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Debt-financed fiscal policy, public capital, and endogenous growth

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Abstract

This study investigates the conflicting effects of a debt-financed fiscal policy in an overlapping generations model with public capital and debt. An accumulation of public capital enhances the production efficiency of private capital, whereas it impedes private capital accumulation by distorting savings allocations through public debt issuance. With a low deficit ratio, the fiscal policy brings steady-state equilibria to an unstable economy. Meanwhile, a debt-financed fiscal policy with a higher deficit ratio causes a fiscal collapse and secular stagnation.

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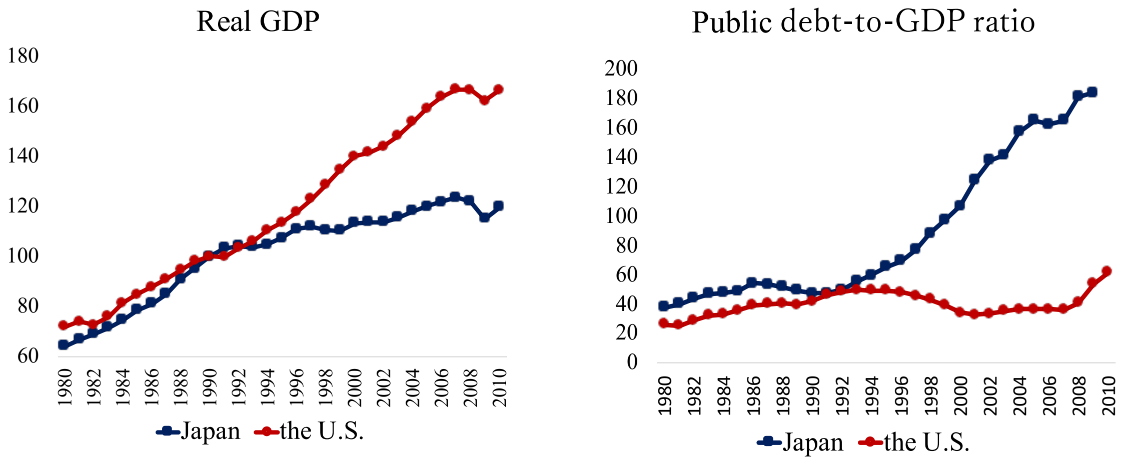
1. Introduction

Today, most advanced economies face a chronic policymaking dilemma: the balance between budget deficits and expansionary fiscal policies. Since the mid-1970s, most OECD countries have faced serious budget deficits and declining economic growth. Figure 1 presents the trends in real GDP and public debt-to-GDP ratio over 1980–2010 in the U.S., marking a relatively stable economic growth, and Japan, suffering from a long recession with the highest public debt-to-GDP ratio among the major advanced economies.



World Bank (2021) World Development Indicators.
OECD (2021) OECD Statistics.

Figure 1. Trends in real GDP and public debt-to-GDP ratio
(Real GDP of both countries is indexed at 100 as of 1990)

Since the seminal paper of Aschauer (1989), productive expenditure on social infrastructure, such as roads, ports, and highways, is widely recognized to contribute to output. According to Modigliani (1961), however, an excessive bond issuance distorts the savings allocation to investment and reduces the economic growth rate by inhibiting capital accumulation. This study aims to present a theoretical analysis of the debt-financed fiscal policy, revealing that a debt-financed public investment does not necessarily have the generally expected positive effects on economic growth.

Most theoretical studies incorporating government bond issuance into their models have focused mainly on long-run fiscal sustainability under a specific policy of interest without necessarily considering the direct impacts of bond issuance on economic growth. Carlberg (1995) and Bräuninger (2005) made the first theoretical contributions on this topic. Subsequently, Yakita (2008), Arai (2011), Teles and Mussolini (2014), Minea and Villieu (2012), and Agénor and Yilmaz (2017) explicitly introduced productive public expenditure and analyzed the long-run dynamics of public debt and growth. Meanwhile, our study investigates the medium-run effects of a debt-financed fiscal policy on endogenous growth in an overlapping generations model, offering a theoretical framework that examines the conditions under which such a policy may cause a serious crowding out, fiscal collapse, and secular stagnation.

2. Model

2.1 Production Sector

Our model assumes an economy that produces a final good with labor, private capital, and productive public capital. Part of government expenditure is accumulated as a production factor and contributes directly to output. Numerous identical firms exist that manufacture a single commodity, and the aggregated production function is presented by the following Cobb-Douglas production function:

$$Y_t = \Phi {Z_t}^{\alpha} {K_t}^{\beta} (EN)^{1-\alpha-\beta}, \tag{1}$$

where Y_t denotes output, Φ, total factor productivity, Z_t, public capital, K_t, private capital, and N, labor. The t index represents the period.[1] We assume that each worker provides one unit of labor inelastically and that N is normalized as one. $\alpha, \beta \in (0,1)$ denote the elasticity of public capital and private capital share, respectively. Following Romer (1986), the average capital per worker, $E \equiv \frac{K_t}{N}$, has a positive spillover effect on labor productivity. Public-private capital ratio is defined as $\Omega \equiv \frac{Z_t}{K_t}$, as in Futagami et al. (1993), and we assume it to be constant considering that the long-run equilibrium level of Z_t remains proportionate to the accumulated private capital, as shown in Yakita (2008). Therefore, the production function can be simplified to an AK-type production function.

$$Y_t = \Phi \Omega^{\alpha} K_t. \tag{2}$$

The goods and factors markets are perfectly competitive. Noting that ${Z_t}^{\alpha}$ and E in production function (1) are externalities for firms, the profit maximization conditions are

$$r_t = \beta \Phi \Omega^{\alpha}, \tag{3}$$

$$w_t = (1-\alpha-\beta) \Phi \Omega^{\alpha} K_t, \tag{4}$$

where r_t and w_t denote the rental price of capital and real wage rate, respectively.

2.2 Household Sector

For the household sector, we assume an overlapping generations model in which individuals

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[1] We assume that one period is approximately 30 years, which is compatible with the overlapping generations model for the household sector.

live for two periods. The representative individual's utility function depends on the consumption per worker in the working and retirement periods, c_t^Y and c_{t+1}^O, respectively.

$$u = (1-\theta)\log c_t^Y + \theta \log c_{t+1}^O, \tag{5}$$

where $\theta \in \left(0, \frac{1}{2}\right)$ denotes an intertemporal weight of utility[2]. Young workers earn wage income, which is partly allocated to consumption, with the remaining being stored as savings. On ageing, individuals do not work; they receive capital income from private and public assets. The return they earn from one asset is equivalent to that from the other under non-arbitrage conditions. The intertemporal budget constraint is $c_t^Y + \frac{c_{t+1}^O}{1+(1-\tau_{t+1})r_{t+1}} = (1-\tau_t)w_t$, where τ_t denotes the constant income tax rate. With utility maximization, savings $s_t = (1-\tau_t)w_t - c_t^Y$ can be represented as follows:

$$s_t = (1-\tau_t)(1-\alpha-\beta)\theta\Phi\Omega^\alpha K_t. \tag{6}$$

2.3 Government Sector

The government balances total revenues and expenditures by adjusting the tax rate τ_t.

$$B_t + \tau_t(Y_t + r_t D_t) = G_t + r_t D_t. \tag{7}$$

The government also spends a share of the national income Y_t as government expenditure G_t, defined as $G_t = gY_t$, where $g \in (0,1)$ is given exogenously. After excluding interest expenses, G_t is divided into productive capital expenditure and nonproductive government consumption, such as social security costs. D_t denotes public debt accumulated in the current period t, and the government pays interest $r_t D_t$ to households. The revenue consists of tax on gross income $Y_t + rD_t$ and a new government bond B_t, defined as $B_t = bY_t$, where $b \in (0,1)$. Defining the public debt-to-capital ratio as $x_t \equiv \frac{D_t}{K_t}$, we obtain $\tau_t = \frac{g-b+\beta x_t}{1+\beta x_t}$, and the savings amount of the whole economy S_t is presented as

$$S_t = s_t N = \frac{\theta(1+b-g)(1-\alpha-\beta)}{1+\beta x_t}\Phi\Omega^\alpha K_t. \tag{8}$$

S_t is divided into public debt and private capital in the subsequent period: $S_t = D_{t+1} + K_{t+1}$.

[2] To consider the endogenous labor supply, we can assume that the representative individual's utility function depends on c_t^Y, c_{t+1}^O, and leisure $l_t \in (0,1)$. The utility maximization problem is: max $u = (1-\theta_1-\theta_2)\log c_t^Y + \theta_1 \log l_t + \theta_2 \log c_{t+1}^O$, $s.t.\ c_t^Y + \frac{c_{t+1}^O}{1+(1-\tau_{t+1})r_{t+1}} = (1-\tau_t)(1-l_t)w_t$, where $\theta_1, \theta_2 \in (0,1)$ and $1-l_t$ is labor supply. By solving this, we obtain $l_t = \theta_1$ and the endogenous labor supply is determined as constant value under the log utility function. Therefore, the main results of our model with inelastically supplied labor still hold even under the overlapping generations model with endogenous labor supply.

2.4 Fiscal Consolidation Policy

We assume that $m \in (0,1)$ denotes the redemption rate of public debt, and the difference between D_{t+1} and $(1-m)D_t$ corresponds to the new government bond B_t. Accordingly, we obtain

$$\frac{D_{t+1}}{D_t} = 1 - m + \frac{b\Phi\Omega^{\alpha}}{x_t}, \tag{9}$$

$$\frac{K_{t+1}}{K_t} = \Phi\Omega^{\alpha}\left[\frac{\theta(1+b-g)(1-\alpha-\beta)}{1+\beta x_t} - b\right] - (1-m)x_t. \tag{10}$$

Transforming the equation $\frac{D_{t+1}}{D_t} = \frac{K_{t+1}}{K_t}$ in the steady state where x_t converges to a certain positive value, we re-define two new functions, $P(x_t, \Omega)$ and $Q(x_t, \Omega)$, as follows:

$$P(x_t, \Omega) \equiv (1-m)(1+x_t) + \frac{b\Phi\Omega^{\alpha}}{x_t}, \tag{11}$$

$$Q(x_t, \Omega) \equiv \Phi\Omega^{\alpha}\left[\frac{\theta(1+b-g)(1-\alpha-\beta)}{1+\beta x_t} - b\right]. \tag{12}$$

$P(x_t, \Omega)$ is a downward convex curve with $\lim_{x_t \to 0} P(x_t, \Omega) = \infty$ and $\lim_{x_t \to \infty} P(x_t, \Omega) = \infty$. $Q(x_t, \Omega)$ is a monotonically decreasing function of x_t with constant intercepts of x_t-axis and P-axis, the latter of which depends on Ω. As Carlberg (1995) and Bräuninger (2005) describe, multiple steady states emerge depending on conditions: a lower stable equilibrium x_L^* and an upper unstable equilibrium x_H^*. When public debt D_t is relatively lower than private capital K_t, x_t converges to x_L^*, and fiscal sustainability can be maintained (Figure 2). If x_t in the initial period is higher than x_H^*, the public debt-capital ratio x_t keeps rising over time and diverges to infinity.

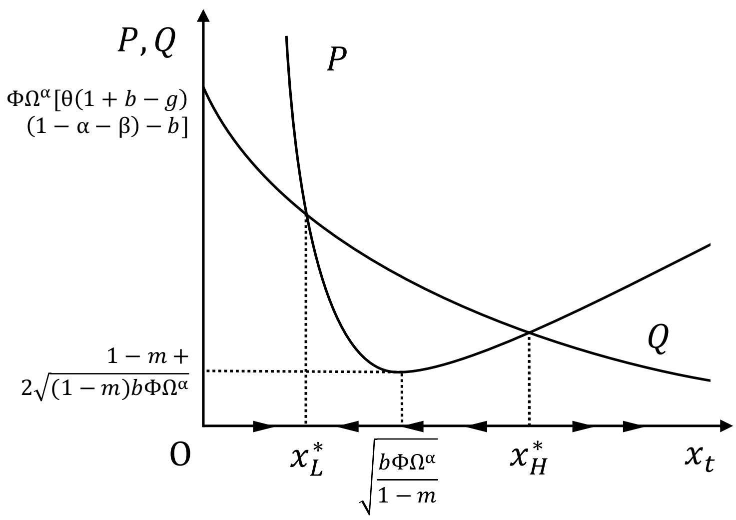

Figure 2. Multiple equilibria and stability

As $\frac{\partial P}{\partial m} < 0$, $\frac{\partial Q}{\partial m} = 0$, and $\frac{\partial\left(\frac{K_{t+1}}{K_t}\right)}{\partial x_t} < 0$, the fiscal consolidation policy leads to sustainable economic growth in this model. A rise in the redemption rate m shifts the function $P(x_t,\Omega)$ downward, while $Q(x_t,\Omega)$ remains unchanged, which lowers x_L^* further and enhances the growth rate. Through a change in savings allocation, the rise in m decreases the share of current savings in refinancing the public debt accumulated in the previous period, thereby indirectly promoting private capital investment.

2.5 Debt-financed Fiscal Policy

While public capital accumulation enhances the production efficiency of private capital and increases output, it also impedes private capital accumulation by distorting savings allocations through public debt issuance. We consider the impact of public investment on economic growth through a change in the public-private capital ratio Ω. Thus,

$$\frac{\partial P}{\partial \Omega} = \frac{\alpha b \Phi}{x_t \Omega^{1-\alpha}} > 0, \tag{13}$$

$$\frac{\partial Q}{\partial \Omega} = \frac{\alpha \Phi}{\Omega^{1-\alpha}}\left[\frac{\theta(1+b-g)(1-\alpha-\beta)}{1+\beta x_t} - b\right]. \tag{14}$$

From Equations (13) and (14), when x_t is close to zero, $\frac{\partial P}{\partial \Omega}$ diverges to infinity, whereas $\frac{\partial Q}{\partial \Omega}$ takes a constant value. Moreover, when x_t is close to the x_t-axis intercept $\tilde{x} = \frac{\theta(1+b-g)(1-\alpha-\beta)-\mathrm{b}}{b\beta}$, $\frac{\partial P}{\partial \Omega}$ takes a constant positive value, whereas $\frac{\partial Q}{\partial \Omega}$ changes to zero. Thus, $\lim_{x_t\to 0}\frac{\partial P}{\partial \Omega} > \lim_{x_t\to 0}\frac{\partial Q}{\partial \Omega}$ and $\lim_{x_t\to \tilde{x}}\frac{\partial P}{\partial \Omega} > \lim_{x_t\to \tilde{x}}\frac{\partial Q}{\partial \Omega}$ hold true. The shift speed of the two functions varies with x_t areas and can be analyzed using the following inequality:

$$\frac{\partial P}{\partial \Omega} \gtreqless \frac{\partial Q}{\partial \Omega} \Leftrightarrow (1+\beta x_t)\left(1+\frac{1}{x_t}\right) \gtreqless \frac{\theta(1+b-g)(1-\alpha-\beta)}{b}. \tag{15}$$

Denoting left and right hand sides by $\Psi_L(x_t)$ and $\Psi_R(b)$, respectively, $\Psi_L(x_t)$ is a downward convex curve with $\lim_{x_t\to 0}\Psi_L(x_t) = \infty$ and $\lim_{x_t\to \infty}\Psi_L(x_t) = \infty$, with Ψ_R being a constant positive value. Let x_t-coordinates satisfying $\Psi_L(x_t) = \Psi_R(b)$ be $\underline{x}(b)$ and $\overline{x}(b)$ ($\underline{x} < \overline{x}$). As Figure 3 shows, in the range of $x_t \in (0,\underline{x})$ and $x_t \in (\overline{x},\infty)$, $P(x_t,\Omega)$ shifts upward faster with public investment than $Q(x_t,\Omega)$ does, and vice versa in the range of $x_t \in (\underline{x},\overline{x})$. In addition, as $\frac{\partial \Psi_R(b)}{\partial b} = -\frac{\theta(1-g)(1-\alpha-\beta)}{b^2} < 0$, the rise in b shifts $\Psi_R(b)$ downward, narrowing the range for $\frac{\partial P}{\partial \Omega} < \frac{\partial Q}{\partial \Omega}$, before eventually making it disappear.

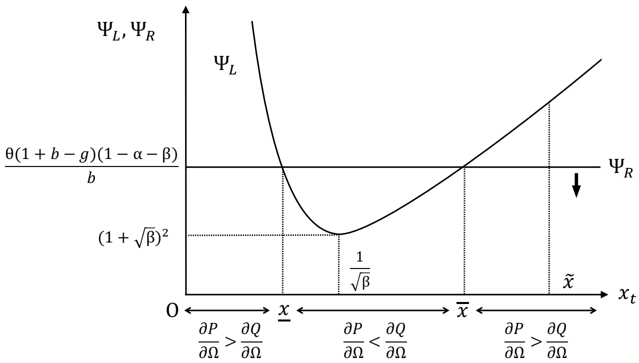

Figure 3. The shift speed's variation

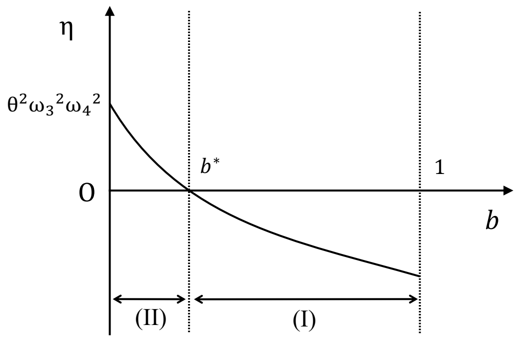

Figure 4. The discriminant equation

Thus, the range of $x_t \in (\underline{x}, \overline{x})$ differs depending on the level of b, and the discriminant equation $\eta(b)$ for $\Psi_L(x_t) = \Psi_R(b)$ is given as follows:

$$\eta(b) = [\omega_1{}^2 - 2\theta\omega_2\omega_3 + \theta^2\omega_3{}^2]b^2 - 2[\theta\omega_2\omega_3\omega_4 - \theta^2\omega_3{}^2\omega_4]b + \theta^2\omega_3{}^2\omega_4{}^2, \quad (16)$$

where $\omega_1 = 1-\beta$, $\omega_2 = 1+\beta$, $\omega_3 = 1-\alpha-\beta$, and $\omega_4 = 1-g$; ω_1, ω_3, and $\omega_4 \in (0,1)$, and $\omega_2 > 1$. From Equation (16), $\eta(0) = \theta^2\omega_3{}^2\omega_4{}^2 > 0$ and $\left.\frac{\partial \eta}{\partial b}\right|_{b=0} = -2\theta\omega_3\omega_4[\omega_2 - \theta\omega_3] < 0$. With $\omega_1{}^2 < \theta\omega_3(1+\omega_4)[2\omega_2 - \theta\omega_3(1+\omega_4)]$[3], we determine that both $\eta(1) < 0$ and $\frac{\partial \eta}{\partial b}|_{b=1} < 0$ hold true. Therefore, in the range of $b \in (0,1)$, the discriminant equation $\eta(b)$ is a monotonically decreasing function of b and crosses the b-axis at $b^* = \frac{\theta\omega_3\omega_4}{\omega_2 - \theta\omega_3 + 2\sqrt{\beta}}$. Figure 4 suggests that when b lies in the range of $b \in (0, b^*)$, the region for $\frac{\partial P}{\partial \Omega} < \frac{\partial Q}{\partial \Omega}$ exists. Meanwhile, when b is higher than the threshold b^*, $\frac{\partial P}{\partial \Omega} > \frac{\partial Q}{\partial \Omega}$ holds true in the whole area of x_t.

Based on the level of b, an economy can be categorized as Type (I) or Type (II). When b is higher than a certain threshold b^*, the economy is classified as Type (I), "public debt acceleration type." With public investment, the public debt ratio $\frac{D_{t+1}}{D_t}$ further exceeds the private capital ratio $\frac{K_{t+1}}{K_t}$, which produces a greater divergence between $P(x_t, \Omega)$ and $Q(x_t, \Omega)$. Consequently, the rise in x_t accelerates over time, making it impossible for fiscal stability to be sustained (Figure 5). As the share of D_{t+1} in the savings increases, less savings are allocated to private capital K_{t+1} in the following period. This implies that this economy faces a serious crowding out, and the rise in x_t directly hinders growth, with private capital

[3] In the numerical simulation in Section 3 based on the U.S. and Japan, this assumption holds true.

K_t not accumulating sufficiently. In this type of economy, a debt-financed fiscal policy aggravates fiscal sustainability and hinders economic growth.

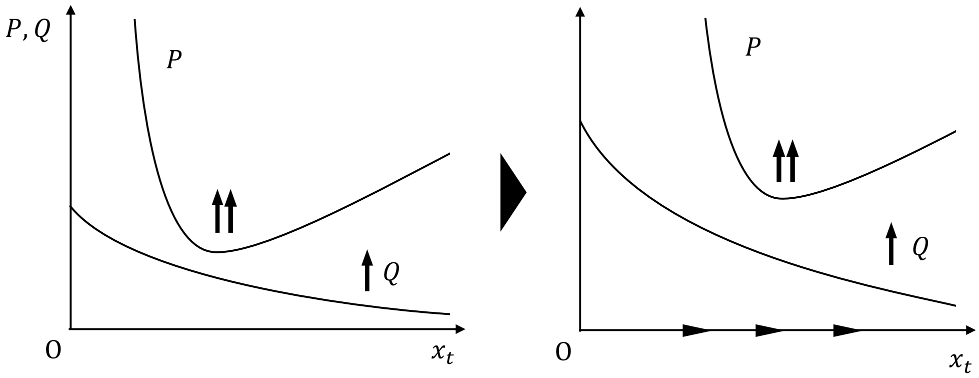

Figure 5. Public debt acceleration-type economy

Meanwhile, it is only in the case of Type (II), "private capital-acceleration type," with $b \in (0, b^*)$ that a debt-financed fiscal policy enhances economic growth and stabilizes the economy. We suppose that the economy has no steady state during the initial period. Although the rise in Ω shifts both functions upward, the shift speed of both functions depends on the x_t areas. With x_t sufficiently greater than zero or smaller than the intercept $\tilde{x}$, the shift speed of $Q(x_t, \Omega)$ is higher than that of $P(x_t, \Omega)$. By contrast, when x_t is close to zero or the intercept, $\frac{\partial P}{\partial \Omega} > \frac{\partial Q}{\partial \Omega}$ holds true, as stated above. Therefore, this twisted shift speed diminishes the gap between the two functions in the middle area of x_t and increases it in other areas. Consequently, the two functions eventually cross, and multiple steady states appear even with no equilibrium in the initial period.

Moreover, the economy simultaneously achieves fiscal stability and economic growth. Figure 6 suggests that a continuous rise in Ω gradually decreases x_L^* and increases x_H^*, which enlarges the stable range of x_t and generates a fiscally more solid economy. In addition, because a lower level of x_L^* increases $\frac{K_{t+1}}{K_t}$, a higher public investment facilitates economic growth through private capital accumulation.

Finally, as $\frac{\partial P}{\partial m} < 0$ and $\frac{\partial Q}{\partial m} = 0$, when the economy is classified as Type (I) with $\frac{\partial P}{\partial \Omega} > \frac{\partial Q}{\partial \Omega}$, the government is expected to execute a fiscal consolidation policy, as well as a public investment to direct the economy to the private capital acceleration type. In the model with public capital and debt, unlike Bräuninger (2005), the level of relative public capital and the redemption rate have significant effects on determining multiple equilibria, fiscal stability, and economic growth.

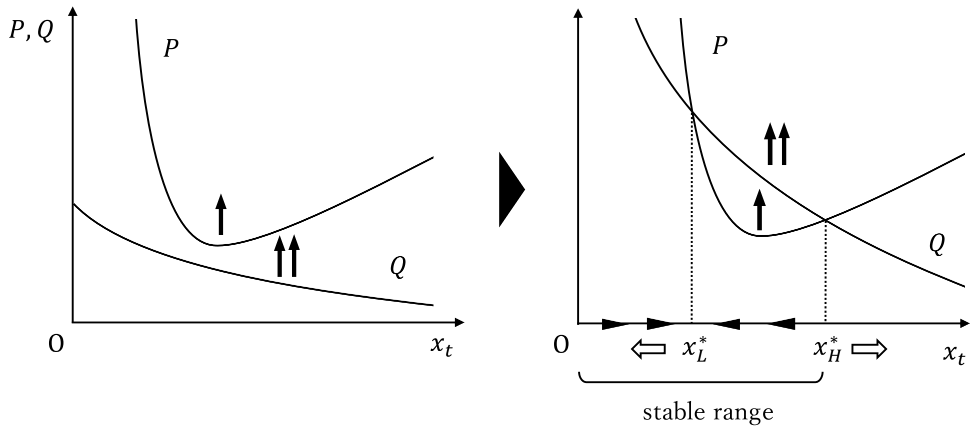

Figure 6. Private capital acceleration-type economy

3. Numerical Simulations

Table 1. Calibration of variables

Variables	The U.S.	Japan
$\boldsymbol{\alpha}$	0.25	
$\boldsymbol{\beta}$	0.25	
$\boldsymbol{b}$	0.03	0.08
$\boldsymbol{g}$	0.2	
$\boldsymbol{\theta}$	0.4	

In this section, we present numerical simulations for two contrasting advanced economies, the U.S. and Japan. Table 1 quantifies the variables, with b and g calculated based on fiscal data over 2012–2019, excluding the fiscal impact of the global financial crisis and the COVID-19 pandemic[4].

When $b = 0.08$, reflecting the severe Japanese fiscal situation, the economy is categorized as Type (I), the public debt acceleration type with no steady state; the burden of the new public bond issue hinders private capital accumulation, and the public debt-to-capital ratio x_t keeps rising over time and diverges to infinity.

Next, when $b = 0.03$, reflecting the fiscal situation of the more stable U.S. economy, the result is a Type (II) economy, the private capital acceleration type. Table 2 presents the results of the numerical simulation of the U.S. economy: as the relative public capital Ω

[4] As both Japan and the U.S. show upward trends in public debt accumulation, the redemption rate m is assumed to be zero, and total factor productivity Φ is set to 15 in this simulation.

increases, multiple equilibria are generated, and the economic growth rate increases with the wider stable range of x_t.

Table 2. Results of numerical simulation of the U.S. economy

Ω	x_L^*	x_H^*	Growth rate
7.5	0.610	0.692	2.69%
8.0	0.552	0.774	2.92%
8.5	0.524	0.825	3.06%
9.0	0.504	0.866	3.17%

In addition, the threshold level of deficit ratio b^*, which determines Type (I) or Type (II), depends on α, β, g, and θ. Table 3 shows that the rise in α, β, and g reduces the stable range of Type (II). As α and β (the elasticity of public capital and private capital share, respectively) increase, less labor share reduces room for sustainable government bond issuance. Similarly, the higher government expenditure ratio g increases the equilibrium tax rate, shrinks disposable income, and leads to the unstabilized public debt acceleration type economy. Meanwhile, the rise in the intertemporal weight θ enables sustainable bond insurance through more saving and, thus, increases the level of b^* and augments fiscal stability. Therefore, the government is expected to enhance household affordability for government bonds by directly or indirectly adjusting the aforementioned variables, as well as to maintain a lower deficit ratio for sustainable economic growth.

Table 3. Threshold values of deficit ratio b^*

	$\boldsymbol{\alpha}$				$\boldsymbol{\beta}$			
	0.15	0.20	0.25	0.30	0.15	0.20	0.25	0.30
$\boldsymbol{b^*}$	0.096	0.087	0.078	0.070	0.114	0.094	0.078	0.065

	$\boldsymbol{g}$				$\boldsymbol{\theta}$			
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
$\boldsymbol{b^*}$	0.088	0.083	0.078	0.073	0.057	0.068	0.078	0.089

4. Conclusion

This study theoretically analyzes a debt-financed fiscal policy and its impact on private capital accumulation and economic growth. In the model, a certain share of government spending is accumulated in the economy as public capital, which directly contributes to output as a production factor. While an increase in public capital enhances the production efficiency of private capital and increases output, it also impedes private capital accumulation by distorting savings allocation through public debt issuance.

Contrary to popular expectations, public investment can have both positive and negative impacts on economic growth when debt-financed. With a relatively low deficit ratio, the fiscal policy promotes capital accumulation and brings the economy to a steady state, even when there is no equilibrium in the initial period. Conversely, when the deficit ratio exceeds a threshold, public investment has serious crowding-out effects and decreases the growth rate. This implies that a fiscal policy that depends heavily on public debt issuance will not promote economic growth but rather cause a fiscal collapse and secular stagnation. The analysis of a threshold of deficit ratio suggests that a policy to enhance household affordability allows for sustainable government insurance and economic growth. These findings make novel contributions to Bräuninger (2005) and Yakita (2008) as the economic consequences of a debt-financed public investment differ depending on the level of deficit ratio, and public capital accumulation can be a critical factor that guarantees the existence of steady-state equilibria.

Appendices

Table A1. Annual fiscal data of the U.S.

(In millions of dollars)

Fiscal year	2012	2013	2014	2015	2016	2017	2018	2019
(1)Nominal GDP	16,197,007	16,784,849	17,527,164	18,238,301	18,745,076	19,542,979	20,611,861	21,433,225
(2)Outlays	3,526,563	3,454,881	3,506,284	3,691,850	3,852,616	3,981,630	4,109,044	4,446,956
(3)Reciepts	2,449,990	2,775,106	3,021,491	3,249,890	3,267,965	3,316,184	3,329,907	3,463,364
(4)Interest payments (net)	220,408	220,885	228,956	223,181	240,033	262,551	324,975	375,158
Government spending/GDP [(2)-(4)]/(1)	20%	19%	19%	19%	19%	19%	18%	19%
Deficit ratio [(2)-(3)]/(1)	7%	4%	3%	2%	3%	3%	4%	5%

The Office of Management and Budget, The White House (2021)
World Bank (2021)

Table A2. Annual fiscal data of Japan

(In 100 millions of yen)

Fiscal year	2012	2013	2014	2015	2016	2017	2018	2019
(1)Nominal GDP	4,943,698	5,072,552	5,182,352	5,327,860	5,368,508	5,475,480	5,481,216	5,524,997
(2)Outlays	903,339	926,115	958,823	963,420	1,000,087	974,547	977,128	1,014,571
(3)Interest payments	98,546	99,027	101,319	101,472	88,278	91,605	90,275	88,502
(4)Public debts	442,440	428,510	412,500	368,630	371,820	343,698	336,922	326,605
Government spending/GDP [(2)-(3)]/(1)	16%	16%	17%	16%	17%	16%	16%	17%
Deficit ratio (4)/(1)	9%	8%	8%	7%	7%	6%	6%	6%

Economic and Social Research Institute, Cabinet Office, Government of Japan (2021)

Table A3. Calibration of variables

Variables	The U.S.	Japan	Grounds
$\boldsymbol{\alpha}$	0.25		Bom and Ligthart (2014) conducted a cross-sectional survey analysis of 67 empirical studies over 1983–2008, finding that the average elasticity of public capital was 0.268 when considering spillover effects across regions over time.
$\boldsymbol{\beta}$	0.25		The capital share in factor values in major industrialized countries, including the U.S. and Japan, has remained stable at approximately one-thirds over 2013–2017.
$\boldsymbol{b}$	0.03	0.08	The deficit ratio is assumed to be 0.03 and 0.08 with the fiscal conditions in the U.S. and Japan, respectively, considering both countries' SNA statistics over 2012–2019.
$\boldsymbol{g}$	0.2		The average government spending–output ratio in both countries is approximately 20% over the 2012–2019 period.
$\boldsymbol{\theta}$	0.4		Evans and Sezer (2004) estimated the long-term time preference rates for the U.S., Japan, and Australia to be 1.5 %, considering disaster risk. Considering that one period lasts 30 years, the estimated time preference rate is $\left(\frac{1}{1.015}\right)^{30} \approx 0.640$. The discount factor satisfies $1: 0.640 = 1 - \theta: \theta$ and can be estimated to be approximately 0.4.

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