents in many developing communities. We then use our theory to propose a strategy to incite people to participate in the provision of public goods, within a context of high mistrust for policymakers, reluctance to contribute, or lack of enforcement of tax collection.

Many public goods are subject to congestion because of the use of private goods, meaning that their consumption involves some level of rivalry. Because of that congestion, preferences involve a *satiation level* above which the marginal benefit of the private good is positive, and below which the marginal benefit is non-positive. This allows us to examine a situation in which agents' selfish choices are socially optimal. We recognize that much is yet to be done to empirically assess the satiation level. We propose a "tâtonnement" process. With an appropriate way to reveal the satiation level, we can provide "*more of the public good than the amount recommended by the social planner*", and avoid compulsory taxes that make local authorities unpopular.

Two main assumptions give rise to our findings. First, the preferences of each consumer are known by other consumers. Each consumer knows that he has the same preference as the others, and hence, the satiation level is known to all. The second assumption is that the satiation level is exogenous, as in Jackson and Nicolo (2004). Concerning the first restriction, it is arguably realistic in a context of local public good. For small communities, identical preferences and common knowledge are a quite reasonable assumption. As discussed by Tiebout, people with the same preferences might move to the same jurisdiction. This information then becomes public knowledge. The second assumption seems stronger because intuition would suggest a satiation level dependent on the amount of the private good consumed or on the population size. Our results may continue to hold if the satiation level depends positively on the amount of the private good consumed. Indeed, the reason why the Nash equilibrium is optimal is that congestion is severe in our setting. A model in which the satiation level is an increasing function of the amount of the private good consumed would have exacerbated congestion.

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A - Proof of the lemma

Proof. We solve the problem for $y - \sum_{j=1}^n x_j \geq \bar{y}$ and for $y - \sum_{j=1}^n x_j < \bar{y}$.

• **Under the constraint of** $y - \sum_{j=1}^n x_j \geq \bar{y}$, the problem of the social planner is

$$\begin{array}{cl} \max\limits_{x_l,\, l=1,\cdots,n} & \sum_{i=1}^n U\left(x_i, K - \sum_{j=1}^n x_j\right) \\ s.t. & K - 2\sum_{j=1}^n x_j \geq \bar{y}. \end{array} \tag{8}$$

The Lagrangian is

$$\mathcal{L}^s = \sum_{i=1}^n U\left(x_i, K - \sum_{j=1}^n x_j\right) + \theta\left(K - 2\sum_{j=1}^n x_j - \bar{y}\right). \tag{9}$$

The first-order conditions are (a) $U_1\left(x_l, K - \sum_{j=1}^n x_j\right) - \sum_{i=1}^n U_2\left(x_i, K - \sum_{j=1}^n x_j\right) - 2\theta = 0$, $\forall l$, (b) $\theta\left(K - 2\sum_{j=1}^n x_j - \bar{y}\right) = 0$, (c) $\theta \geq 0$. Agents are symmetric, so, $x_l^s = x^s$, $\forall l$.

Case 1: $\theta > 0$. Then $K - 2nx^s = \bar{y}$, that is $x_l^s = \frac{K-\bar{y}}{2n}$, $\forall l$, and $y^s = \frac{K+\bar{y}}{2}$. Since $\frac{\partial U}{\partial x_l} > 0$, $\forall l$, we should have $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - U_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) > 0$ (I_1). Also, from the first-order conditions, we have $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - nU_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) > 0$ (I_2). We can show that (I_2) implies (I_1).

Case 2: $\theta = 0$. Then $K - 2nx^s = \bar{y}$ or $K - 2nx^s > \bar{y}$. If $K - 2nx^s = \bar{y}$ then $x_l^s = \frac{K-\bar{y}}{2n}$, $\forall l$, $y^s = \frac{K+\bar{y}}{2}$, and $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - nU_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) = 0$. If $K - 2nx^s > \bar{y}$ then $\exists\ \varepsilon > 0$ such that $K - 2nx^s = \bar{y} + \varepsilon$. So, $x_l^s = \frac{K-\bar{y}-\varepsilon}{2n}$, $y^s = \frac{K+\bar{y}+\varepsilon}{2}$, and $U_1\left(\frac{K-\bar{y}-\varepsilon}{2n}, \frac{K+\bar{y}+\varepsilon}{2}\right) - nU_2\left(\frac{K-\bar{y}-\varepsilon}{2n}, \frac{K+\bar{y}+\varepsilon}{2}\right) = 0$. Among the two solutions (for $K - 2nx^s > \bar{y}$ and for $K - 2nx^s = \bar{y}$), the social planner will choose the one that provides the highest social utility. If $x_l^s = \frac{K-\bar{y}}{2n}$, $\forall l$, and $y^s = \frac{K+\bar{y}}{2}$, then the social utility is $W^s = nU\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$. If $x_l^s = \frac{K-\bar{y}-\varepsilon}{2n}$ and $y^s = \frac{K+\bar{y}+\varepsilon}{2}$, then the social utility is $W_\varepsilon^s = nU\left(\frac{K-\bar{y}-\varepsilon}{2n}, \frac{K+\bar{y}+\varepsilon}{2}\right)$. $W_\varepsilon^s - W^s = nU\left(\frac{K-\bar{y}-\varepsilon}{2n}, \frac{K+\bar{y}+\varepsilon}{2}\right) - nU\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$. Let $\nu = \frac{\varepsilon}{2n}$. We can write $W_\varepsilon^s - W^s = nU\left(\frac{K-\bar{y}}{2n} - \nu, \frac{K+\bar{y}}{2} + n\nu\right) - nU\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$, or $\frac{1}{n}\left(W_\varepsilon^s - W^s\right) = U\left(\frac{K-\bar{y}}{2n} - \nu, \frac{K+\bar{y}}{2} + n\nu\right) - U\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$. We are going to compare $U\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$ and $U\left(\frac{K-\bar{y}}{2n} - \nu, \frac{K+\bar{y}}{2} + n\nu\right)$. Suppose that ν is small, that is, ν is an infinitesimal variation at $U\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$. If ν is infinitesimal we can write $U\left(\frac{K-\bar{y}}{2n} - \nu, \frac{K+\bar{y}}{2} + n\nu\right) = U\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - \nu U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) + n\nu U_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$, using the total differential formula. We know that $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - nU_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) = 0$. Hence, $U\left(\frac{K-\bar{y}}{2n} - \nu, \frac{K+\bar{y}}{2} + n\nu\right) = U\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right)$. So, $W_\varepsilon^s - W^s = 0$. The two solutions provide the same utility. In other words, if it happens that ε exits, it would not lead to solution yielding more utility.

• **Under the constraint of** $y - \sum_{j=1}^n x_j \leq \bar{y}$, the problem of the social planner is

$$\begin{array}{cl} \max\limits_{x_l,\, l=1,\cdots,n} & \sum_{i=1}^n U\left(x_i, K - \sum_{j=1}^n x_j\right) \\ s.t. & K - 2\sum_{j=1}^n x_j \leq \bar{y}. \end{array} \tag{10}$$

The Lagrangian is

$$\mathcal{L}^s = \sum_{i=1}^{n} U\left(x_i, K - \sum_{j=1}^{n} x_j\right) + \eta\left(\bar{y} - K + 2\sum_{j=1}^{n} x_j\right). \tag{11}$$

The first-order conditions are (a) $U_1\left(x_l, K - \sum_{j=1}^n x_j\right) - \sum_{i=1}^n U_2\left(x_i, K - \sum_{j=1}^n x_j\right) + 2\eta = 0$, $\forall l$, (b) $\eta\left(\bar{y} - K + 2\sum_{j=1}^n x_j\right) = 0$, (c) $\eta \geq 0$. Agents are symmetric, so, $x_l^s = x^s$, $\forall l$.

Case 1: $\eta > 0$**.** Then $K - 2nx^s = \bar{y}$, that is $x_l^s = \frac{K-\bar{y}}{2n}$, $\forall l$, and $y^s = \frac{K+\bar{y}}{2}$. Since $\frac{\partial U}{\partial x_l} > 0$, $\forall l$, we should have $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - U_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) > 0$. Also, from the first-order conditions, we have $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - nU_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) < 0$.

Case 2: $\eta = 0$**.** Then $K - 2nx^s = \bar{y}$ or $K - 2nx^s < \bar{y}$. If $K - 2nx^s = \bar{y}$, then $x_l^s = \frac{K-\bar{y}}{2n}$, $\forall l$, $y^s = \frac{K+\bar{y}}{2}$, and $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - nU_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - \alpha\left(K - 2\bar{y}\right) = 0$. If $K - 2nx^s < \bar{y}$ then $\exists\, \varepsilon > 0$ such that $K - 2nx^s = \bar{y} - \varepsilon$. So, $x_l^s = \frac{K-\bar{y}+\varepsilon}{2n}$, $y^s = \frac{K+\bar{y}-\varepsilon}{2}$. From the first-order conditions, we have $U_1\left(\frac{K-\bar{y}-\varepsilon}{2n}, \frac{K+\bar{y}+\varepsilon}{2}\right) - nU_2\left(\frac{K-\bar{y}-\varepsilon}{2n}, \frac{K+\bar{y}+\varepsilon}{2}\right) - \alpha\left(K - 2\bar{y} - 2\varepsilon\right) = 0$ (I_3). Also, since $\frac{\partial U}{\partial x_l} \leq 0$, $\forall l$, we have $U_1\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) - U_2\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) \leq 0$ (I_4). But (I_4) contradicts (I_3).

In conclusion, the social planner outcome is the one that is given by (5).

B - Proof of the proposition

Each agent i chooses an amount x_i of the private good to maximize his utility. As we said, we solve the problem for $y - \sum_{j=1}^n x_j \geq \bar{y}$ and for $y - \sum_{j=1}^n x_j < \bar{y}$.

• **Under the constraint of** $y - \sum_{j=1}^n x_j \geq \bar{y}$, the problem of agent i is

$$\begin{aligned} &\max_{x_i} U\left(x_i, y\right) \\ &s.t. \quad y - \textstyle\sum_{j=1}^n x_j \geq \bar{y} \\ &\qquad\;\; y + \textstyle\sum_{j=1}^n x_j = K. \end{aligned} \tag{12}$$

Using the second constraint we can rewrite the problem as follows.

$$\begin{aligned} &\max_{x_i} U\left(x_i, K - \textstyle\sum_{j=1}^n x_j\right) \\ &s.t \quad K - 2\textstyle\sum_{j=1}^n x_j \geq \bar{y}. \end{aligned} \tag{13}$$

For agent i, the Lagrangian is

$$\mathcal{L}^i = U\left(x_i, K - \sum_{j=1}^{n} x_j\right) + \lambda_i\left(K - 2\sum_{j=1}^{n} x_j - \bar{y}\right). \tag{14}$$

The first-order conditions are (a) $U_1\left(x_i, K - \sum_{j=1}^n x_j\right) - U_2\left(x_i, K - \sum_{j=1}^n x_j\right) - 2\lambda_i = 0$, $\forall i$, (b) $\lambda_i\left(K - 2\sum_{j=1}^n x_j - \bar{y}\right) = 0$, $\forall i$, (c) $\lambda_i \geq 0$, $\forall i$.

Agents are symmetric, so at equilibrium, $x_i \equiv x_i^* = \bar{x}$ and $\lambda_i = \lambda$, $\forall i$. We discuss, at the end of the proof, the symmetry and the uniqueness of the Nash equilibrium.

Case 1: $\lambda > 0$**.** Then $K - 2n\bar{x} = \bar{y}$, that is, $x_i^* = \frac{K-\bar{y}}{2n}$, $\forall i$, and $y^* = \frac{K+\bar{y}}{2}$. From the first-order conditions, and since $\frac{\partial U}{\partial x_i} > 0$, we should have $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - U_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) = 2\lambda$ and $U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - U_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) > 0$.

Case 2: $\lambda = 0$**.** $K - 2\sum_{j=1}^n x_j = \bar{y}$ or $K - 2\sum_{j=1}^n x_j > \bar{y}$. Since $\lambda = 0$ the first order conditions imply $\frac{\partial U}{\partial x_i} = 0$. But $K - 2\sum_{j=1}^n x_j \geq \bar{y}$ implies $\frac{\partial U}{\partial x_i} > 0$. So, we cannot have $\lambda = 0$. In conclusion, for $y - \sum_{j=1}^n x_j \geq \bar{y}$, the solution is

$$x_i^* = \frac{K-\bar{y}}{2n} \quad \forall i, \quad \text{and} \quad y^* = \frac{K+\bar{y}}{2}, \text{with} \tag{15}$$

$$U_1\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) - U_2\left(\frac{K-\bar{y}}{2n}, \frac{K+\bar{y}}{2}\right) > 0. \tag{16}$$

• **Let us turn now to the constraint of** $y - \sum_{j=1}^n x_j < \bar{y}$. We look at the problem with the constraint of $y - \sum_{j=1}^n x_j \leq \bar{y}$. For agent i, the Lagrangian is $\mathcal{L}^i = U\left(x_i, K - \sum_{j=1}^n x_j\right) + \mu_i\left(\bar{y} - K + 2\sum_{j=1}^n x_j\right)$. The first-order conditions are 1) $U_1\left(x_i, K - \sum_{j=1}^n x_j\right) - U_2\left(x_i, K - \sum_{j=1}^n x_j\right) + 2\mu_i = 0$, 2) $\mu_i\left(\bar{y} - K + 2\sum_{j=1}^n x_j\right) = 0$, and $\mu_i \geq 0$, $\quad \forall i$. Let us remember that agents are symmetric.

Case 1: $\mu_i > 0 \quad \forall i$**.** $K - 2\sum_{j=1}^n x_j = \bar{y}$. The first-order conditions tell that $U_1\left(x_i, K - \sum_{j=1}^n x_j\right) - U_2\left(x_i, K - \sum_{j=1}^n x_j\right) = -2\mu_i < 0$. It implies that $\frac{\partial U}{\partial x_i} < 0$. This is impossible because $K - 2\sum_{j=1}^n x_j = \bar{y}$.

Case 2: $\mu_i = 0 \quad \forall i$**.** We then have $K - 2\sum_{j=1}^n x_j < \bar{y}$ because we cannot have $K - 2\sum_{j=1}^n x_j = \bar{y}$; it would contradict the first-order conditions. $K - 2\sum_{j=1}^n x_j < \bar{y}$ implies $\exists\ \varepsilon > 0$ such that $K - 2\sum_{j=1}^n x_j = \bar{y} - \varepsilon$. So, the solution would be

$$x_i^* = \frac{K-\bar{y}+\varepsilon}{2n}, \forall i, \quad \text{and} \quad y^* = \frac{K+\bar{y}-\varepsilon}{2}. \tag{17}$$

From the first-order conditions, a necessary condition to have this solution is

$$U_1\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) - U_2\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) = 0. \tag{18}$$

Let us analyze the solution we have found in (17). If $\varepsilon > 0$ exists, then the utility of the agent is $U^\varepsilon = U\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right)$. If the solution in (17) maximizes the utility, it should provide at least a local maximum. So, we are going to see how U^ε changes if we decide to move a little bit (infinitesimal, i.e, a very small deviation) from the solution. In other words, we look at the sign of $\frac{\partial U^\varepsilon}{\partial \varepsilon}$. Indeed, $\frac{\partial U^\varepsilon}{\partial \varepsilon} = \frac{1}{2n}U_1\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) - \frac{1}{2}U_2\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right)$. In other words, $2n\frac{\partial U^\varepsilon}{\partial \varepsilon} = U_1\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) - nU_2\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right)$. Using the condition in (18), we can write $2n\frac{\partial U^\varepsilon}{\partial \varepsilon} = -(n-1)U_2\left(\frac{K-\bar{y}+\varepsilon}{2n}, \frac{K+\bar{y}-\varepsilon}{2}\right) < 0$. So, at the equilibrium point, when we consider an infinitesimal decrease in ε, there is an increase in the utility. In other words, the outcome in (17) is not a local maximum and thus cannot be the solution.

In conclusion, the non-cooperative outcome is the one that is given by (15). Condition (6) is exactly that of (16).

We now discuss the symmetry and the uniqueness of the Nash equilibrium. Indeed, at the optimum, the constraint of $K - 2\sum_{j=1}^n x_j \geq \bar{y}$ is bounded, otherwise the marginal utility of the private good is positive and some agents are interested in increasing their consumption of the private good. On the other hand, λ_i is the shadow price of the resource K, that is the change in $U\left(x_i, K - \sum_{j=1}^{n} x_j\right)$ at the optimum due to an increase in K. Here, all the agents face the same constraint (resource and variables), and have the same utility function with symmetric arguments. Therefore, the shadow price is the same for all the consumers. Moreover, the first-order condition in (a) tells us that the derivative of $U(.)$ with respect to x_i is equal to 2λ. Since the derivative of $U(.)$ is monotonic then, at the optimum, x_i is unique and is the same for all the consumers.