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Per unit versus ad valorem taxes under strategic bilateral trade

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Abstract

This paper compares ad valorem and per-unit taxes in a bilateral market where all traders have market power. To do so, we use a simple prototype of strategic market games, namely bilateral oligopoly models, and show that ad valorem taxation welfare-dominates per-unit taxation under strategic bilateral trade. Moreover, ad valorem and per-unit taxes have qualitatively different effects on strategic equilibrium offers.

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1 Introduction

The comparison between per-unit and ad valorem taxes remains one of the oldest issues in public finance. It is well-known that per-unit and ad valorem taxes are equivalent in a perfectly competitive environment. This equivalence breaks down under imperfect competition (Suits and Musgrave, 1953). Indeed, Suits and Musgrave (1953) show that ad valorem taxation is welfare superior to per-unit taxation in a monopolistic market. Since Suits and Musgrave (1953), literature has developed on the comparison of these two taxes under various market structures, such as monopoly markets (Aiura and Ogawa, 2019; Blackorby and Murty, 2007; Skeath and Trandel, 1994), and oligopoly markets (Wang et al., 2018; Kotsogiannis and Serfes, 2014; Anderson et al., 2001; Delipalla and Keen, 1992). However, there is no consensus in the literature concerning the welfare superiority of ad valorem taxation over per-unit taxation. While some studies, such as those by Anderson et al. (2001), Skeath and Trandel (1994), and Delipalla and Keen (1992), argue that ad valorem taxation may dominate per-unit taxation in welfare terms, other works, including those by Grazzini (2006), Kotsogiannis and Serfes (2014), Wang et al. (2018), and Chu and Wu (2023), show that per-unit taxation may dominate ad valorem taxation in welfare terms.

Specifically, using a general equilibrium model in which *few* traders behave *strategically* and a *large* number of traders behave *competitively*, Grazzini (2006) shows that per-unit taxation welfare dominates ad valorem taxation when the number of competitive traders is sufficiently high compared to the number of strategic traders. In the model of Grazzini (2006), these two kinds of taxes are compared in exchange economies with production where the behavior of traders is asymmetric, as *few* traders have market power while a *large* number of traders are price takers.

In this paper, we propose to extend Grazzini's contribution by considering only a pure exchange economy and strategic behavior between *all* traders. In contrast to Grazzini (2006), traders on both market sides can interact strategically and have market power. To simplify, we consider a simple prototype of strategic market games called *bilateral oligopoly model*, first introduced by Gabszewicz and Michel (1997).[1] In a bilateral oligopoly, there are two types of traders, each type being endowed with only one commodity. All traders behave strategically by manipulating the price by restricting the supplies they send to the market. Thus, each agent decides how much of a good to trade on the market to acquire the other good and maximize its utility. There is a trading post that aggregates all the agents' offers and allocates the amounts exchanged to each agent in proportion to its offer. The resulting equilibrium is a non-cooperative equilibrium, i.e., a Cournot-Nash equilibrium (CNE thereafter). The above-mentioned literature focuses on the comparison of per-unit and ad valorem taxes in economies operating under partial equilibrium or general equilibrium with imperfect competition in which strategic competition occurs within a given sector, i.e., competition is modeled at the sectoral level. In this paper, we are considering strategic interactions that occur *within* and *between* on both market sides. Within this framework, we introduce two kinds of taxes, *namely* per-unit and ad valorem taxes. In both cases, the product of the tax is used to finance a given public expenditure. We show that ad valorem and

[1] Strategic market games have been introduced by Shapley and Shubik (1977).

per-unit taxes have qualitatively different effects on strategic equilibrium offers, and ad valorem taxation welfare dominates per-unit taxation under neutral revenue.

The remainder of this paper is organized as follows. In Section 2, we introduce the model. In Section 3, we study the effects of per-unit and ad valorem taxes. In Section 4, we compare these two kinds of taxes. In Section 5, we conclude.

2 The model

We consider a pure exchange economy named $\mathcal{E}$, with two types of agents, labeled 1 and 2, and two *divisible commodities*, labeled X and Y. Let p_X(resp. p_Y) be the unit price of commodity X (resp. Y), with $p_Y = 1$ (commodity Y is the numeraire). Let $\mathcal{I}_1$ and $\mathcal{I}_2$ be the set of agents type 1 and 2 respectively, with $\mathcal{I}_1 = \{1, \ldots, n\}$ and $\mathcal{I}_2 = \{n+1, \ldots, 2n\}$, where $\mathcal{I}_1 \cap \mathcal{I}_2 = \emptyset$. The endowments of each type of trader are given by:

$$w_i = \begin{cases} (1,0), & i \in \mathcal{I}_1; \\ (0,1), & i \in \mathcal{I}_2. \end{cases} \tag{1}$$

Following Grazzini (2006), all traders have the same utility function defined by:

$$u_i(x_i, y_i) = x_i y_i, \tag{2}$$

To $\mathcal{E}$, we associate the bilateral oligopoly game Γ. The traders' strategy consists of offering only a fraction of the commodity they initially hold. Let q_i (resp. b_i) be the amount or supply of commodity X (resp. Y) that the trader of type 1 (resp. type 2) offers in exchange for commodity Y (resp. X). Therefore, the strategy sets are given by:

$$\mathcal{Q}_i = \{q_i \in \mathbb{R}_+ : 0 \leq q_i \leq 1\}, i \in \mathcal{I}_1; \tag{3}$$

$$\mathcal{B}_i = \{b_i \in \mathbb{R}_+ : 0 \leq b_i \leq 1\}, i \in \mathcal{I}_2. \tag{4}$$

Given a strategy profile $(\mathbf{q}, \mathbf{b}) = (q_1, \ldots, q_n; b_{n+1}, \ldots, b_{2n})$, with $(\mathbf{q},\mathbf{b}) \in \mathcal{Q} \times \mathcal{B}$ where $\mathcal{Q} = \prod_{i=1}^{n} \mathcal{Q}_i$ and $\mathcal{B} = \prod_{i=n+1}^{2n} \mathcal{B}_i$, the market clearing price satisfies the following condition: $\sum_{k \in \mathcal{I}_2} b_k = p_X \sum_{k \in \mathcal{I}_1} q_k$, which may be written by:

$$p_X = \frac{\sum_{k \in \mathcal{I}_2} b_k}{\sum_{k \in \mathcal{I}_1} q_k}. \tag{5}$$

The resulting allocations are given by:

$$(x_i, y_i) = \left(1 - b_i, \frac{\sum_{k \in \mathcal{I}_2} b_k}{\sum_{k \in \mathcal{I}_1} q_k} q_i\right), i \in \mathcal{I}_1; \tag{6}$$

$$(x_i, y_i) = \left(\frac{\sum_{k \in \mathcal{I}_1} q_k}{\sum_{k \in \mathcal{I}_2} b_k} b_i, 1 - b_i\right), i \in \mathcal{I}_2. \tag{7}$$

Then, the payoffs are defined by $\pi_i : \mathcal{Q} \times \mathcal{B} \to \mathbb{R}_+, \pi_i\left(q_i, \mathbf{q}_{-i}; \mathbf{b}\right) = u_i\left(1 - q_i, p_X q_i\right), i \in \mathcal{I}_1$, and $\pi_i : \mathcal{Q} \times \mathcal{B} \to \mathbb{R}_+, \pi_i\left(\mathbf{q}; b_i, \mathbf{b}_{-i}\right) = u_i\left(\frac{1}{p_X} b_i, 1 - b_i\right), i \in \mathcal{I}_2$ and are given by: $\pi_i\left(q_i, \mathbf{q}_{-i}; \mathbf{b}\right) = u_i\left(1 - b_i, \frac{\sum_{k \in \mathcal{I}_2} b_k}{\sum_{k \in \mathcal{I}_1} q_k} q_i\right)$, $i \in \mathcal{I}_1$ and by $\pi_i\left(\mathbf{q}; b_i, \mathbf{b}_{-i}\right) = \left(\frac{\sum_{k \in \mathcal{I}_1} q_k}{\sum_{k \in \mathcal{I}_2} b_k} b_i, 1 - b_i\right)$, $i \in \mathcal{I}_2$.

The strategy profile $(\tilde{\mathbf{q}}, \tilde{\mathbf{b}}) \in \prod_{i=1}^{n} \mathcal{Q}_i \times \prod_{i=n+1}^{2n} \mathcal{B}_i$, is a Cournot-Nash equilibrium of the game Γ, if for each trader $i \in \mathcal{I}_1$, $\pi_i(\tilde{q}_i, \tilde{\mathbf{q}}_{-i}, \tilde{\mathbf{b}}) \geq \pi_i(q_i, \tilde{\mathbf{q}}_{-i}, \tilde{\mathbf{b}})$, $\forall q_i \in \mathcal{Q}_i$, and for each trader $i \in \mathcal{I}_2$, $\pi_i(\tilde{\mathbf{q}}, \tilde{b}_i, \tilde{\mathbf{b}}_{-i}) \geq \pi_i(\tilde{\mathbf{q}}, b_i, \tilde{\mathbf{b}}_{-i})$, $\forall b_i \in \mathcal{B}_i$.

3 The effects of taxes

We now turn to studying two taxes: per-unit taxation and ad-valorem taxation.

First, consider that, when exchange takes place, a uniform per-unit tax $\nu \in (0,1)$ is levied on the supply of goods X and Y. Given a vector of strategies $(\mathbf{q{,}b}) \in \prod_{i\in\mathcal{I}_1} \mathcal{Q}_i \times\prod_{i\in\mathcal{I}_2} \mathcal{B}_i$ and a uniform per unit tax ν, a new market price p_X is determined, and the resulting post-tax allocation is given by $(x_i, y_i) = (1 - q_i, p_X q_i - \nu q_i)$, for trader type 1, and by $(x_i, y_i) = \left(\frac{1}{p_X} b_i - \nu b_i, 1 - b_i\right)$, for trader type 2. Let Γ_ν be the new strategic market game. The product of the tax is used to finance some exogenous government expenditure and is equal to $R_\nu = \nu \sum_{k\in\mathcal{I}_1} q_k + \nu \sum_{k\in\mathcal{I}_2} b_k$. From (5), the allocations are given by: $(\tilde{x_i}, \tilde{y_i}) = \left(1 - q_i, \frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} q_k} q_i - \nu q_i\right)$, for trader type 1 and by $(\tilde{x_i}, \tilde{y_i}) = \left(\frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k} b_i - \nu b_i, 1 - b_i\right)$, for trader type 2. Then, the payoffs are given by:

$$\pi_i(q_i, q_{-i}, b_i, \nu) = (1 - q_i) \left(\frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} \S q_k} q_i - \nu q_i\right), \; i \in \mathcal{I}_1; \tag{8}$$

$$\pi_i(q_i, b_i, b_{-i}, \nu) = \left(\frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k} b_i - \nu b_i\right) (1 - b_i), \; i \in \mathcal{I}_2. \tag{9}$$

Second, consider that, when exchange takes place, a uniform ad valorem tax $\tau \in (0,1)$ is levied on the supply of goods X and Y. Given a vector of strategies $(\mathbf{q{,}b}) \in \prod_{i\in\mathcal{I}_1} \mathcal{Q}_i \times\prod_{i\in\mathcal{I}_2} \mathcal{B}_i$, a new market price p_X is determined. Trader i's revenue from sales obtains as $p_X(1-\tau)q_i = \frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} q_k}(1-\tau)q_i$, for each trader type 1, and as $\frac{1}{p_X}(1-\tau)b_i = \frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k}(1-\tau)b_i$, for each trader type 2. The resulting post-tax allocation is given by $(x_i, y_i) = (1 - q_i, p_X(1 - \tau)q_i)$, for trader type 1, and by $(x_i, y_i) = \left(\frac{1}{p_X}(1 - \tau)b_i, 1 - b_i\right)$, for trader type 2. Let Γ_τ be the new strategic market game. The product of the tax is used to finance some exogenous government expenditure and is equal to $R_\tau = \tau \sum_{k\in\mathcal{I}_1} q_k + \tau \sum_{k\in\mathcal{I}_2} b_k$. From (5), the allocations are given by: $(\tilde{x_i}, \tilde{y_i}) = \left(1 - q_i, \frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} q_k}(1 - \tau)q_i\right)$, for trader type 1 and by $(\tilde{x_i}, \tilde{y_i}) = \left(\frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k}(1 - \tau)b_i, 1 - b_i\right)$, for trader type 2. Then, the payoffs are given by:

$$\pi_i(q_i, q_{-i}, b_i, \tau) = (1 - q_i) \left(\frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} q_k} q_i(1 - \tau)\right), \; i \in \mathcal{I}_1; \tag{10}$$

$$\pi_i(q_i, b_i, b_{-i}, \tau) = \left(\frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k} b_i(1 - \tau)\right) (1 - b_i), \quad i \in \mathcal{I}_2. \tag{11}$$

Let us now turn to the computation of the CNE.

Proposition 1. The CNE with per unit and ad valorem taxes is given respectively by:

$$\tilde{q}(\nu) = \tilde{b}(\nu) = \frac{n-1-n\nu}{n(1-\nu)+n-1-n\nu}; \tag{12}$$

$$\tilde{q}(\tau) = \tilde{b}(\tau) = \frac{n-1}{2n-1}, \tag{13}$$

where $\nu < \frac{n-1}{n}$.

Proof. See appendix A

From (12), we deduce that when $\nu = 0$, the strategic offers under both tax mechanisms coincide. However, for $\nu \in (0,1)$, strategic offer under ad valorem tax is higher than strategic offer under per unit tax. In addition, (12) shows that strategic supply decreases with per-unit tax, and depends non-linearly on it, i.e., $\frac{\partial \tilde{q}(\nu)}{\partial \nu} = \frac{\partial \tilde{b}(\nu)}{\partial \nu} = -\frac{n}{(n(1-\nu)+n-1-n\nu)^2} < 0$. In contrast, (13) shows that the ad valorem tax does not affect strategic equilibrium offers, i.e., $\frac{\partial \tilde{q}}{\partial \tau} = \frac{\partial \tilde{b}}{\partial \tau} = 0$.

We now determine tax revenue at the CNE with taxation.

From proposition 1, total tax revenue under per-unit and ad valorem taxes is given respectively by:

$$R_\nu = \nu \sum_{k \in \mathcal{I}_1} q_k + \nu \sum_{k \in \mathcal{I}_2} b_k = \frac{2n\nu(n-1-n\nu)}{n(1-\nu)+n-1-n\nu}; \tag{14}$$

$$R_\tau = \tau \sum_{k \in \mathcal{I}_1} q_k + \tau \sum_{k \in \mathcal{I}_2} b_k = \frac{2n\tau(n-1)}{2n-1}. \tag{15}$$

We finally determine the allocations and payoffs for each type of trader.

Under the per-unit tax, the allocations of each type of trader are given by: $(\tilde{x}_i, \tilde{y}_i) = \left(\frac{n(1-\nu)}{n(1-\nu)+n-1-n\nu)}, \frac{(n-1-n\nu)(1-\nu)}{n(1-\nu)+n-1-n\nu}\right), i \in \mathcal{I}_1$ and $(\tilde{x}_i, \tilde{y}_i) = \left(\frac{(n-1-n\nu)(1-\nu)}{n(1-\nu)+n-1-n\nu}, \frac{n(1-\nu)}{n(1-\nu)+n-1-n\nu)}\right), i \in \mathcal{I}_2$. Then, the payoffs are given by:

$$\tilde{\pi}_i(\nu) = \left(\frac{n(1-\nu)}{n(1-\nu)+n-1-n\nu)}\right)\left(\frac{(n-1-n\nu)(1-\nu)}{n(1-\nu)+n-1-n\nu}\right), i \in \mathcal{I}_1 \cup \mathcal{I}_2. \tag{16}$$

Under ad valorem tax, the allocations of each type of trader are given by: $(\tilde{x}_i, \tilde{y}_i) = \left(\frac{n}{2n-1)}, \frac{(n-1)(1-\tau)}{2n-1}\right)$, for $i \in \mathcal{I}_1$ and $(\tilde{x}_i, \tilde{y}_i) = \left(\frac{(n-1)(1-\tau)}{2n-1}, \frac{n}{2n-1)}\right)$, for $i \in \mathcal{I}_2$. Then, the payoffs are given by:

$$\tilde{\pi}_i(\tau) = \left(\frac{n}{2n-1}\right)\left(\frac{(n-1)(1-\tau)}{2n-1}\right), i \in \mathcal{I}_1 \cup \mathcal{I}_2. \tag{17}$$

4 Per unit taxation versus ad valorem taxation

In this section, we analyze the incidence of an *ad valorem* and *per unit taxes* on agents' welfare. As in Grazzini (2006), we consider the shift from ad valorem to per-unit taxes that raise an equal amount of tax revenue. To perform our analysis, we use *revenue-neutral* per-unit and ad valorem taxes in the Cournot-Nash equilibrium as a basis for comparison. From (14) and (15), the value of τ, which is used as a basis of comparison, is equal to:

$$\tau^* = \frac{\nu(2n-1)(n-1-n\nu)}{(n-1)[2n(1-\nu)-1]}. \tag{18}$$

By substituting (18) into (17), we have:

$$\tilde{\pi}_i^A \equiv \tilde{\pi}_i(\tau^*) = \frac{n(n-1)}{(2n-1)^2}\left(\frac{n(n-1)(1-\nu)+(n-1-n\nu)[(1-\nu)(n-1)-n\nu]}{(n-1)[2n(1-\nu)-1]}\right); \tag{19}$$

$$\tilde{\pi}_i^P \equiv \tilde{\pi}_i(\nu) = \left(\frac{n(1-\nu)}{2n(1-\nu)-1}\right)\left(\frac{(n-1-n\nu)(1-\nu)}{2n(1-\nu)-1}\right). \tag{20}$$

where $\tilde{\pi}_i^A \equiv \tilde{\pi}_i(\tau^*)$ and $\tilde{\pi}_i^P \equiv \tilde{\pi}_i(\nu)$ are respectively the welfare functions under ad valorem and per-unit taxes.

We can compare the welfare level in each taxation mechanism from (19) and (20). Then the utility difference, $\Delta = \tilde{\pi}_i^A - \tilde{\pi}_i^P$ between ad valorem and per-unit taxes is given by:

$$\Delta = \frac{(1-\nu)(n-1)\Phi[A\Psi-\gamma]+A\Psi[\gamma\Phi+n\nu]}{(n-1)\Psi^2} = \frac{\Delta_1}{\Delta_2}. \tag{21}$$

where $A \equiv \frac{n(n-1)}{(2n-1)^2}$, $\Phi = n-1-n\nu$, $\Psi = n(1-\nu)+n-1-n\nu$ and $\gamma = n(1-\nu)$. With $4 \leq n < \infty$, $\Psi = \gamma + \Phi$, and $\nu < \frac{n-1}{n}$. The sign of Δ depends on the sign of Δ_1. So we can rewrite Δ_1 as follows:

$$\begin{aligned}\Delta_1 =&(1-\nu)(n-1)(n-1-n\nu)\left[\frac{n(n-1)}{(2n-1)^2}[2n(1-\nu)-1]-n(1-\nu)\right]+\\ &\frac{n(n-1)}{(2n-1)^2}\left[2n(1-\nu)-1\right]\left[n(1-\nu)(n-1-n\nu)+n\nu\right].\end{aligned} \tag{22}$$

Proposition 2. In strategic bilateral trade with Cournot competition, an ad valorem taxation welfare dominates per unit taxation under a revenue-neutral.

Proof. See appendix B

Proposition 2 shows that ad valorem taxation welfare dominates per-unit taxation when both market sides behave strategically. In contrast to Grazzini (2006), who compares these two kinds of taxes in an asymmetric oligopoly, since as few traders have market power while a large number of traders are price takers, in this paper we consider a symmetric oligopoly since all traders behave strategically by manipulating the price by restricting the supplies they send to the market. In this paper, we consider an economy without production in which all traders have market power. Our result is similar to the existing literature, notably those of Anderson et al. (2001), Skeath and Trandel (1994), Delipalla and Keen (1992) and Suits and Musgrave (1953), who show that when firms engage in Cournot or monopoly competition, an ad valorem taxation welfare dominates per-unit taxation. However, in Delipalla and Keen (1992), the shift from one tax to another leaves total tax payments unchanged at the initial equilibrium price but is not fully revenue-neutral. Likewise, Denicolò and Matteuzzi (2000) and Anderson et al.

(2001) compare these two taxes under identical industry output, which assumes that the average industrial output (total output) under both tax regimes is identical. In this paper, we compare per-unit and ad valorem taxes under *neutral-revenue*, which implies that revenues from ad valorem and per-unit taxes are equal.

5 Conclusion

This paper examines the effects of taxation by analyzing the impacts of per-unit and ad valorem taxes in a general equilibrium setting with strategic interaction, where agents of both market sides have market power and behave strategically. We show that an ad valorem tax welfare dominates per-unit tax under neutral revenue. However, per-unit and ad valorem taxes have qualitatively different effects on strategic equilibrium offers. While ad valorem taxation does not impact strategic offers, the per-unit taxation reduces strategic offers at equilibrium.

One possible extension is to consider the case of Bertrand's competition between oligopolists.

6 Appendix

6.1 Appendix A: Proof of Proposition 1

We determine the CNE by encompassing the two taxation mechanisms. The oligopoly equilibrium is the simultaneous solution to the $2n$ problems given by:

$$\begin{cases} max\ \pi_i(q_i, q_{-i}, b_i, \tau, \nu) = (1-q_i)\left(\frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} q_k}(1-\tau)q_i - \nu q_i\right),\ i \in \mathcal{I}_1; \\ \\ max\ \pi_i(q_i, b_i, b_{-i}, \tau, \nu) = \left(\frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k}(1-\tau)b_i - \nu b_i\right)(1-b_i),\ i \in \mathcal{I}_2. \end{cases} \tag{A1}$$

The CNE satisfies the $2n$ first-order sufficient conditions given by $\frac{\partial \pi_i(q_i,q_{-i},b_i,\tau,\nu)}{\partial q_i} = 0$, for $i \in \mathcal{I}_1$ and by $\frac{\partial \pi_i(q_i,b_i,b_{-i},\tau,\nu)}{\partial b_i} = 0$, for $i \in \mathcal{I}_2$, which can be rewritten as follows:

$$\begin{cases} -\left(\frac{\sum_{k\in\mathcal{I}_2} b_k}{\sum_{k\in\mathcal{I}_1} q_k}q_i - \nu q_i\right) + (1-q_i)\left(\frac{\sum_{k\in\mathcal{I}_2} b_k(\sum_{k\in\mathcal{I}_1} q_k - q_i)}{\left(\sum_{k\in\mathcal{I}_1} q_k\right)^2} - \nu\right) = 0; \\ \\ (1-b_i)\left(\frac{\sum_{k\in\mathcal{I}_1} q_k(\sum_{k\in\mathcal{I}_2} b_k - b_i)}{\left(\sum_{k\in\mathcal{I}_2} b_k\right)^2} - \nu\right) - \left(\frac{\sum_{k\in\mathcal{I}_1} q_k}{\sum_{k\in\mathcal{I}_2} b_k}b_i - \nu b_i\right) = 0. \end{cases} \tag{A2}$$

Since all traders are identical, the strategies of trader types 1 and 2 are type-symmetric. We have: $\tilde{q}_i(\tau,\nu) = \tilde{q}(\tau,\nu)$, for trader $i \in \mathcal{I}_1$ and $\tilde{b}_i(\tau,\nu) = \tilde{b}(\tau,\nu)$, for trader $i \in \mathcal{I}_2$, with $\sum_{k\in\mathcal{I}_1} q_k = n\tilde{q}$ and $\sum_{k\in\mathcal{I}_2} b_k = n\tilde{b}$. Then, we obtain:

$$\begin{cases} \frac{\tilde{q}}{1-\tilde{q}}(1-\nu) = \frac{n-1}{n} - \nu, i \in \mathcal{I}_1; \\ \\ \frac{n-1}{n} - \nu = \frac{\tilde{b}}{1-\tilde{b}}(1-\nu), i \in \mathcal{I}_2. \end{cases} \tag{A3}$$

The solution to (A3) yields (12)-(13).■

6.2 Appendix B: Proof of Proposition 2

From (22), we know that Δ_1 is given by:

$$\begin{aligned}\Delta_1 =&(1-\nu)(n-1)(n-1-n\nu)\left(\frac{n(n-1)}{(2n-1)^2}[2n(1-\nu)-1]-n(1-\nu)\right)+\\ &\frac{n(n-1)}{(2n-1)^2}\Bigg[2n(1-\nu)-1\Bigg]\Bigg[n(1-\nu)(n-1-n\nu)+n\nu\Bigg].\end{aligned} \tag{B1}$$

It can be rewritten as:

$$\begin{aligned}\Delta_1 =&\left(\frac{n(n-1)}{(2n-1)^2}\right)\Bigg[2n(1-\nu)-1\Bigg]\Bigg[(1-\nu)(n-1-n\nu)(2n-1)\Bigg]+\\ &\left(\frac{n(n-1)}{(2n-1)^2}\right)\Bigg[2n(1-\nu)-1\Bigg]n\nu-n(1-\nu)^2(n-1)(n-1-n\nu).\end{aligned} \tag{B2}$$

Equation (B2) can be rewritten as:

$$\begin{aligned}\Delta_1 =&\left(\frac{n(n-1)}{(2n-1)^2}\right)\Bigg[[2n(1-\nu)-1](1-\nu)(n-1-n\nu)(2n-1)+\\ &n\nu[2n(1-\nu)-1]-(1-\nu)^2(2n-1)^2(n-1-n\nu)\Bigg].\end{aligned} \tag{B3}$$

In addition, we can show that, $(1-\nu)^2(2n-1)^2(n-1-n\nu) = (1-\nu)^2(2n-1)^2[2n(1-\nu)-1]-n(1-\nu)^3(2n-1)^2$. Then, we obtain:

$$\begin{aligned}\Delta_1 =&\left(\frac{n(n-1)}{(2n-1)^2}\right)\Bigg[[2n(1-\nu)-1](1-\nu)(n-1-n\nu)(2n-1)+\\ &n\nu[2n(1-\nu)-1]-(1-\nu)^2(2n-1)^2[2n(1-\nu)-1]+n(1-\nu)^3(2n-1)^2\Bigg].\end{aligned} \tag{B4}$$

Some computation shows that,

$$\begin{aligned}\Delta_1 =&\left(\frac{n(n-1)}{(2n-1)^2}\right)\Bigg[[2n(1-\nu)-1](1-\nu)(2n-1)\nu(n-1)-\\ &n[2n(1-\nu)-1](1-\nu)(2n-1)+n\nu[2n(1-\nu)-1]+n(1-\nu)^3(2n-1)^2\Bigg].\end{aligned} \tag{B5}$$

From (B5), we can show that,

$$\begin{aligned}\Delta_1 =&\left(\frac{n(n-1)}{(2n-1)^2}\right)\Bigg[[2n(1-\nu)-1](1-\nu)(2n-1)\nu(n-1)+\\ &n\nu[2n(1-\nu)-1]+n(1-\nu)(2n-1)[\nu^2(2n-1)+\nu]\Bigg].\end{aligned} \tag{B6}$$

It's easy to see that, $\Delta_1 > 0$, as $\nu \leq \frac{n-1}{n}$.■

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