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Exploring transitional and asymptotic impacts of subsistence consumption on wealth inequality in an AK growth model

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Abstract

This study analytically examines an AK model with heterogeneous agents - differentiated by their rates of time preference and intertemporal elasticity of substitution - to explore how the introduction of a subsistence level of consumption can mitigate wealth inequality during the transitional dynamics toward an asymptotic balanced growth path. The mechanism driving this result is that subsistence consumption induces a time-varying intertemporal elasticity of substitution, which alters individuals' lifetime consumption allocation decisions. Specifically, it encourages all agents to favor current consumption, with the wealth-dominating agent being the most affected. As a result, wealth distribution becomes more egalitarian during the transition. However, this more equitable distribution does not persist in the long run, as the impact of subsistence needs diminishes once wealth reaches sufficiently high levels.

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1 Introduction

Wealth distribution is a critical topic in economic growth theory. Frank Ramsey's 1928 paper was among the first to address this issue, conjecturing that the economy would eventually divide into two groups: those with high patience (low time preference) who would accumulate all the wealth, and those with lower patience who would only earn enough to subsist (Ramsey, 1928). This conjecture, known as Ramsey's conjecture, was validated by Becker (1980) and Sorger and Mitra (2013), who confirmed that patience and savings rates are key to long-term wealth distribution.

Studies such as Nakamura (2014), Tsukahara (2016), and Tsukahara (2017) explore wealth distribution within the standard AK model. Nakamura (2014) extends the model by assuming two groups of individuals who differ in their rate of time preference (TP) and intertemporal elasticity of substitution (IES). The model shows that, despite lacking transition dynamics, wealth distribution on the balanced growth path (BGP) is jointly determined by TP and IES. Interestingly, individuals with lower patience (higher TP) can still dominate wealth if their IES is sufficiently high, suggesting that a higher tendency to delay consumption can counterbalance impatience in influencing the wealth distribution. This contrasts with Ramsey's conjecture, which emphasizes patience alone. Another noteworthy point is that, unlike Ramsey's conjecture—based on the Ramsey-Cass-Koopmans model—the group with decreasing wealth share still retains some capital, albeit a very small share, as time progresses. Following this literature, Tsukahara (2016) and Tsukahara (2017) further explore this by examining the roles of love of wealth and habit formation, showing that impatience can dominate wealth distribution under certain conditions.

Our study builds on Nakamura (2014) by incorporating a subsistence level of consumption into the AK model. This addition offers new insights, as it shifts the model from an immediate BGP to an asymptotic balanced growth path (ABGP), with transitional dynamics leading to this state. While wealth distribution along the ABGP is determined by TP and IES, subsistence consumption plays a crucial role in reducing wealth inequality during the transition. It creates time-varying IES effects, particularly impacting those who consume less currently, which leads to more equitable wealth distribution during the transition compared to Nakamura's results. As time progresses, the impact of subsistence needs diminishes, aligning our results with Nakamura's BGP findings.

Incorporating a subsistence level of consumption into an AK model offers a promising and reasonable approach. For instance, Strulik (2010) shows that an AK model with subsistence needs can effectively explain long-term trends in developed economies, such as rising savings rates and growth rates. Similarly, Choi and Shim (2022) demonstrate that adding a subsistence level of consumption to the standard real business cycle model can better capture relative volatility across countries, with its influence being more significant in lower-income countries. Moreover, empirical evidence indicates that the IES tends to increase with wealth levels,[1] and introducing subsistence consumption could account for this, as discussed in Chatterjee and Ravikumar (1999) and Álvarez Peláez and Díaz (2005).

The study is organized as follows: Section 2 details the model, Section 3 characterizes the equilibrium, Section 4 presents the main results, and Section 5 concludes.

[1] See Atkeson and Ogaki (1996) among many others.

2 The Model

This section outlines the key components of a continuous-time AK model with a subsistence level of consumption, specifically addressing household behavior and production.

2.1 Household

At any time $t \geq 0$, the population consists of a large number of agents, normalized to unity. These agents are divided into two groups based on their rates of TP: an impatient group H, with a fraction $\lambda \in (0, 1)$, and a patient group L, with a fraction $1 - \lambda$. The impatient group has a common rate of time preference ρ_H, while the patient group has a rate ρ_L, with $\rho_H \geq \rho_L > 0$. Within each group, agents share identical preference parameters and endowments, allowing their decisions to be represented by a standard representative agent model.

The instantaneous utility function for the representative agent $i \in \{H, L\}$ is a variant of the standard constant relative risk aversion (CRRA) utility function with subsistence consumption incorporated, expressed as:

$$u(c_i) = \frac{(c_i - c_{min})^{1-\frac{1}{\epsilon_i}}}{1 - \frac{1}{\epsilon_i}}, \tag{1}$$

where $\epsilon_i > 0$ denotes the intertemporal elasticity of substitution (IES), and $c_{min} \geq 0$ represents the subsistence level of consumption.[2]

By incorporating subsistence consumption, the model captures the feature that utility is derived only from consumption exceeding c_{min}. This approach is widely employed in economic growth models to account for minimum consumption requirements [see, e.g., Chatterjee and Ravikumar (1999), Steger (2000), Strulik (2010), Alonso-Carrera and Raurich (2018), and, Antony and Klarl (2023)].

The budget constraint for agent i is:

$$\dot{k}_i = r k_i - c_i, \tag{2}$$

where r denotes the interest rate (net of depreciation). Additionally, we assume that initial capital holdings are equal among agents, i.e., $k_H(0) = k_L(0)$. This assumption is crucial for two reasons. First, it aligns with the setup in Nakamura (2014), allowing us to compare our results with his result. Second, by having an equal distribution of initial capital, time-varying IES will reflect the interaction between different IES values and c_{min} without introducing variability from different initial capital choices $(k_L(0), k_H(0))$.[3] Importantly, the initial capital for the aggregate economy is $k(0) = \lambda k_H(0) + (1 - \lambda) k_L(0)$.

Given the instanteneous utility function (1), agent i chooses the consumption profile $(c_i(t))_{t \geq 0}$ to maximize $\int_0^\infty e^{-\rho_i t} u(c_i) dt$ subject to the budget constraint (2) and initial wealth

[2] For clarity, the time index t is omitted unless necessary.

[3] Chatterjee and Ravikumar (1999) demonstrate that, in an AK model with a subsistence level of consumption, an asymmetric distribution of initial wealth could affect inequality dynamics. Since our focus is not on this aspect, we exclude this channel.

$k_i(0) > 0$. Standard optimal control theory implies that the consumption profile satisfies the budget constraint and the following conditions:

$$\frac{\dot{c}_i}{c_i} = \left(\frac{c_i - c_{min}}{c_i}\right)\epsilon_i(r - \rho_i) \tag{3}$$

and

$$\lim_{t\to\infty} e^{-\rho_i t}(c_i - c_{min})^{-\frac{1}{\epsilon_i}} k_i = 0. \tag{4}$$

Condition (3) is the Euler equation, governing optimal lifetime consumption. The IES for agent i is time-varying:

$$IES_i(t) \equiv \left(\frac{c_i - c_{min}}{c_i}\right)\epsilon_i, \tag{5}$$

which is strictly positive and bounded above by ϵ_i. As c_i increases, $IES_i(t)$ rises and approaches ϵ_i. Condition (4) is the transversality condition, ensuring rational terminal values.

2.2 Production

In the standard AK model, the aggregate production function is: $y = Ak$, where $A > 0$ is the productivity parameter net of depreciation. Profit maximization leads to:

$$r = A. \tag{6}$$

3 Asymptotic Balanced Growth Path and Transitional Dynamics

This section characterizes the dynamic equilibrium of the economy. Given initial capital levels $k_H(0)$ and $k_L(0)$, the dynamic equilibrium paths for individual capital and consumption are:

$$k_i(t) = \frac{c_{min}}{A} + \left[k_i(0) - \frac{c_{min}}{A}\right]e^{\epsilon_i(A-\rho_i)t}, \tag{7}$$

and

$$c_i(t) = c_{min} + \Big[A - \epsilon_i(A - \rho_i)\Big]\left[k_i(0) - \frac{c_{min}}{A}\right]e^{\epsilon_i(A-\rho_i)t}, \tag{8}$$

where $i \in \{H, L\}$ and $t \geq 0$. The detailed mathematical derivations for these equations are provided in Appendix A in supplementary material.

We impose certain parameter restrictions to ensure clarity. First, we specify that $\epsilon_i \in (0, 1)$, consistent with the analysis by Nakamura (2014). Second, for perpetual growth, initial capital and productivity must be sufficiently large so that individual income covers subsistence, $Ak_i(0) > c_{min}$, and the marginal product of capital exceeds the rate of time preference, $A > \rho_i$. These conditions lead to the following assumption:

Assumption 1: Let $k_i(0) = k_j(0) > \frac{c_{min}}{A} \geq 0$, $A > \rho_i$, and $\epsilon_i \in (0, 1)$ for all $i, j \in \{H, L\}$.

Notably, the aggregate variables c and k are given by $c = \lambda c_H + (1-\lambda)c_L$ and $k = \lambda k_H + (1-\lambda)k_L$.

Unlike the standard AK model, the equilibrium paths (7) and (8) depict transitional dynamics rather than balanced growth paths as long as $c_{min} > 0$. This is due to the non-constant growth rates of capital and consumption over time. Specifically, during the transitional phase, the growth rates are given by:

$$\frac{\dot{k}_i}{k_i} = \frac{\epsilon_i(A-\rho_i)}{1 + \frac{c_{min}e^{-\epsilon_i(A-\rho_i)t}}{Ak_i(0)-c_{min}}} \quad \text{and} \quad \frac{\dot{c}_i}{c_i} = \frac{\epsilon_i(A-\rho_i)}{1 + \frac{c_{min}e^{-\epsilon_i(A-\rho_i)t}}{\left[A-\epsilon_i(A-\rho_i)\right]\left[k_i(0)-\frac{c_{min}}{A}\right]}},$$

for $i \in \{L, H\}$. These expressions can be easily obtained by log-differentiating (7) and (8). Interestingly, as the economy grows, the influence of c_{min} gradually diminishes, leading these growth rates to converge asymptotically to a constant. This convergence illustrates the existence of the Asymptotic Balanced Growth Path (ABGP).

The ABGP emerges as the limiting behavior when the impact of c_{min} becomes negligible. Over time, as c_{min} loses significance, the system asymptotically approaches the ABGP, characterized by stabilized and constant growth rates. This contrasts with the framework of Nakamura (2014), where the economy immediately jumps to a BGP. In our model, however, the presence of subsistence consumption introduces transitional dynamics, leading to a gradual convergence toward the ABGP, thus capturing the influence of subsistence consumption on the economy's growth trajectory.

4 Re-examining Wealth Distribution

In this section, we first characterize the dynamics of wealth distribution. We then revisit Nakamura (2014) with a detailed analysis. Finally, we highlight our findings on the impact of subsistence consumption on wealth inequality, focusing on both transitional and long-term effects.

4.1 Wealth Distribution Dynamics

Given $k = \lambda k_H + (1-\lambda)k_L$, we start with log-differentiating this expression: $\frac{\dot{k}}{k} = \lambda \frac{k_H}{k}\frac{\dot{k}_H}{k_H} + (1-\lambda)\frac{k_L}{k}\frac{\dot{k}_L}{k_L}$. Define $s_H \equiv \lambda \frac{k_H}{k}$ and $s_L \equiv (1-\lambda)\frac{k_L}{k}$ as the wealth shares of individuals of type H and L, respectively, noting that $s_H + s_L = 1$. The growth rate of aggregate capital is:

$$\frac{\dot{k}}{k} = s_H \frac{\dot{k}_H}{k_H} + (1-s_H)\frac{\dot{k}_L}{k_L}. \tag{9}$$

Log-differentiating s_H yields: $\frac{\dot{s}_H}{s_H} = \frac{\dot{k}_H}{k_H} - \frac{\dot{k}}{k}$. Also, we know from the previous section that $\frac{\dot{k}_i}{k_i} = \epsilon_i(A-\rho_i)\left[1 + \frac{c_{min}e^{-\epsilon_i(A-\rho_i)t}}{Ak_i(0)-c_{min}}\right]^{-1}$ for $i \in \{H, L\}$. Substituting these into (9) leads to the following lemma describing the dynamics of s_H:

Lemma 1. *Given $s_H(0)$, the wealth share $s_H(t)$ evolves according to*

$$\dot{s_H} = \Omega(t)(s_H(t)) - \Omega(t)(s_H(t))^2 \tag{10}$$

where $\Omega(t) \equiv \left[\frac{\epsilon_H(A-\rho_H)}{1+\frac{c_{min}}{A[k_H(0)-\frac{c_{min}}{A}]e^{\epsilon_H(A-\rho_H)t}}} - \frac{\epsilon_L(A-\rho_L)}{1+\frac{c_{min}}{A[k_L(0)-\frac{c_{min}}{A}]e^{\epsilon_L(A-\rho_L)t}}} \right]$.

4.2 Nakamura (2014) Revisitation

The result presented in Nakamura (2014) is a special case of our model when $c_{min} = 0$. With zero subsistence needs, k_i will strictly increase over time for both types of agents, ensuring that no single type will hold the entire capital stock. Also, without subsistence needs Ω remains time-invariant, given by $\Omega = \epsilon_H(A - \rho_H) - \epsilon_L(A - \rho_L)$. The evolution of the state s_H is described by the differential equation:

$$\dot{s_H} = \Omega s_H - \Omega s_H^2. \tag{11}$$

This equation corresponds to Equation (14) in Nakamura (2014), and as noted in his paper, the result can be derived using this equation.

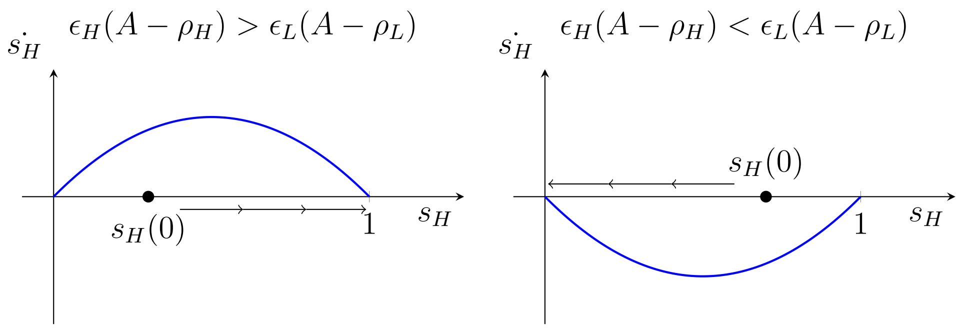

Figure 1: *Phase diagram analysis of the differential equation $\dot{s_H} = \Omega s_H - \Omega s_H^2$ when $c_{min} = 0$. The left plot shows convergence to 1 for $\Omega > 0$, and the right plot shows convergence to 0 for $\Omega < 0$.*

The wealth share dynamics can be analyzed in two primary ways. The first approach involves examining the phase diagram, as shown in Figure 1, which illustrates the trajectory of wealth shares over time depending on the value of Ω. The left panel demonstrates the case where $\Omega > 0$, leading to convergence toward complete dominance by agent of type H. The right panel illustrates the case where $\Omega < 0$, leading to convergence toward zero wealth share for the agent of this type.

Alternatively, a more explicit analysis involves deriving a closed-form solution for the distributional dynamics. Since the differential equation (11) is a Bernoulli equation with a constant coefficient, solving it with an appropriate substitution provides the closed-form solution:

$$s_H(t) = \frac{s_H(0)}{s_H(0) + (1 - s_H(0))e^{-(\eta_H - \eta_L)t}} \tag{12}$$

where $\eta_i = \epsilon_i(A-\rho_i)$ for $i \in \{H, L\}$. This solution confirms that if $(\eta_H - \eta_L) > (<)\ 0$, then $s_H(t) \to 1\ (0)$. In the case where $(\eta_H - \eta_L) = 0$, the wealth share remains time-invariant, with $s_H(t) = s_H(0)$ for all $t \geq 0$. Interestingly, even though both agents accumulate wealth over time, if the impatient group of agents has a sufficiently high IES, their wealth will become overwhelmingly large, though it will not entirely dominate the economy. For detailed derivations, please refer to Appendix B in supplementary material.

Based on the discussion above, we restate Nakamura's result as follows:

Proposition 1. $\big[$*Nakamura (2014)*$\big]$ *Assuming Assumption 1 holds and* $c_{min} = 0$*, the economy immediately jumps to a BGP where the most (least) impatient household will own almost all of the capital in the long run if* $\epsilon_H(A-\rho_H) - \epsilon_L(A-\rho_L) > (<) 0$.

4.3 Subsistence Consumption and Wealth Inequality

Let us now present the main results of this study. Assuming $c_{min} > 0$, the general solution to (10) is given by:

$$s_H(t) = \frac{s_H(0)}{s_H(0) + (1-s_H(0))\left[\frac{1+\Phi e^{-\eta_L t}}{1+\Phi e^{-\eta_H t}}\right] e^{-(\eta_H-\eta_L)t}} \tag{13}$$

where $\Phi = \frac{c_{min}}{A[k_H(0) - \frac{c_{min}}{A}]} = \frac{c_{min}}{A[k_L(0) - \frac{c_{min}}{A}]}$ and $s_H(0) \in (0,1)$ is given. The mathematical derivation of this result is provided in Appendix B in supplementary material. Notably, the expressions in (12) and (13) will be identical when $c_{min} = 0$.

The difference between (13) and (12) arises from the term $\left[\frac{1+\Phi e^{-\eta_L t}}{1+\Phi e^{-\eta_H t}}\right]$. Therefore, it is crucial to analyse this term to understand how it influences the system's dynamics. The following lemma details its properties; see Appendix C in supplementary material for a full proof.

Lemma 2. *Suppose that* $c_{min} > 0$*. The term* $\left[\frac{1+\Phi e^{-\eta_L t}}{1+\Phi e^{-\eta_H t}}\right] \equiv V(t)$ *is well-defined for* $t \in [0, \infty)$ *, with the following properties:*

(i) ***Boundary Behavior:*** $V(0) = 1$ *and* $\lim_{t\to\infty} V(t) = 1$.

(iii) ***Interior Behavior:*** *For* $t \in (0, \infty)$, $V(t)$ *exhibits an upward U-shaped curve if* $\eta_H > \eta_L$*, a U-shaped curve if* $\eta_H < \eta_L$*, and a constant value* $V(t) = 1$ *if* $\eta_H = \eta_L$.

Given Lemma 2, the comparison of the solution paths (12) and (13) leads to the following proposition:

Proposition 2. *Assuming Assumption 1 holds and* $c_{min} > 0$*, subsistence needs influence wealth distribution as follows:*

(i) ***Transitional Dynamics:*** *For any finite period (excluding* $t = 0$*), subsistence needs make wealth distribution more egalitarian if* $\epsilon_H(A-\rho_H) - \epsilon_L(A-\rho_L) \neq 0$.

(ii) ***ABGP:*** *In the long run, the wealth distribution converges to the result stated in Proposition 1.*

To clarify our main findings, two crucial points will be explained. The first point concerns the long-run implications. Proposition 1 ensures that the introduction of subsistence consumption plays no role in determining the long-run wealth distribution along the ABGP. Specifically, the long-run wealth distribution is consistent with Nakamura (2014), where the wealth share dynamics are jointly determined by the rate of TP and the IES, but not by subsistence consumption. As wealth becomes sufficiently large, the impact of subsistence needs diminishes, causing the wealth dynamics in the presence of subsistence consumption to converge to those observed in its absence.

The second point concerns transitional dynamics. During the transition, the rate of TP, IES, and subsistence consumption each play a significant role. Specifically, while the rate of TP and IES determine the direction of wealth distribution, subsistence consumption influences its magnitude. This analysis is divided into two cases.

Case 1: $\epsilon_H(A-\rho_H) - \epsilon_L(A-\rho_L) > 0$

Here, the wealth share of the impatient agent increases over time but remains lower than it would without subsistence needs. When $c_{min} = 0$, equation (8) implies $c_H(0) < c_L(0)$. With subsistence consumption, both agents increase their initial consumption due to a reduced periodic IES. Specifically, the impatient agent's ultimate IES is scaled down more:

$$0 < \left(1 - \frac{c_{min}}{c_H(0)}\right) < \left(1 - \frac{c_{min}}{c_L(0)}\right).$$

Thus, $dc_H(0) > dc_L(0)$ at $t = 0$, leading to decreased savings and a more egalitarian wealth distribution at a small but positive time τ as $s_H(\tau, c_{min} = 0) - s_H(\tau, c_{min} > 0)$ becomes positive. As long as

$$\left(1 - \frac{c_{min}}{c_H(t)}\right) < \left(1 - \frac{c_{min}}{c_L(t)}\right),$$

the gap $s_H(\tau, c_{min} = 0) - s_H(\tau, c_{min} > 0)$ widens. Eventually, as the consumption growth rate of the impatient agent surpasses the patient agent, the gap will start to decrease and converge to zero as t approaches infinity.

Case 2: $\epsilon_H(A-\rho_H) - \epsilon_L(A-\rho_L) < 0$

In this case, the wealth share of the impatient agent decreases over time but remains higher than it would without subsistence needs. When $c_{min} = 0$, equation (8) implies $c_H(0) > c_L(0)$. With subsistence consumption, both agents increase their initial consumption, but the ultimate IES of the patient agent is scaled down more:

$$0 < \left(1 - \frac{c_{min}}{c_L(0)}\right) < \left(1 - \frac{c_{min}}{c_H(0)}\right).$$

Thus, $dc_L(0) > dc_H(0)$ at $t = 0$, leading to a more egalitarian wealth distribution at a small positive time τ as $s_H(\tau, c_{min} = 0) - s_H(\tau, c_{min} > 0)$ becomes negative. As long as

$$\left(1 - \frac{c_{min}}{c_L(t)}\right) < \left(1 - \frac{c_{min}}{c_H(t)}\right),$$

the gap $s_H(\tau, c_{min} = 0) - s_H(\tau, c_{min} > 0)$ widens. However, due to the higher consumption growth rate of the patient agent, the gap will start to decrease and converge to zero as t approaches infinity.

In sum, for any finite $t > 0$, the introduction of subsistence consumption initially promotes a more equal distribution of wealth. This occurs because the impact of subsistence consumption on the IES affects the consumption-saving decisions of both agents, leading to a more egalitarian wealth distribution in the short run. However, subsistence consumption plays a diminishing role as time progresses. Accordingly, the long-term wealth distribution is primarily determined by the rate of TP and the EIS, consistent to the case without subsistence consumption. Figure 2 captures this result.

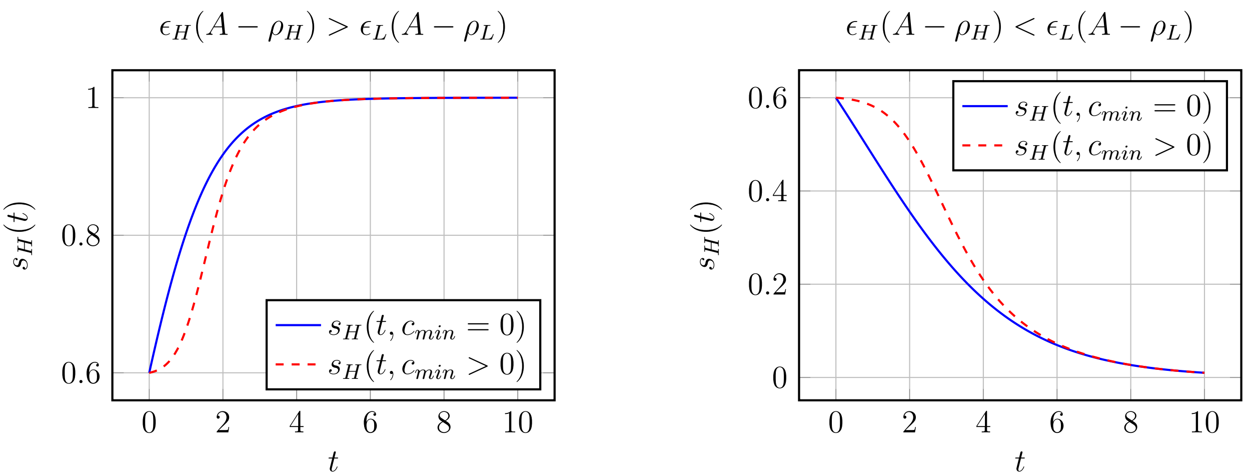

Figure 2: *Time paths of the wealth share for the impatient agents' group.*

These findings should be interpreted cautiously. While it is revealed that wealth inequality becomes less pronounced in a more realistic setting, the inequality will eventually become as large as it would be without subsistence needs. This warrants that policymakers recognize that while inequality may be less observable at the early stages of economic development, it does not imply that such unfairness will never occur.

5 Conclusions

This study examines the effects of subsistence consumption on wealth distribution within an AK model, where agents are differentiated by their rates of TP and IES. We find that introducing a subsistence level of consumption promotes a more equitable wealth distribution during the transitional dynamics. This is primarily due to the time-varying IES induced by subsistence needs, which alters saving behavior over time. However, as wealth accumulates and IES stabilizes, the influence of subsistence consumption diminishes, leaving long-run wealth distribution unaffected by it. Consequently, the asymptotic distribution of wealth aligns with the results of Nakamura (2014), where wealth shares are driven predominantly by TP and IES.

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