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A note on income risks and their implications for wealth concentration

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Abstract

Income risks are not accurately captured by standard AR processes that are common in the literature. This paper proposes a simple stochastic process which matches several moments in the data including the cross-sectional distribution of income and the distribution income risk, and can be easily used in models with uninsurable income risk. Incorporating this process into an off-the-shelf OLG model leads to a rise in wealth concentration narrowing the gap between traditional models and the data. However, the right tail of the wealth distribution remains significantly thinner than the data.

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1 Introduction

When markets are incomplete, individuals who face income uncertainty will save to smooth their future consumption. This precautionary saving motive is the backbone of many economic analyses and an important contributing factor to the higher saving rates of the top earners (Carroll, 1998; Castaneda et al., 2003). Most quantitative models of uninsured earnings risks use autoregressive processes to capture income risks through the idiosyncratic labor productivity shocks. However, this approach faces serious challenges. Empirically speaking, due to data gathering issues such as undersampling and top-coding, most surveys are not reliable; especially not for top earners in the economy (Heathcote et al., 2009; DeNardi & Fella, 2017). Additionally, there is growing evidence that such processes do not fully capture all relevant properties of the data (Blundell et al., 2015; Arellano et al., 2017). Finally, this approach has proven insufficient in accounting for the high saving rates of top earners, which is an important determinant of wealth concentration (Aiyagari, 1994; Huggett, 1996).

Guvenan et al. (2021) use confidential administrative data of U.S. tax payers to measure various aspects of income risks, which resolves concerns about top-coding or sampling issues in known surveys. They document that the distribution of income changes -as a measure of risk- has substantial negative skewness and excess kurtosis, which are not typically reproduced by AR processes. In other words, in any given period, a large fraction of workers see very small changes in their earnings while few experience very large negative shocks. Also, they report that for high-earners large positive changes are transitory and large negative changes are persistent while the exact opposite is true for low-earners. These facts, which are not captured by standard income processes, have important implications for individuals' savings decisions.

This paper extends the method proposed by Kaplan et al. (2018) to develop a stochastic process that consists of two *modified* AR processes. The parameters of these processes are determined by simulated method of moments to reproduce the cross sectional distribution of income, the distribution of income changes, and the skill-dependence of income profiles. Then, I approximate this stochastic process with a discrete Markov process of order one, which is the standard framework in macroeconomic models. The resulting Markov process not only matches targeted moments, it also captures several non-targeted moments. An important observation in this simulation is that households on both ends of the distribution (extremely poor or extremely rich) face relatively higher risks. However, majority of these changes, especially for top earners, appear to be small in size. Instead, those in the middle of the distribution are relatively more likely to experience large changes in their income. This is consistent with the data as reported in the literature. As an example, I incorporate this income process in an otherwise standard stochastic OLG production economy with skilled and unskilled workers. This simulation shows that when income risks are more consistent with the data, wealth concentration rises compared to standard models. However, this rise is still not enough to reproduce the empirical saving rates of top earners, which results in less wealth concentration compared to the data.

2 Income Process

Each worker’s labor income in period t is the sum of two independent random components; i.e., $e_t = e_t^1 + e_t^2$ where each component follows a modified AR process with ρ^i being the autocorrelation coefficient of the i -th component and ε^i as the i.i.d shock¹

$$e_t^i = \rho^i e_{t-1}^i + \lambda_{1,t}^i \lambda_{2,t}^i \varepsilon_t^i \quad \text{where} \quad \varepsilon_t^i \sim \left| N\left(0, \sigma^{i2} (\lambda_{2,t}^i, \lambda_{3,t}^i)\right) \right| \quad (1)$$

Three features separate this process from an AR(1). First is the infrequent arrival of i.i.d. random shocks, ε^i . The occasional arrival of i.i.d. shocks is effective in generating a high kurtosis in the distribution of earnings changes, which is an important trait of the data. I assume the arrival of shocks to each component follows a Poisson process with rate γ_1^i . The second modification is that the absolute value of i.i.d. shocks are drawn from a *folded* normal distribution. Since all shocks can not be positive, another random i.i.d. variable, $\lambda_{2,t}^i$, which takes two values $\{-1, 1\}$, determines the sign of each shock at any given period. The probability of a negative income shock in the i -th component is denoted by γ_2^i . This combination helps generate both the excess kurtosis and negative skewness in the distribution of income changes that are observed in the data. Lastly, the variance of the underlying normal distributions from which i.i.d. shocks are drawn is itself stochastic. This implies that the likelihood of facing larger shocks is not the same over different periods. Two random variables, $\lambda_{2,t}^i$ and $\lambda_{3,t}^i$, jointly determine this variance where $\lambda_{3,t}^i$ is a binary random variable that takes 1 with probability γ_3^i . This third feature essentially means that some periods ($\lambda_{3,t}^i = 0$) are ordinary and some are extraordinary ($\lambda_{3,t}^i = 1$). In extraordinary periods, the standard deviation of the underlying distribution for positive and negative shocks is potentially different, which creates the possibility of sudden large income changes in either direction. This, though rare, is another trait of the data. For simplicity, I assume that extraordinary periods for both components have the same likelihood of occurring; i.e., $\gamma_3^1 = \gamma_3^2$.

These features make the stochastic process in [equation 1](#) more suitable for capturing unconventional aspects of the data. I use three sets of empirical targets to determine these parameters. The first set of targets are moments of cross-sectional distribution of income, which are measured in the Survey of Consumer Finances (SCF) in 2013. The second set are the moments of the distribution of income changes as reported by [Güvenen et al. \(2021\)](#). They report the averages of several moments between 1978 and 2013. These include the first four moments of one-year and five-year income shocks as well as the likelihood of experiencing

¹Assuming that the two components are independent is conventional in the -quantitative macroeconomic-literature. However, it is important to note that some influential empirical studies have argued for considering two components that are “correlated”. A main reason for this consideration is the observed heterogeneity in *persistence* of various shocks in the data ([Meghir & Pistaferri, 2004](#); [Güvenen, 2007](#)). Though the two components of [equation 1](#) are independent, as I will explain shortly, it allows for the shocks to vary in terms of their arrival rates and their persistence, which accommodates what correlated components aim to achieve.

small and large shocks. Finally, the third set of moments are the population share of skilled workers across the income distribution in the SCF as well as the average ratio of earnings of skilled and unskilled workers (‘earnings ratio’). These provide a total of 29 empirical targets that are used to determine the value of all parameters for both types of workers. I simulate a large sample of workers and use the simulated method of moments (SMM) to find the best value of parameters that provide the best match of targeted moments.²

Table 1 reports the empirical targets as well as their simulated counterparts. The second column corresponds to the calibrated stochastic process in equation 1. The last two columns report the results of two Markov chain approximations. In the third column the approximated Markov process has 100 states.³ Apart from the number of states, using a discrete Markov chain is the standard practice in the literature. Therefore, this approximation is consistent with how income risks are modelled in macroeconomics with one obvious advantage: this process matches higher-order moments and other data traits better than a simple AR process without any major additional computational costs. This approximation, however, does not fully capture the skewness and kurtosis of five-year changes. To have a better fit, the last column reports another approximation where the two components of equation 1 are separately discretized with 75 states for ε^1 and 25 states for ε^2 . It should be noted that using this approximation adds to the computational cost of solving a model as the dimension of the state space will increase.⁴

Is this stochastic process a reliable framework to capture income risks? To answer this question, Table 2 reports several empirical moments that were not targeted in the calibration process. Particularly, this table focuses on the right tail of the distribution to examine whether the right tail of the income distribution is implausibly stretched. In that case, the simulated distribution should overestimate these moments. For comparison, one may consider two of the most influential theories of wealth inequality. In Castaneda et al. (2003) the ratio of earnings of the top four percent of workers to the average is more than 900, which is an overestimation Benhabib et al. (2017) while in Cagetti & DeNardi (2006), this ratio is less than 10, which is an underestimation. In the simulated sample of this paper, this ratio is nearly 500, which falls between the two values.

Lastly, it is worth noting that all parameters of equation 1 were determined in a numerical optimization with the sole purpose of matching the empirical targets in table 1. However, the results indicate a pattern that clearly differentiates e_t^1 from e_t^2 , and provides a link between equation 1 and the extant literature. The literature often decomposes workers’

²I assume that the arrival rate of ordinary shocks and the variance of the underlying distribution of ordinary shocks are the same for both types of workers, which reduces the number of undetermined parameters to 24. Also, the calibrated values for the variance of ordinary shocks for both components, and the variance of extraordinary shocks for the second component were virtually identical. Table 4 in the appendix reports the values of these parameters.

³Starting from the mean, the states are log-distanced towards each end. This is motivated by the fact that most individuals do not see large changes in their income in a given period (high kurtosis), and that the density of worker is low around both tails of the distribution.

⁴Table 6 and 7 report approximated processes with 10 states for both skilled and unskilled workers.

Table 1: Stochastic Income Process

	Data	Stochastic Process	Approx.(1)*	Approx.(2)*
mean of 1-year changes	0.00	0.00	0.00	0.00
std of 1-year changes	0.51	0.51	0.51	0.52
skewness of 1-year changes	-1.07	-1.08	-1.10	-1.01
kurtosis of 1-year changes	14.93	15.09	15.63	14.34
mean of 5-year changes	0.00	0.00	0.00	0.00
std of 5-year changes	0.78	0.81	1.05	0.82
skewness of 5-year changes	-1.25	-1.21	-0.46	-1.11
kurtosis of 5-year changes	9.51	9.62	5.69	9.27
$\Pr(e_t - e_{t-1} < 0.05)$	0.31	0.28	0.28	0.25
$\Pr(e_t - e_{t-1} < 0.10)$	0.49	0.49	0.47	0.32
$\Pr(e_t - e_{t-1} < 0.20)$	0.67	0.67	0.67	0.61
$\Pr(e_t - e_{t-1} < 0.50)$	0.83	0.82	0.83	0.82
$\Pr(e_t - e_{t-1} < 1.00)$	0.93	0.93	0.93	0.93
share of top 60 percent	97.10	94.47	94.41	94.46
share of top 40 percent	86.70	85.64	85.38	85.53
share of top 20 percent	66.50	68.48	67.35	67.78
share of top 10 percent	49.60	52.62	49.67	50.87
share of top 5 percent	37.20	39.45	37.11	38.00
share of top 1 percent	18.80	19.03	17.86	18.36
Gini coefficient	0.670	0.657	0.646	0.650
skilled share in 0-20	0.20	0.21	0.19	0.20
skilled share in 20-40	0.23	0.21	0.18	0.20
skilled share in 40-60	0.21	0.25	0.23	0.23
skilled share in 60-80	0.38	0.35	0.32	0.33
skilled share in 80-90	0.54	0.52	0.59	0.53
skilled share in 90-95	0.69	0.67	0.81	0.73
skilled share in 95-99	0.80	0.78	0.74	0.78
skilled share in 99-100	0.89	0.87	0.87	0.85
earnings ratio	3.02	3.03	3.11	3.12

* The third column approximates this stochastic process with a discrete Markov process with 100 states, and the fourth column approximates with two Markov processes with 75 states for the first component and 25 states for the second component.

Table 2: Ratio of Average to the Median and Mean of Income

	Average to Median		Average to Mean	
	Data	Simulation	Data	Simulation
Top 10 (%)	6.8300	11.530	4.8500	4.6300
Top 5 (%)	10.090	17.070	7.160	6.860
Top 1 (%)	24.530	39.070	17.410	15.710
Top 0.1 (%)	82.330	91.950	58.450	36.980
Top 0.01 (%)	302.69	305.81	214.90	122.98

earnings to a *permanent* and a *transient* component where the permanent component is essentially a unit-root process, and the transient component is an i.i.d. random variable. In this framework, shocks to the permanent component have lasting impacts while shocks to the transient component fade away quickly as the transient component has no persistence. Parameters of [equation 1](#) can be interpreted in a similar manner⁵. The first component, e_t^1 , is significantly more persistent than e_t^2 . This is evident from the fact the coefficient ρ_1 is greater than ρ_2 . Unlike what is conventional in the literature, neither component is perfectly permanent or perfectly transient; rather, both are persistent albeit to very different degrees. Therefore, shocks to the first component will have noticeably longer-lasting effects. Additionally, shocks to the second component are more frequent than the first component ($\gamma_1 > \gamma_2$) while, on average, they are more likely to be smaller ($\sigma^1(1, 0) > \sigma^2(1, 0)$). In other words, the first component is more persistent, in most periods does not see any shocks and when it does, the shocks are likely to be large and have long-lasting effects. Therefore, the first component could represent ‘big’ events that occur rarely but significantly affect the future -*e.g.*, a promotion. On the other hand, the second component is less persistent and is subject to small shocks which occur more frequently. Therefore, e_t^2 possibly represents ‘marginal’ events that may be smaller in size, may occur several times during a worker’s lifetime, and have short-lasting effects -*e.g.*, a bonus or a pay cut.

3 Implications for Wealth Concentration

As an application, I study the implications of this stochastic process for the precautionary savings of top earners and the wealth concentration. The economic environment is a standard stochastic OLG economy similar to [Castaneda et al. \(2003\)](#) and [Heathcote et al. \(2014\)](#).

Workers. A worker is identified by her skill level ($i \in \{U, S\}$), her age, her labor productivity z and her assets a . Age has three stages: youth, middle-aged, and elderly. As standard, the transition of workers between these ages is exogenous and replicates the demographics of the US economy. Each period, young and middle-age workers draw an idiosyncratic labor productivity and supply their labor in a competitive market, and the

⁵[Table 4](#) reports the values of all parameters for both skilled and unskilled workers.

elderlies receive a constant social security benefit. Workers maximize their life time utility, and discount future at rate β . Finally, workers pay income taxes $\tau(y)$, which following [Heathcote et al. \(2014\)](#) is defined as $\tau(y) = y - a_1 y^{a_2}$. Thus, workers' problem is given by:⁶

$$V(i, j, z, a) = \max_{\{c, a'\}} \frac{c^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}V(i, j', z', a')$$

subject to

$$c + a' \leq y - \tau(y) + a \tag{2}$$

$$y = w_i z n + r a$$

Production Sector

A representative firm hires labor and capital in perfectly competitive markets and uses a standard CRS technology to produce a homogeneous consumption good. The capital share in the production technology is γ . The effective total supply of labor by unskilled and skilled workers - N_u and N_s - form the aggregate supply of labor through a CES aggregator where the elasticity of substitution between skilled and unskilled workers is $\frac{1}{1+\rho}$ and the weight assigned to the total unskilled labor supply is denoted by α .⁷

$$Y = AK^\gamma N^{1-\gamma} \quad \text{where} \quad N = \left[\alpha N_u^{-\rho} + (1-\alpha) N_s^{-\rho} \right]^{-\frac{1}{\rho}} \tag{3}$$

The aggregate supply of labor for both types of workers -skilled and unskilled- is the sum of effective units of labor that they supply. To clarify how these quantities are computed in the model, we should note that individuals' labor income follows the stochastic process described in [equation 1](#). This will ensure that the distribution of risk in the model matches the moments reported in [table 1](#), which is essential for the quantitative analysis of the paper. Conceptually, these earnings are the product of three components: an equilibrium wage rate for each type of workers (w_i), the worker's idiosyncratic labor productivity (z), and hours supplied, which are normalized to 1 ($n = 1$). Thus, given the workers' earnings and the wage rate of their respective type, the worker's labor productivity, z , can be obtained by $z = \frac{e}{w_i}$.⁸ The effective supply of labor for such a worker is z . In other words, each worker of type i , with a random draw e supplies z effective units of labor in the market. The aggregate supply of each type, N_i , is, then, the sum of all effective labor units of workers of that type. Both equilibrium wage rates are determined iteratively following the standard procedures in the literature, and their ratio $\frac{w_u}{w_s}$, which represents skill premium in the model, is a target in the calibration of the model.

⁶The elderly solve a similar problem but instead of labor income they receive the retirement benefit, R_b .

⁷See [Katz & Murphy \(1992\)](#) and [Heathcote et al. \(2013\)](#) for more details.

⁸This is equivalent of drawing a random individual productivity z when the wage rate is w_i , which will result in labor income $e = w_i z(1)$.

Government. A government that keeps a balanced budget, collects income taxes and pays a constant social security benefit, R_b , to retirees. Equation 4 gives the government’s tax revenue where $\mu(\cdot)$ is the steady state distribution of workers in the economy.

$$T = \sum_{i \in \{u,s\}} \sum_j \int_{a \otimes z} \mu(i, j, z, a) \tau(y) da \otimes z \quad (4)$$

3.1 Calibration and Analysis

Calibration. Given the standard structure of the model, most parameters are borrowed from the literature except for α, β, a_1 and R_b . Four empirical targets identify these parameters. To determine α , I use Heathcote et al. (2013) where they estimate the skill premium to be 0.31 between 1980 and 1984, and 0.52 between 2001 and 2005. I use these two data points and extrapolate to find the skill premium in 2013, which results in a skill premium of 0.62 in the model. To determine β , I target a risk-free rate of 3 percent. Finally, both a_1 and R_b are determined to match the ratio of total social security benefits to output, and the ratio of the U.S. government expenditure to output, which are, on average, nearly 5 and 20 percent, respectively. This fully characterized the model economy. Table 5 reports the parameter values.

Table 3: Wealth Concentration in the Benchmark Economy

		Quantiles					Top Percentile		
	Gini	1st	2nd	3rd	4th	5th	90-95	95-99	99-100
U.S.	0.85	-0.7	0.60	3.20	9.80	87.00	12.10	27.50	35.40
model	0.75	0.00	1.68	5.52	14.97	77.54	16.11	25.65	18.52

Analysis. The introduced income process, consistent with the data, implies that the top earners face more downward -than upward- risk. But is this enough to account for their higher saving rates and the wealth concentration observed in the data? To answer this question, Table 3 compares the distribution of wealth in the model with the U.S. data in 2013. As this table shows, the right tail of the distribution in the model is thinner than the data. But this is noticeably thicker than standard models. For comparison, in seminal papers such as Aiyagari (1994) and Huggett (1996) -and many other models with uninsured income risks- the wealth share of the top one percent in the model is less than one-third of its value in the data. However, as table 3 shows, with this stochastic process the wealth share of the top one percent in the model is slightly more than half of its value in the data. This improvement over earlier models confirms the criticism about the risk modelling, and shows that the gap in wealth concentration between those models and the data was partly due to their inability to capture the income risks for top earners. The higher downward risk that top earners face creates a stronger saving motive for them, which leads to more wealth

concentration. Therefore, since such risks are accounted for in this paper, the concentration of wealth in the model is much closer to the data. However, even this correcting does not fully explain the data.

4 Conclusion

Top earners save more than the average household, which is why wealth is even more concentrated than income in the data. However, many incomplete market models with uninsured income risk are unable to account for this difference as the standard methods of modelling income risks are unable to accurately capture income risks; especially for top earners. I design a flexible stochastic income process that matches several moments in the data including the distribution of income risk. A discrete Markov chain approximation, which is easy to incorporate in macroeconomic models, is provided. As an example, I study the implications of this process for wealth concentration as under this process, consistent with the data, top earners face more downward risk, which strengthens their precautionary saving motive. Incorporating this process in an off-the-shelf stochastic OLG economy shows a better modelling of income risks increases the concentration of wealth closer to the data as compared to traditional frameworks; although even this correction falls short of explaining the thick right tail of the wealth distribution.

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Appendix

Table 4: Parameters of the Income Process

Parameter	Description	Unskilled	Skilled
<i>i</i> = 1			
ρ	persistence rate	0.9776	0.9750
γ_1	arrival rate of shocks	0.1312	0.1312
γ_2	arrival rate of negative shocks	0.7950	0.5997
γ_3	arrival rate of extraordinary shocks	0.1150	0.2250
$\sigma(1,0)$	std of ordinary shocks	0.5300	0.5300
$\sigma(1,1)$	std of extraordinary positive shocks	1.8300	2.0000
$\sigma(-1,1)$	std of extraordinary negative shocks	1.1000	0.9000
<i>i</i> = 2			
ρ	persistence rate	0.2500	0.2500
γ_1	arrival rate of shocks	0.9700	0.9700
γ_2	arrival rate of negative shocks	0.7950	0.5997
γ_3	arrival rate of extraordinary shocks	0.1150	0.2250
$\sigma(1,0)$	std of ordinary shocks	0.0600	0.0600
$\sigma(1,1)$	std of extraordinary positive shocks	0.8200	0.8200
$\sigma(-1,1)$	std of extraordinary negative shocks	1.0000	1.0000

Table 5: Parameters of the Model Economy

Parameter	Description	Value	Target
β	time discount factor	0.91	$r = 0.035$
σ	elasticity of substitution	1.35	Literature
δ	capital depreciation rate	0.07	Literature
γ	capital share of output	0.37	Literature
α	weight of unskilled labor in CES aggregator	0.38	skill prem. = 0.62
ρ	elasticity of substitution btw workers	-0.33	Katz & Murphy (1992)
a_1	coeff. of income tax function	0.701	$\frac{G}{GDP} = 0.15$
a_2	progressivity of income tax function	0.941	Heathcote et al. (2014)
R_b	retirement benefits	0.095	$\frac{R_b}{GDP} = 0.05$

Table 6: Income Process for Skilled Workers

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
z_1	0.0001	0.9999	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
z_2	2.1e-4	0.7265	0.2731	2.1e-4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
z_3	0.0000	0.0095	0.7906	0.1944	0.0047	1.3e-4	4.9e-4	8.5e-5	0.0000	0.0000
z_4	0.0000	5.5e-4	0.0467	0.7625	0.1606	0.0035	0.0196	0.0059	4.8e-4	2.8e-6
z_5	0.0000	9.7e-5	0.0057	0.1082	0.6849	0.0361	0.1234	0.0373	0.0040	2.7e-5
z_6	0.0000	2.8e-5	0.0031	0.0455	0.2854	0.1539	0.4321	0.0705	0.0091	5.7e-5
z_7	0.0000	3.5e-5	0.0023	0.0250	0.1169	0.0240	0.6566	0.1576	0.0171	1.9e-4
z_8	0.0000	5.7e-6	0.0011	0.0076	0.0346	0.0049	0.1251	0.7509	0.0741	0.0015
z_9	0.0000	0.0000	4.2e-4	0.0025	0.0076	0.0011	0.0286	0.2562	0.6859	0.0175
z_{10}	0.0000	0.0000	0.0000	0.0015	0.0012	1.5e-4	0.0031	0.0467	0.5336	0.4135
$\ln(z)$	0.0000	3.2770	8.2586	10.849	12.197	12.689	13.057	13.960	15.473	18.009

Table 7: Income Process for Unskilled Workers

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
z_1	0.0001	0.9999	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
z_2	5.1e-4	0.7185	0.2809	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
z_3	0.0000	0.0036	0.8087	0.1862	0.0013	0.0000	7.4e-5	1.6e-5	0.0000	0.0000
z_4	0.0000	1.2e-4	0.0278	0.8223	0.1384	0.0017	0.0079	0.0015	1.0e-4	0.0000
z_5	0.0000	1.4e-5	0.0025	0.0782	0.7765	0.0426	0.0845	0.0142	0.0011	8.2e-6
z_6	0.0000	0.0000	0.0013	0.0279	0.2210	0.1859	0.5280	0.0329	0.0026	6.4e-5
z_7	0.0000	0.0000	9.8e-4	0.0150	0.0889	0.0179	0.7540	0.1177	0.0052	6.1e-5
z_8	0.0000	0.0000	3.5e-4	0.0043	0.0229	0.0035	0.0909	0.8417	0.0357	5.0e-4
z_9	0.0000	0.0000	8.8e-5	0.0011	0.0041	5.2e-4	0.0165	0.1813	0.7902	0.0059
z_{10}	0.0000	0.0000	0.0000	0.0000	0.0004	0.0000	0.0027	0.0346	0.3944	0.5677
$\ln(z)$	0.0000	3.6874	9.2486	12.074	13.511	14.025	14.389	15.283	16.774	19.264