Economics Bulletin

Volume 44, Issue 1

The multi-scale analysis of dynamic transmission volatility of carbon prices

Ange Nsouadi
LARES, University Marien Ngouabi Of Congo

Virginie Terraza
DEM, University of Luxembourg, Faculty of Law, Economics and Finance

Abstract

The implementation of the EU ETS in 2005 led to the establishment of a price that enables manufacturers to realize the impact of their activities on the environment clean. There are no items in this day, since the creation of the European carbon market, which has focused on the analysis of volatility transmission between different investment horizons. The purpose of this paper is to fill this gap in the literature. we analyze the volatility of the price of carbon quota (EUA), by studying linear and nonlinear causal relationships of wavelet components between the different volatilities that we captured at different time scales. we initially conducted the decomposition of the EUA price volatility at different time-frequency interval using a wavelet approach. Our study will be to examine whether the volatility is transmitted from the high-frequency structure of the carbon price in the low frequency. Our results show an intra-structural dependance in carbon price volatility. We detect instability in the volatility of carbon and observe the existence of a bidirectional relationship from high frequency traders to low frequency traders. Our study showed that high-frequency shocks yields carbon price can have a significant impact beyond their Fontiers and touch the low frequency structure associated with long-term traders

Paper #9 in Special issue "In memory of Pr. Michel Terraza"
Citation: Ange Nsouadi and Virginie Terraza, (2024) "The multi-scale analysis of dynamic transmission volatility of carbon prices", *Economics Bulletin*, Volume 44, Issue 1, pages 399-415
Contact: Ange Nsouadi - ange_nsouadi@hotmail.com, Virginie Terraza - virginie.terraza@uni.lu.
Submitted: September 28, 2021. **Published:** March 30, 2024.

Economics Bulletin

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Special issue “In memory of Professor Michel Terraza”

1 Introduction

Just after the Kyoto protocol in 1995, a large platform containing the European carbon market quota was implemented to fight against the emission of greenhouse gas emissions. The balance between supply fixed carbon quota and flexible demand gave birth to a carbon price.

The purpose of this article is to analyze the structure of the quota carbon price by studying the causal relationship between different returns and volatilities captured at different time scales. We investigate whether a relationship exists between the different frequency bands of the carbon price quota containing the behavior of different players in the market. We investigate if there is a transfer of volatility between the bands from agent activity as they speculate in the short, medium, and long term.

The study conducted by Nsouadi et al (2013) has shown that stakeholders agents on the carbon market do not have the same behavior. Based on these results, we want to contribute to the work on the determinants of the carbon price made by Mansanet-Bataller et al. (2007) and Alberola et al. (2008) checking if there are interactions between the bands containing the behavior of different agents to determine the CO2 price.

So far, no research articles have focused on the return and volatility transmission between the different frequency of bands of the CO2 price. This paper aims to fill this gap in the literature. We analyze the causal relationships and the nature (linear, and non-linear) between different trading frequencies.

In this study, we use the wavelet methodology to investigate the dependence structure of EUA (European Union Allowance) at different time scales. The wavelet multiscale decomposition allows simultaneous analysis in the time and frequency domain.

The powerful wavelet analysis approach is model-free and it also helps us uncover interactions that the other econometric model cannot easily provide. Multiscale wavelet decomposition could become a valuable means of exploring and forecasting the complex dynamics of economic time series, as it allows for temporal and frequency analysis at the same time.

The wavelet analysis has been applied in several areas of economics. Davidson et al.(1997) have applied wavelet to find semi-parametric regression for study commodity price behavior. Connor and Rossiter (2005) are precursors for estimating price correlations based on scale decomposition of time series using wavelets on commodity markets. Recently, Naccache, T (2011) has used wavelets to study the correlation between oil prices and economic activity.

The "palette" of causality tests include the linear Granger test (Granger,1969) which assumes a parametric, linear model for the conditional mean, and, Hiemstra and Jones (1994) which proposed a causality-in-probability test for nonlinear dynamic relationships which is applied to the residuals of vector autoregressions and it is based on the conditional correlation integrals of lead-lag vectors of the variables.
Our study shows the existence in some cases of a bidirectional causal relationship between the frequency bands while it is sometimes unidirectional depending on whether the relationship is linear or not.

The remainder of the paper is organized as follows: in sections 2 and 3, we present the wavelet analysis and the data description. In sections 4 and 5, we analyze the wavelet decomposition of EUA price and we test causality by frequency bands. Section 6 concludes with some policy implications that discuss the relevance of our approach for markets policy.

2 Wavelet Analysis

Transitional series analysis of different lengths requires the use of temporal atoms whose supports are of varying sizes. The wavelet transform is the best tool to decompose signals by a family of translated and dilated wavelets. The study of economic data through wavelet is therefore to decompose the time series into components associated with different scales of resolution. A wavelet $\psi \in L^2(R)$ is a function with zero mean:

$$\int \psi(t)dt = 0. \tag{1}$$

It is normalized to $\|\psi\|= 1$, and centered in the neighborhood of $t = 0$. A family of time-frequency atoms is obtained by dilating the wavelet ψ by a factor s, and the u by translating:

$$\psi_{u,s}(t) = \frac{1}{\sqrt{s}}\psi(\frac{t-u}{s}) \tag{2}$$

These atoms are Standard 1: $\|\psi_{u,s}\|= 1$. However, any function $f(t) \in L^2(R)$ can be represented by the wavelet series expansion as follows:

$$f(t) = \sum_k \upsilon_{j.k}\phi_{j.k}(t) + \sum_k \omega_{j.k}\psi_{j.k}(t) + ... + \sum_k \omega_{j.k}\psi_{j.k}(t) + ... + \sum_k \omega_{j.k}\psi_{j.k}(t). \tag{3}$$

Where the coefficients $\upsilon_{j.k} = \sum_k \phi_{jk} f(t)$ and $\omega_{jk} = \sum_k \psi_{jk} f(t)$ are respectively coefficients and scale wavelet coefficients and ϕ_{jk}, ψ_{jk} are scaling functions and wavelets that meet the following conditions:

$$\begin{cases} \int \phi_{j.k}(t)\phi_{j.k*}(t)dt & = & \delta_{k,k*} \\ \int \psi_{j.k}(t)\psi_{j*.k*}(t)dt & = & \delta_{j,j*}\delta_{k,k*} \\ \int \psi_{j.k}(t)\psi_{j.k*}(t)dt & = & 0, \forall\ j,\ k \end{cases} \tag{4}$$

$\delta_{j,k}$ is the Kronecker coefficient (Kronecker delta). The scaling function, also known as the "father wavelet", is presented as follows:

$$\phi_{j.k}(t) = 2^{\frac{-j}{2}}\phi(\frac{t-2^jk}{2^j}), \tag{5}$$

The wavelet function, also called "mother wavelet" is defined as follows:

$$\psi_{j.k}(t) = 2^{\frac{-j}{2}}\psi(\frac{t-2^jk}{2^j}), \tag{6}$$

Equation (6) depends on two parameters, the scale (or frequency) and time. The scale or dilation factor j controls the length of the wavelet, while the translation or location parameter or location k indicates the portion of each non-zero vector wavelet basis.

Where k ranges from 1 to the number of coefficients in the specified level and J is the number of multiresolution levels, (scales).

The multiresolution analysis (Mallat, 1989) of a signal $f(t)$ into orthogonal components at different resolutions or scales can be defined as:

$$f(t) = S_j(t) + D_j(t) + D_{j-1}(t) + \ldots + D_1(t) \tag{7}$$

Each coefficient sets s_J , d_J , d_{J-1}, ... d_1 is called a crystal, where coefficients from level j = 1 . . . J are associated with scale $[2^{j-1}, 2^j]$.
Here, we apply the maximum overlap discrete wavelet transform (MODWT) as it allows us to explore any sample size, align the coefficients with the original data and calculate the wavelet variance and covariance effectively at different scales. Like the discrete wavelet transform (DWT), MODWT produces a set of time-dependent wavelet and scaling coefficients with basis vectors associated with a location t and scale j=$[2^{j-1}, 2^j]$ for each decomposition level $j = 1, \ldots J_0$. However, the MODWT is nonorthogonal and has a high level of redundancy, retaining downsampled values at each level of the decomposition that would be discarded by the DWT.

The coefficients $S_{j.k}$, (smooth) represent the smooth behaviour of the signal at the coarse scale 2^J (trend). The coefficients $d_{j.k}$, (details) coefficients represent deviations from the trend; $d_{j.k}$,$d_{j-1.k}$,...,$d_{1.k}$ capture the deviations from the coarsest to finest scale. Smooth and detail component coefficients, $S_{j.k}$ and $d_{j.k}$, are found by integrating over time, dt,

$$S_{j.k} = \int \phi_{j.k} f(t) dt \tag{8}$$

$$D_{j.k} = \int \psi_{j.k} f(t) dt \tag{9}$$

To capture data, the wavelet basis function is stretched (or compressed) in accordance with the scale parameter. To extract frequency information, we have a wide window of information on the performance of movements of low frequencies and a narrow window of information yields high frequency movements.
For more details, note that the scaling function integrates to 1 and is designed to reconstruct the smooth parts and low frequency signal, while the wavelet function integrates to 0 and it allows to write parts and detailed high frequency signal. Thus, by applying an analysis of several resolutions broken level j, we can obtain a complete reconstruction of the signal partitioned into a set of j frequency components, so that each element corresponds to a specific range of frequencies.

3 Data description

Our study focused on daily data from ECX[1] for the period from 24 March 2008 until 19 October 2012 or 1010 comments on the future price of quota carbon (EUA) (Figure

[1]European Climate Exchange : www.theice.com/emissions.jhtml

5 in Appendix). To study the different relationships that can exist between different investment horizons, we will link the meter to various risk factors. To do this calculation return is, therefore, necessary to take into account sudden changes in our price. The return performance of prices P_t at t and $t-1$, is defined as follows:

$$R_t = Ln(\frac{P_t}{P_{t-1}}) \tag{10}$$

Where P_t is the closing level on day t. The volatility of EUA is defined as the absolute value of the returns $V_t = |R_t|$ as defined in Jensen and Whitcher (2000) and Gencay and al (2002). The table I presents the descriptive statistics for the return and volatility of carbon price.

Table I: Descriptive Statistics: Weekly Returns and volatility of EUA

	Return of EUA	Volatility of EUA
Mean	-0.000597	0.0191
Median	0.000000	0.0139
Maximum	0.245247	0.245247
Minimum	-0.116029	0.00000
Std.Dev	0.027118	0.019244
Skewness	0.563177	2.972282
Kurtosis	10.90372	2.972282
Jarque-Bera	2679.631	2.972282
Probability	0.000000	0.000000
Sum	-0.602567	19.27881
Sum Sq.Dev	0.741293	0.373296
Observations	1009	1009

Considering descriptive statistics, we obtained positive coefficients of asymmetry of return distributions and volatility, indicating that the increases in the market price of carbon quota are more likely than decreases. It is the presence of leptokurticity (the distribution of returns has a value of kurtosis higher than 3) in our return series while we observe the presence of platykurtic distributions in the volatility series. There is thus a rejection of the normality assumption by the Jarque-Bera test and find ourselves in the same conditions to financial data.

4 Wavelet decomposition of EUA price

In this section, we investigate the effect of different levels of relationships that may exist between different investment horizons. We offer a comprehensive study of carbon allowance prices dynamics across and within all scales both for returns and volatility via the multiscale decomposition methodology. This study is based on discrete wavelet transform (DWT), in particular, the manual overlap discrete wavelet transforms (MODWT) in multi-resolution analysis (MRA). If we consider a time series X with an arbitrary sample size N, the j^{th} level wavelet ($\widetilde{W}_j$) and scaling ($\widetilde{V}_j$) are defined as:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{L_1-l} \widetilde{h}_{j,l} X_{t-lmodN} \tag{11}$$

$$\widetilde{V}_{j,t} = \sum_{l=0}^{L_1-l} \widetilde{g}_{j,l} X_{t-lmodN} \tag{12}$$

Where $\widetilde{h}_{j,l} = \frac{h_{j,t}}{2^{\frac{j}{2}}}$ are the wavelet filters MODWT, and $\widetilde{g}_{j,l} = \frac{g_{j,t}}{2^{\frac{j}{2}}}$ filters scales.
the MRA provides an additive decomposition through MODWT which is as follows:

$$X = \sum_{j=1}^{j0} \widetilde{D}_j + \widetilde{S}_{j_0} \tag{13}$$

Where $\widetilde{D}_{j,t} = \sum_{l=0}^{N-l} \widetilde{h}_{j,l} \widetilde{W}_{j,t+lmodN}$ and $\widetilde{S}_{j,t} = \sum_{l=0}^{N-l} \widetilde{g}_{j,l} \widetilde{W}_{j,t+lmodN}$

According to equation (13), at a scale of j, we obtain a set of coefficient D_j, each with the same number of samples (N) as in the original signal (X). These coefficients capture the detail at each scale local fluctuations throughout a time series. The set of values S_{j_0} smooth or provided the overall trend of the original signal.

Adding D_j to S_{j0} for $j = 1, 2, 3, \ldots, j_0$ gives an approximation more accurate to the original signal. This additive form of reconstruction allows us to validate each of these subsets (D_j, S_{j_0}) separately and add validation to generate a single overall deduction.

Each scale corresponds to a frequency interval and it is associated with a range of time horizons that span from several days to one year[2]. To sum up, the EUA returns series are decomposed at scale level j = 7, therefore containing up to yearly frequencies, while the volatility series are analyzed up to the $j = 4$ scale, which is associated with a frequency range of $[0.8; 1.6]$ weeks [3].

Using daily data on the first level represents the dynamics of the 1-2 day period, as demonstrated by Benhmad (2012,2013), the second scale represents the dynamics of the 2-4 day period, while the scales and 3,4,5,6 represent dynamic 4 to 8, 8 to 16, 16 to 32, 32 to 64 daily periods. This representation shows the time scale interpretation of wavelet multiresolution analysis; each time scale corresponds to a specific dealing frequency of a category of traders at the carbon market.

[2]For instance, the detail D_2 is associated with a frequency range of 48 days $[0.8; 1.6]$ weeks, while D_4 with approximately one month. Scale level $j = 7$ corresponds to a cycle length between 2.1 and 4.3 quarters, or equally between a semester and a yearly variation. Thereafter the notation D_j corresponds to the MODWT details, to enhance readability

[3]Also in economic terms it is reasonable to investigate causality relationships for the returns from daily to yearly frequencies, whereas up to monthly variations for the volatility. In real-world applications, quarterly or yearly volatility is not interesting for the economic analysis of high-frequency (daily) EUA series, nor traded in EUA markets, as opposed to daily, weekly and monthly volatility. On the contrary, the causality analysis of the returns up to yearly variations can be very useful in detecting EUA market linkages with macroeconomic fundamentals and in producing multi-step ahead return or price forecasts

Table II describes the wavelet scales according to appropriate time horizons, providing an overview of the relationship between frequency bands and time scales for time series.

Table II: Multiresolution analysis on different levels of scales

Time horizons		
	Wavelet scales	frequency days
	D_1	1-2 days
	D_2	2-4 days
	D_3	4-8 days
	D_4	8-16 days
	D_5	16-32 days
	D_6	32-64 days
	D_7	64-128 days
	A_7	>128 days

The wavelet decomposition allows us to locate the presence of a trader behavior with different time horizons. The different scales include fundamentalists (low frequency) that trade on longer time horizons, short-term traders (high frequency), and finally court-termites which the market in the medium term (average frequency). Each class trader can have a homogeneous behavior, but it is the combination of these classes in all levels that generates the time series overall. Therefore, the underlying dynamics are heterogeneous due to the interaction of all trader classes at different time scales.

Figure 1: Carbon Price Returns

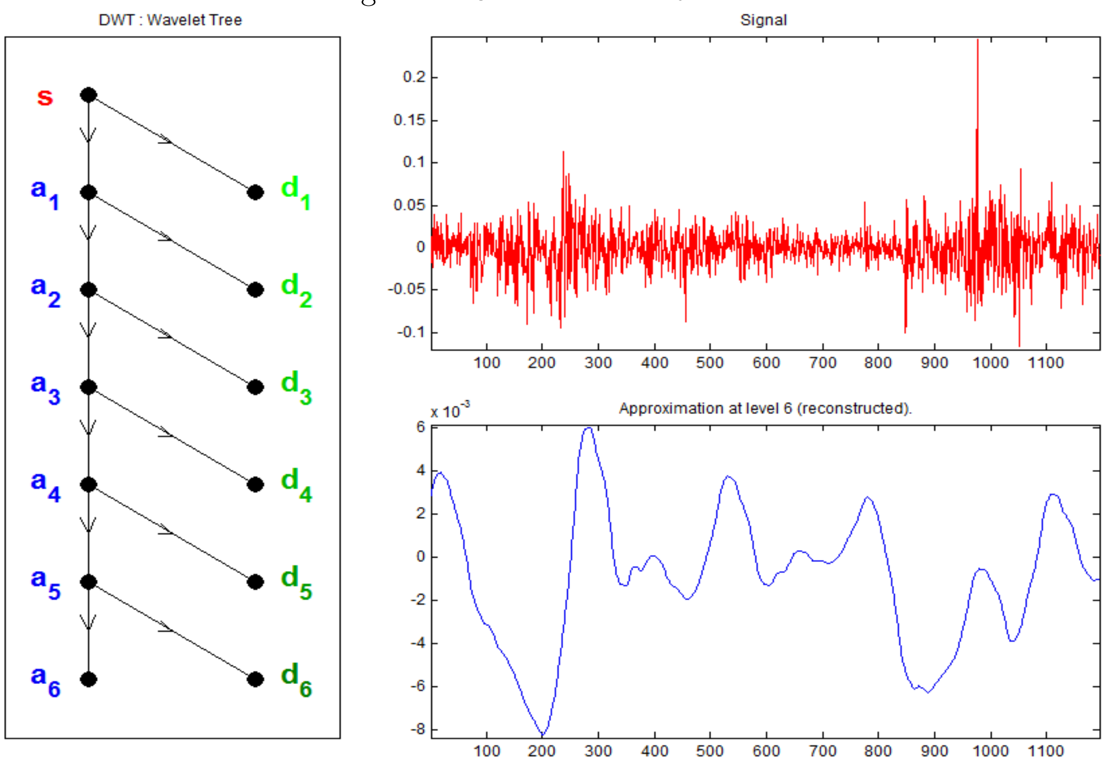


Figure 2 below presents the wavelet decomposition plot of carbon price returns.

Figure 2: Carbon Price returns wavelet decomposition

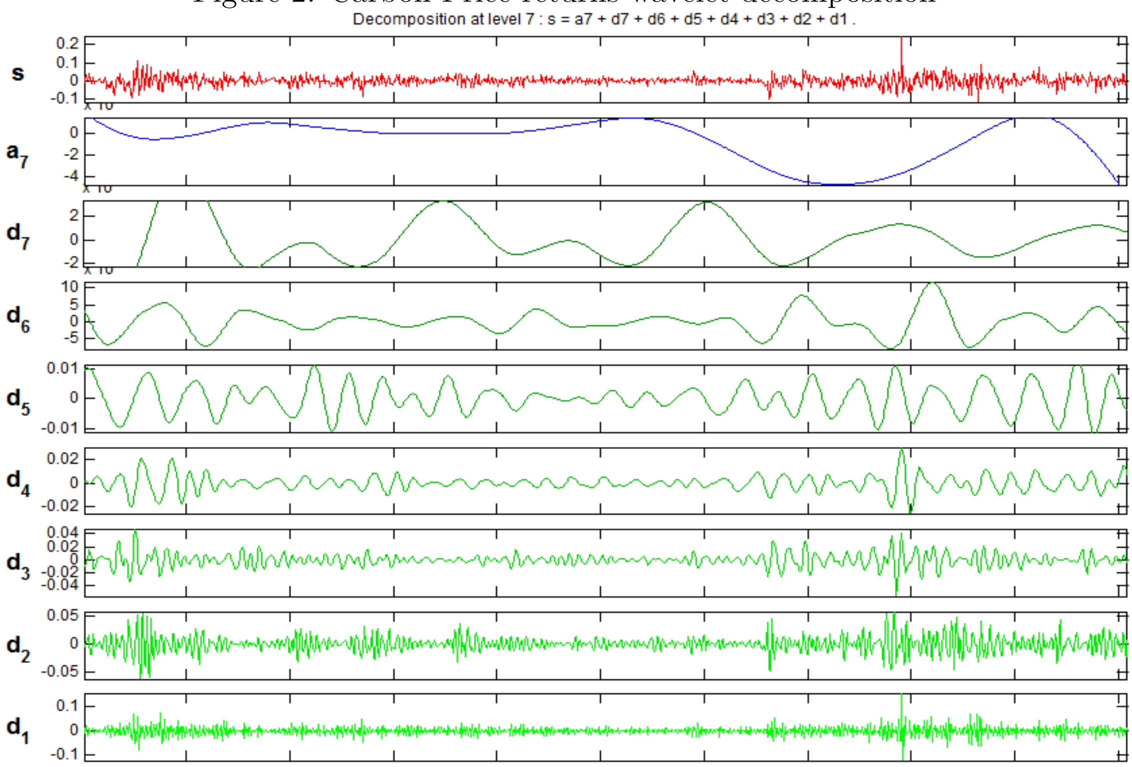


Figure 3: Carbon Price Volatility

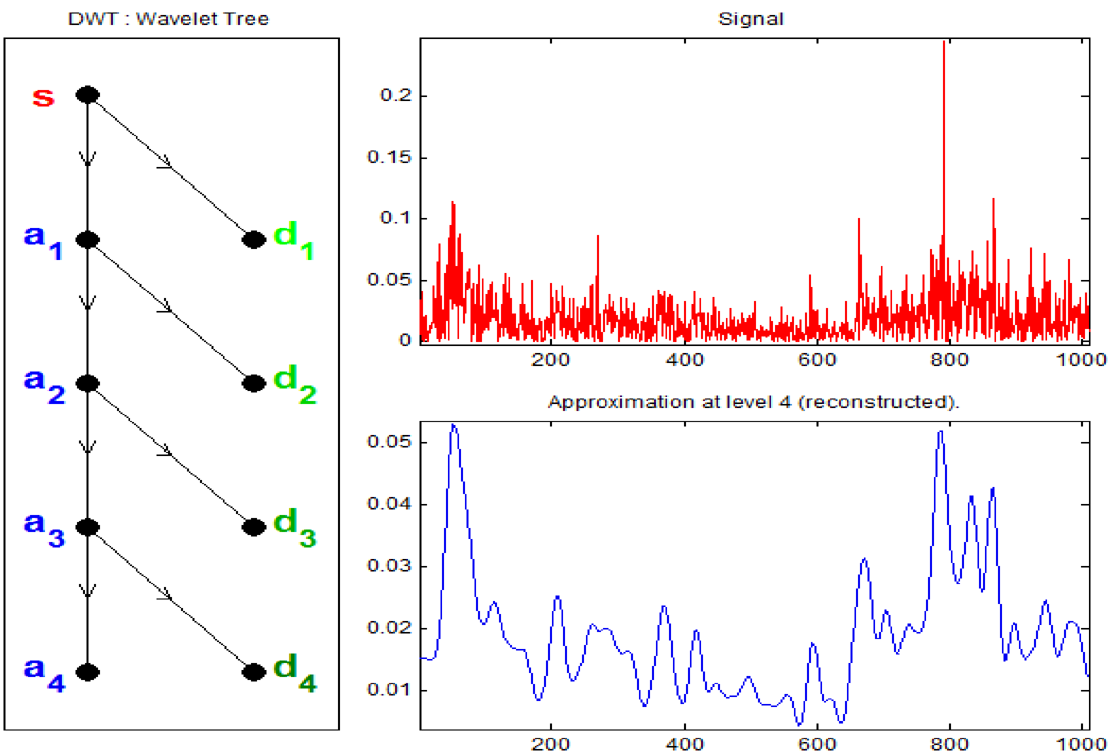
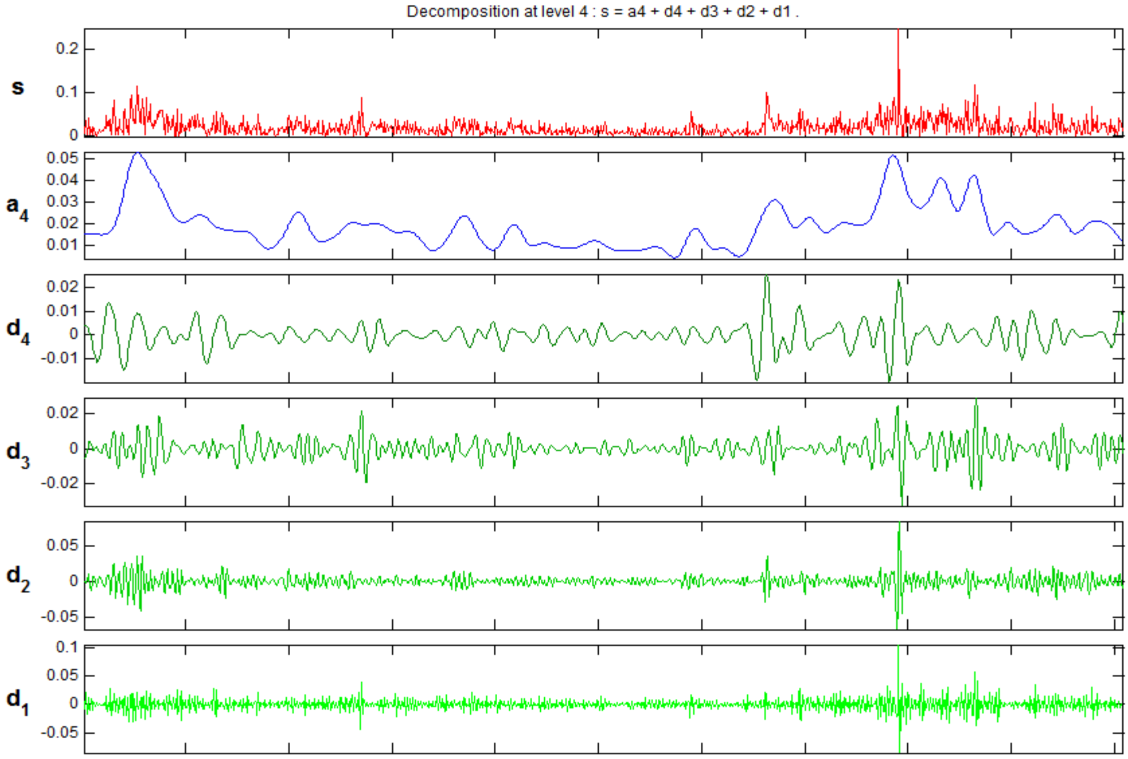


Figure 4 presents the wavelet decomposition plot of carbon price volatility.

Figure 4: Carbon Price Volatility wavelet decomposition



Observation of figures 1, 2, 3, 4 present the MODWT wavelet approximation and details estimated from the Daubechies class, for the returns and volatility series of EUA. To distill information from the wavelet components, it is crucial to recall that nonzero wavelet coefficients correspond to activity in a particular range of frequencies over time. Consequently, when the details are rapidly changing, this implies that its corresponding frequency interval contains important information about the original process.
These levels can be represented as follows:

- EUA returns series show that for all periods, there are significant differences in high- and low-scale dynamics. All components display a cyclical pattern with fairly high oscillation amplitude. Fundamentally, there is notable "activity" on high scales at all levels.

- The increased variability is particularly evident in the series of volatility in the price of carbon quota, which is associated with oscillations of 2-4 days period, but also in the second scale, third and fourth, corresponding to oscillations with a period of the order 3,4, which represent dynamic 4 to 8, 8 to 16 daily periods respectively. Thus, we observe a strong fluctuation of the volatility of each scale, which refers to a large speculative activity on different investment horizons.

The question we ask is to know if there is a transmission of volatility between the different levels (high, medium, and low) of the EUA price. For example, for EUA return (Figure 6 in Appendix), the first three finest scales (i.e., D_1, D_2, D_3) mostly affect the dynamics appearing in the raw data (S), while for EUA volatility (Figure 4) all scales seem to contribute to the raw series variability. Similarly, the detail D_1 of the EUA volatility (Figure 4) in all periods dominates the aggregate raw series oscillation amplitude, whereas other frequency components embed lower information. It should be noted that an impact at low frequency (long-term component), could lead to an answer for a short period of the high-frequency band.

5 Causality testing by frequency band

In this section, we study the causal relationships between different EUA returns and volatilities price captured at different frequency bands by using linear Granger causality and the nonlinear causality of Hiemstra and Jones (1994). The linear Granger causality test works best when the true causal relationship is linear but loses a lot of power when this is no longer the case. To overcome this drawback, Hiemstra and Jones (1994) proposed a causality-in-probability test for nonlinear dynamic relationships which is applied to the residuals of vector autoregressions and is based on the conditional correlation integrals of lead-lag vectors of the variables.

In this article, Granger causality and Hiemstra and Jones test are employed on the original and the disaggregated wavelet components to investigate the linear and nonlinear dynamic linkages at various scales. These tests can detect the nature and direction of causality explored for each scale component of the return and volatility series that we compare against the causalities results of the aggregate series.

The results of the frequency-domain test for the aggregate series can be comparatively analyzed against those of up to the a4 scale of the wavelet transform for the return and volatility series.

Our results of the causality tests between EUA returns and volatility price decomposed into seven and four frequency bands respectively are reported in Tables III and IV. We consistently choose the lag that gave the most significant result.[4]

The causality tests (see Tables III and IV) on returns and volatility of carbon allowance prices show that, in many cases, nonlinear causality tests give different results than the standard Granger non-causality test.

[4]The program used in this exercise, E-views, always tests for the null-hypothesis that variable x does not Granger-cause variable y. E-views will thus calculate the F-statistic of the regression under the null hypothesis that $a_{21} = a_{22} = *** = a_{2j} = 0$ and provides the probability that the null hypothesis is accepted. For probability values lower than 0.05(5%),we consider the Null hypothesis rejected and hence consider that variable x does Granger cause variable y.

Causality tests results between difference frequency bands of EUA of the EU ETS. 2 lags considered. The black color denote statistically significance at 5% level.

EUA return price	Frequency Bands								
	Tests	D_1	D_2	D_3	D_4	D_5	D_6	D_7	A_7
D_1									
	Linear		**0.0106**	0.9301	0.9891	0.9810	0.9984	0.9980	0.9968
	Nonlinear		0.06399	**0.02536**	**0.02825**	**0.00247**	**0.00777**	0.40701	0.17302
D_2									
	Linear	**0.0211**		0.1642	0.2170	0.8051	0.9711	0.9842	0.9998
	Nonlinear	**0.00119**		0.22789	**0.00291**	**0.00442**	0.22937	0.45350	0.30473
D_3									
	Linear	0.9982	**0.0246**		**0.0362**	0.3698	0.9928	0.9732	0.9822
	Nonlinear	**0.00515**	**0.01926**		**0.00043**	**0.00386**	**0.00327**	0.13930	0.36616
D_4									
	Linear	0.9998	0.9654	**0.0020**		**0.0002**	**0.0138**	0.1720	0.9631
	Nonlinear	**0.00219**	**0.00515**	**0.00638**		**0.00004**	**0.00194**	0.36452	0.34597
D_5									
	Linear	0.9998	0.9967	0.9743	0.1163		**0.0387**	**0.0288**	0.6419
	Nonlinear	**0.00249**	**0.01258**	**0.04092**	**0.00292**		0.05249	0.27057	**0.01558**
D_6									
	Linear	0.9995	0.9989	0.9985	0.8292	**0.0036**		**1.E-15**	**5.E-07**
	Nonlinear	**0.02739**	**0.01139**	**0.01676**	**0.00012**	0.05664		0.47988	0.37756
D_7									
	Linear	0.9990	0.9974	0.9987	0.8367	0.2773	**3.E-05**		**6.E-20**
	Nonlinear	0.20928	0.39866	0.27169	0.26937	0.06115	**0.00084**		**0.00061**
A_7									
	Linear	0.9983	0.9971	0.9975	0.9347	0.7350	0.7966	0.0733	
	Nonlinear	**0.04560**	0.46128	**0.02643**	**0.00458**	0.14301	**0.00121**	0.21917	

Table III: Tests of linear and nonlinear return causality (P-values)

Causality tests results between difference frequency bands of EUA of the EU ETS. 2 lags considered. The black color denote statistically significance at 5% level.

EUA volatility price	Frequency Bands					
	Tests	D_1	D_2	D_3	D_4	A_4
D_1						
	Linear		**0.0003**	0.6264	0.5916	0.9975
	Nonlinear		0.09882	0.11363	**0.03820**	0.39452
D_2						
	Linear	**1.E-05**		0.0825	0.1358	0.9288
	Nonlinear	**0.00452**		0.43886	0.11060	0.39884
D_3						
	Linear	0.9778	**0.0620**		**0.0324**	0.6813
	Nonlinear	0.07825	**0.01305**		0.07716	0.10913
D_4						
	Linear	0.9935	0.9550	**2.E-06**		0.3232
	Nonlinear	0.09074	0.21799	0.30146		**0.03658**
A_4						
	Linear	0.9999	0.9973	0.9582	0.6601	
	Nonlinear	**0.00469**	**0.00737**	0.06259	**0.00791**	

Table IV: Tests of linear and nonlinear Volatility causality (P-values)

We observe an instability of causality test in the different frequency bands.
On the return series, the linear test of Granger causality reveals a bi-directional relationship between D_1 which corresponds to the trading behavior of intraday traders or noise traders, and D_2; it is nonexistent between D_1 and other bands D_3, D_4, D_5, D_6, D_7, and A_7.
The nonlinear causality test, however, reveals a bidirectional relationship between D_1 and other bands D_3, D_4, D_5, and D_7.

We observe a unidirectional causality in the linear and nonlinear cases from D_3 and D_2. While it is bidirectional in the nonlinear case between D_3 and D_4, D_5, D_6 and it is unidirectional from A_7 to D_3.

The nonlinear causality reveals a bidirectional relationship between D_4 and bands D_5, D_6, then it is unidirectional in the linear case from D_4 to D_5, D_6. However, the bands D_5, D_6 and D_7 reveal a linear bidirectional relationship, it is against unidirectional from D_5 to A_7 in the nonlinear case.

We see an unstable causality and different nature between the different bands' returns.

For bands of volatility, we observe a bi-dimensional linear (in both directions) transfer of volatility between D_1 and D_2, whereas it is absent between bands D_3 , D_4, A_4. We also observe a linear unidirectional transfer of volatility between bands from D_3 to D_2 and D_4 and also from D_4 to D_3.

The nonlinear causality reveals a unidirectional relationship from D_1 to D_4,from D_4 to A_4, and then from A_4 to D_1, D_2 and D_4.

In summary, the study detects an instability transfer of volatility between different frequency bands.

6 Conclusion

The aim of the paper was two-fold: first to test for the existence of causal relationships among the different EUA prices returns and volatilities captured at different time scales, and then the nature of causality (linear and nonlinear) was investigated on different time scales of the return and volatility series by using the wavelet-based approach.

This study attempted to probe into the micro-foundations of across-scale heterogeneity in the return and variability pattern, based on trader behavior with different time horizons and information flow across time scales. The trading pattern of fundamentalists is reflected at the highest approximation wavelet scale, while at lower scales short-term traders and market makers operate. Each trader class may possess a homogeneous behavior, but the aggregate underlying market dynamics are heterogeneous due to the interaction of all trader classes at different time scales. In such a market, a low-frequency shock infiltrates through all scales, while a high-frequency shock runs out quickly and might have no impact whatsoever in the long-run dynamics.

The propagation properties of this heterogeneous-driven behavior were investigated, the causality structure from low-to high frequency was identified, and the implications for the flow of information across time scales in the European carbon market (EU ETS) were inferred.

Our study shows the existence in some cases of a bidirectional causal relationship between the frequency bands while it is sometimes unidirectional depending on whether the relationship is linear or not. We observed that the behavior of agents on short-term have an impact on the medium and long term and vice versa. There is a transfer of volatility and return between the different frequency bands of the carbon price quota. We prove that high-frequency shocks could have an impact outside their boundaries and reach the long-term traders and vice versa low and medium-frequency shocks could also have an impact outside their boundaries and reach the short and medium-term traders.

The different agents involved in the carbon market must integrate into the determinants of the price of quota carbon this analysis. They also take into account these results according to their investment horizon.

7 Appendix

Figure 5: Price Carbon

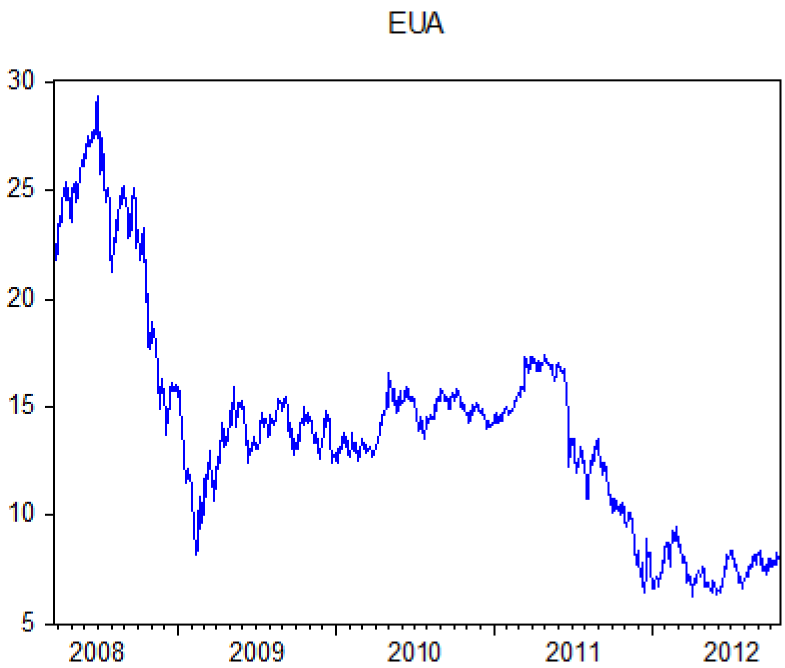
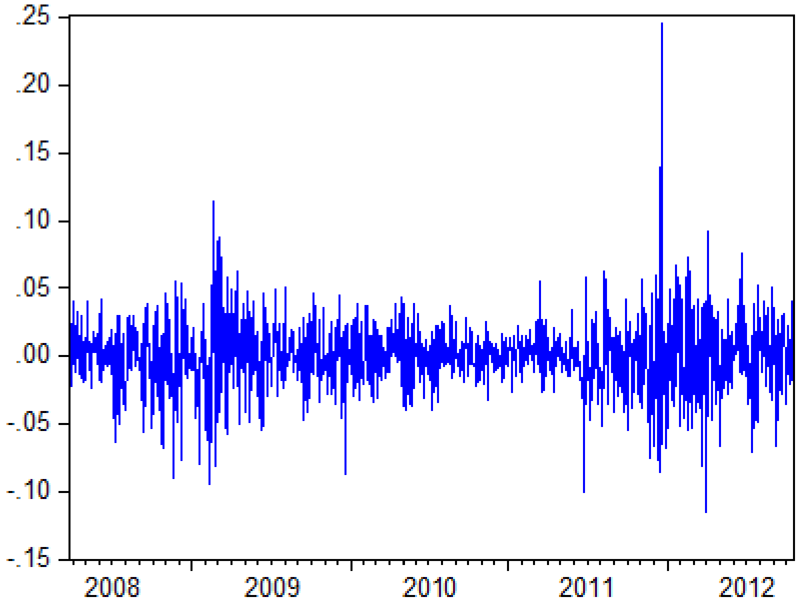


Figure 6: Price Carbon Return



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