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Demand for education investment in a model with uncertainty

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Abstract

Many reports of the relevant literature describe studies of education investment. As undertaken by Glomm and Ravikumar (1992), we also consider public and private school education in the study described herein. However, unlike the related literature, our model setting incorporates private tutoring and uncertainty about the productivity of human capital accumulation. Based on Bearse, Glomm and Patterson (2005), our paper presents consideration of uncertainty about education results and presents an examination of how demand for education investment is determined. Results demonstrate that, because of uncertainty about education results, demand for education investment is less than in a case with no uncertainty.

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1. Introduction

Many studies have examined education investment. Glomm and Ravikumar (1992) presented fundamental work on this topic after conducting a study with a human capital accumulation model that incorporated education investment. By that study, Glomm and Ravikumar (1992) show that private education, where parents pay education costs for their children, is associated with a higher human capital growth rate and greater inequality than in a case with public education. Glomm and Ravikumar (1992) assume a model with a Cobb–Douglas human capital accumulation function and a logarithmic utility function. Glomm and Ravikumar (1992), Glomm and Kaganovich (2003), Cardak (2004) and others assume a logarithmic utility function. By virtue of these assumptions, the result can be derived easily. As one might expect, some studies have sought avoidance of the assumptions of a Cobb–Douglas human capital accumulation function and a logarithm utility function. Glomm (1997), Glomm and Ravikumar (2003), and Bearse, Glomm and Patterson (2005) assume a Constant Relative Risk Averse (CRRA) utility function. Moreover, Bearse, Glomm and Patterson (2005) assume a Constant Elasticity of Substitution (CES) human capital accumulation function.

The aim of our study is to set a model with uncertainty for education investment and to examine how demand for education investment is determined. Some papers incorporate uncertainty about education. Oshio and Yasuoka (2009), Andersson and Konrad (2002), and Brodaty, and Gary-Bobo and Prieto (2014), based on the expected income that can be gained from education investment, incorporate uncertainty of education and household consideration of demand for education investment. One can consider that a utility function by which an increase in the income level, but not an increase in the human capital accumulation level, raises the utility level.[1] For such a function, a redistributive policy with a lump-sum transfer, as considered by Boadway, Marceau and Marchand (1992), reduces demand for education investment.

Although some studies examine how education uncertainty affects demand for education investment, no model exists for uncertainty of education in a model incorporating school education and private tutoring. Based on work by Bearse, Glomm and Patterson (2005), who consider both school education and private tutoring, we examine how education uncertainty affects demand for additional levels of education. Compared with the no-uncertainty model, uncertainty reduces demand for private tutoring in the public school education model. In the case of private school education, demand for private school education decreases.

The remainder of this paper is presented with an explanation of the basic model in Section 2, with an examination of how education uncertainty affects education investment. Section 3 concludes the

[1] Being different from the uncertainty of consumption, the uncertainty of achievement of education investment is greater than the uncertainty of consumption because the education period is a very long time (from pre-elementary school to university) and because the achievement of education investment depends on some aspects such as private tutoring, friendship and others. Moreover, the parents are unable to anticipate the achievement deriving from education investment in children.

paper.

2. Model

As described herein, we demonstrate how demand for education investment is determined. As education investment, one can consider school education (public school education, private school education) and private tutoring. We consider school education of two types: public school education and private school education.

We consider a two-period model: t and $t+1$ period. The preferences for consumption and education for children can be presented as

$$u_t = \frac{c_t^{1-\gamma}}{1-\gamma} + E_t \frac{h_{t+1}^{1-\gamma}}{1-\gamma}, 0 < \gamma \tag{1}$$

where c_t and h_{t+1} respectively stand for the consumption of the parents and the human capital stock of children. The utility function assumed for this study is not a logarithm utility function but a Constant Relative Risk Averse (CRRA) utility function.

Next, we consider the Constant Elasticity of Substitution (CES) function as the human capital accumulation function. First, we consider the case of public school education. Then, the human capital accumulation function is assumed as presented below.

$$\begin{cases} h_{t+1}^H = (1+x)h_{t+1}^M \\ h_{t+1}^M = A\left(\alpha q_t^\rho + (1-\alpha)Q_t^{u\rho}\right)^{\frac{1}{\rho}} \\ h_{t+1}^L = (1-x)h_{t+1}^M \end{cases} \quad \text{with probability} \quad \begin{matrix} \pi \\ 1-2\pi \\ \pi \end{matrix} \tag{2}$$

We assume that $\rho < 1$, $0 < \alpha < 1$, $0 < x < 1$, and $0 < \pi < 1$. This setting resembles that used by Lord and Rangazas (1998). Therein, x denotes the ability of education to provide goods such as competency and academic achievement. Here, x is known by the household ex ante. However, the household can not know the type of child. The household considers the type of child with probability. Also, q_t and Q_t^u respectively denote private tutoring and public school education.

In the case of private school education, parents pay for private school education. Human capital accumulation is assumed as[2]

$$\begin{cases} h_{t+1}^H = (1+x)h_{t+1}^M \\ h_{t+1}^M = AQ_t^r \\ h_{t+1}^L = (1-x)h_{t+1}^M \end{cases} \quad \text{with probability} \quad \begin{matrix} \pi \\ 1-2\pi \\ \pi \end{matrix}, \tag{3}$$

where Q_t^r denotes private school education.

Next, we consider the budget constraint. Public school education is financed by taxation. Parents need not pay for school education. Nevertheless, they must pay for education investment for private tutoring. Therefore, the budget constraint of parents is

$$c_t + q_t = (1-\tau)h_t. \tag{4}$$

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[2] This setting is based on Bearse, Gloom and Patterson (2005).

In that equation, τ and h_t respectively denote the tax rate for financing public school education and the human capital stock of parents, which denotes the labor income of parents.

The budget constraint of the parents in the case of private school education is

$$c_t + Q_t^r = h_t. \quad (5)$$

The optimal demand for private tutoring q_t in the case of public school education to maximize utility (1) subject to constraints (2) and (4) is given to satisfy the following equation.

$$\left(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma}\right)A^{1-\gamma}\left(\alpha q_t^{\rho} + (1-\alpha)Q_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-1}\alpha q_t^{\rho-1} = \left((1-\tau)h_t - q_t\right)^{-\gamma} \quad (6)$$

With $1-\gamma-\rho < 0$, the left-hand side of (6) decreases with an increase in q_t. The right-hand side of (6) increases with an increase in q_t.[3] Subsequently, one can obtain the unique solution of q_t, shown by the following figure.

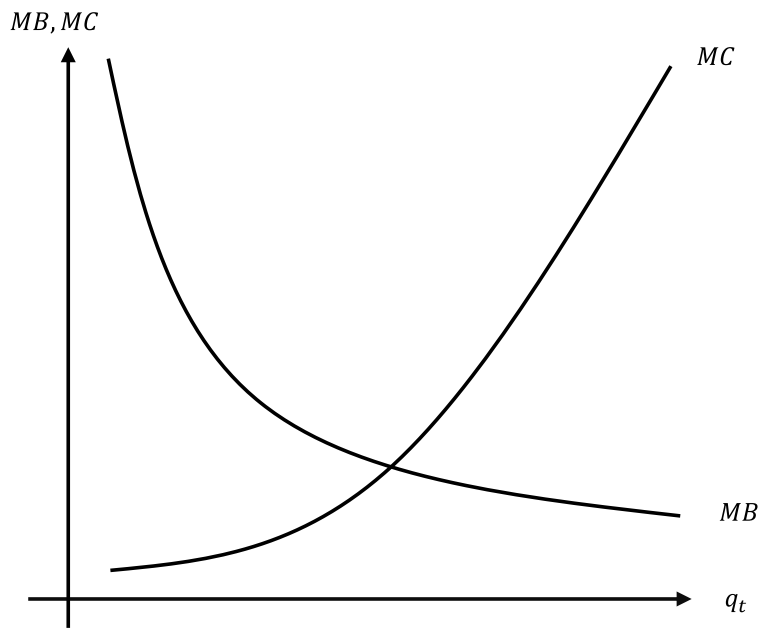

Fig. 1 Optimal demand for private tutoring.

[3] We calculate $\frac{dMB}{dq_t}$ of the left-hand side of (6). Thereby, we obtain $\left(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma}\right)A^{1-\gamma}\alpha\left((\rho-1)\left(\alpha q_t^{\rho} + (1-\alpha)Q_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-1} q_t^{\rho-2} + \alpha(1-\gamma-\rho)\left(\alpha q_t^{\rho} + (1-\alpha)Q_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-2} q_t^{2(\rho-1)}\right)$. Actually, $(\rho-1)$ in the first term of the bracket is always negative. Also, $(1-\gamma-\rho)$ in the second term of the bracket is negative if $1-\gamma-\rho < 0$ and then the left-hand side of (6) is negative. Even if $1-\gamma-\rho > 0$, as long as $(\rho-1)\left(\alpha q_t^{\rho} + (1-\alpha)Q_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-1} q_t^{\rho-2} + \alpha(1-\gamma-\rho)\left(\alpha q_t^{\rho} + (1-\alpha)Q_t^{u\rho}\right)^{\frac{1-\gamma}{\rho}-2} q_t^{2(\rho-1)} < 0$, the left-hand side of (6) is negative. This condition is consistent with the decrease in the marginal utility of q_t.

Therein, MB and MC respectively denote the left-hand side of (6) and the right-hand side of (6): MB is regarded as the marginal utility of private tutoring; MC represents the marginal cost of private tutoring. An increase in private tutoring reduces consumption. The utility of consumption decreases. Then, the demand for private tutoring is determined such that the marginal utility of private tutoring is equal to the marginal cost of private tutoring.

The difference between work presented by Bearse, Glomm and Patterson (2005) and our work is the examination of the uncertainty of education results, which is human capital accumulation. Bearse, Glomm and Patterson (2005) do not consider uncertainty related to human capital accumulation. By contrast, we address uncertainty of human capital accumulation and examine how education investment is changed by uncertainty.

First, we examine the effects of π. With $\pi = 0$, no uncertainty about human capital accumulation exists. We consider the case of $\pi > 0$. Then we calculate $\frac{dM}{d\pi}$ of the left-hand side of (6). If the following inequality holds, then the slope of MB shifts downward as shown by Fig.2. Demand for private tutoring e_t decreases.[4] Also, the households increase consumption c_t.

$$(1+x)^{1-\gamma} + (1-x)^{1-\gamma} < 2. \tag{7}$$

If one considers the logarithm utility function to set $\gamma = 1$, then the change of π does not affect private tutoring q_t.

Second, we examine the effects of x. An increase in x engenders an increase in the variance of human capital accumulation. We calculate $\frac{dMB}{dx}$ of the left-hand side of (6). If the following inequality holds, then the slope of MB shifts down; also, demand for private tutoring q_t decreases. The households increase consumption c_t.

$$\pi(1-\gamma)((1+x)^{-\gamma} - (1-x)^{-\gamma}) < 0. \tag{8}$$

The sign of the left-hand side of (6), that is MB, is determined by parameter γ. As long as $\gamma < 1$, inequality (8) holds. The MB curve shifts downward. Then the demand for private tutoring q_t decreases, as shown in Fig. 2. By contrast, with $\gamma > 1$, the inequality of (8) does not hold. Then the MB curve shifts upward: an increase in x raises demand for private tutoring.

Consequently, we can establish the following proposition.

Proposition

For public school education, with $(1+x)^{1-\gamma} + (1-x)^{1-\gamma} < 2$, an increase in π reduces private tutoring q_t. With $\gamma < 1$, an increase in x reduces private tutoring q_t.

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[4] If inequality (7) holds, then an increase in π decreases the value of MB for any q_t. As shown by Fig. 2, MB shifts downward. Then the intersection of MC and MB moves leftward. That is, demand for private tutoring q_t decreases.

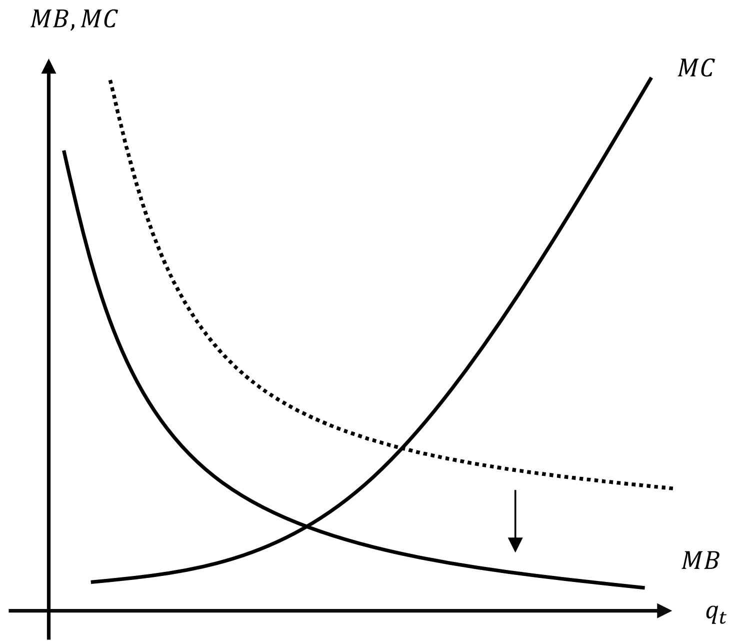

Fig. 2 Shift downward and demand for private tutoring.

If one considers the logarithm utility function to set $\gamma = 1$, then the uncertainty does not affect private tutoring. The logarithm utility function is assumed in many related papers because of the tractability for the model. Results produced by our study show attention for the model setting if uncertainty is considered.

Results obtained for public schools are roughly equivalent to those obtained in the case of private schools.

The optimal allocations of private school education maximize utility function (1) subject to constraints (3) and (5). Private tutoring is represented as

$$Q_t^r = \frac{h_t}{1+X^{-\frac{1}{\gamma}}A^{-\frac{1-\gamma}{\gamma}}}. \quad , \tag{9}$$

where X is

$$X = (\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma}). \tag{10}$$

An increase in X raises demand for the private school education Q_t^r. If inequality (7) holds, then the level of $(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma})$ decreases and private school education Q_t^r decreases. In addition, if inequality (8) holds, then the level of $(\pi(1+x)^{1-\gamma} + (1-2\pi) + \pi(1-x)^{1-\gamma})$ decreases. Consequently, private school education Q_t^r decreases.

3. Conclusions

For this study, we set a model of education investment with uncertainty about education productivity. Because of uncertainty about ability, parents reduce their demands for education investment compared to the no-uncertainty model. Therefore, education investment can be less than the social optimal level because of uncertainty. Then an increase in investment for public school education can be supported. Bearse, Glomm and Patterson (2005) derive the political equilibrium with the median voter theorem. In our study based on Bearse, Glomm and Patterson (2005), we can obtain the political equilibrium with the median voter theorem for the level of public school education, as demonstrated by Bearse, Glomm and Patterson (2005).[5]

[5] Our paper includes no consideration of endogenous school choice between public schools and private schools. However, our paper presents consideration of the endogenous school choice as shown by Bearse, Glomm and Patterson (2005). Intuitively, an increase in public school education investment increases the share of households choosing public school education. Examining how uncertainty affects school choice is important. This topic shall be considered for future research.

References

Andersson, F. and Konrad, K. (2002) "Human Capital Investment and Globalization in Extortionary states" WZB Discussion Paper, No. FS IV 02-01, Wissenschaftszentrum Berlin für Sozialforschung (WZB), Berlin.

Bearse, P., Glomm, G. and Patterson, D.M. (2005) "Endogenous Public Expenditures on Education" *Journal of Public Economic Theory* **7**(4), 561-577.

Boadway, R., Marceau, N. and Marchand, M. (1992) "Investment in Education and the Time Inconsistency of Redistributive Tax Policy" Working Paper 860, Economics Department, Queen's University.

Brodaty, T., Gary-Bobo, R.J. and Prieto, A. (2014) "Do Risk Aversion and Wages Explain Educational Choices?" *Journal of Public Economics* **117**(C), 125-148.

Cardak, B.A. (2004) "Education Choice, Endogenous Growth and Income Distribution" *Economica* **71**(281), 57-81.

Glomm, G. (1997) "Parental Choice of Human Capital Investment" *Journal of Development Economics* **53**(1), 99-114.

Glomm, G. and Kaganovich, M. (2003) "Distributional Effects of Public Education in an Economy with Public Pensions" *International Economic Review* **44**(3), 917-937.

Glomm, G. and Ravikumar, B. (1992) "Public versus Private Investment in Human Capital Endogenous Growth and Income Inequality" *Journal of Political Economy* **100**(4), 818-834.

Glomm, G. and Ravikumar, B. (2003) "Public Education and Income Inequality" *European Journal of Political Economy* **19**, 289-300.

Lord, W. and Rangazas, P. (1998) "Capital Accumulation and Taxation in a General Equilibrium Model with Risky Human Capital" *Journal of Macroeconomics* **20**(3), 509-531.

Oshio, T. and Yasuoka, M. (2009) "How Long Should We Stay in Education If Ability Is Screened?" *Metroeconomica* **60**(3), 409-431.