

## **Submission Number:EB-15-00712**

This is an appendix file, which is intended to be available to readers.

# Appendix

## A Details of the computation

The solution algorithm follows a modified version of Maliar, Maliar and Valli (2010). The state space for household capital,  $k_i$ , is discretized by 100 grids in the range  $[-\phi, 1000]$ . The upper bound is chosen to be sufficiently high so that the households do not reach the upper bound in the simulated paths. The number of grids is chosen to be sufficiently high so that a further increase of the grid number will not change the simulated mean capital. To capture the curvature of the policy functions, we take the grids densely toward  $-\phi$ . Specifically, we set  $(k_i + \phi)^{0.25}$  to be equally spaced. The state space for the mean capital is discretized by four grids.

Given the approximated law of motion for the joint distribution of the capital holding and employment state, we obtain a policy function by iteration of the Euler equation. To evaluate the policy function at the forecasted mean capital in the next period, we interpolate the policy function in mean capital by the cubic spline method.

Once the policy function is obtained, we simulate the equilibrium path with 10,000 households for 3,000 periods. In each simulation period, the policy function is interpolated at the current mean capital level by the spline method, and the interpolated policy function, which is evaluated at the current mean capital and aggregate state, is further fitted by a quadratic function for each employment state. Fitting by the higher-degree polynomial functions does not alter the results. The fitted function is then used to compute the capital holding for each household in the next period. We use the simulated mean capital path for the last 2,000 periods to estimate the law of motion. The convergence criterion for the value function iteration is  $1.e-8$  in the sup norm. The convergence criterion for the law of motion is  $1.e-10$  for all coefficients.

## B Calibration

We assume that the unemployment rate follows an exogenous regime-switching process of labor policy. In this study, we set the Jobs and Growth Tax Relief Reconciliation Act (JGTRRA) in 2003 as our calibration target policy. The Economic Growth and Tax Relief Reconciliation Act (EGTRRA) in 2001 and JGTRRA are collectively called the Bush tax cuts. The JGTRRA is a policy that consists of tax reductions in both labor and capital incomes, and it has been successful in reducing unemployment and increasing the level of consumption (House and Shapiro (2006)).

We set the mean interval of policy changes as two years, considering that the U.S. general elections are held at that interval, and that it took two years after EGTRRA to implement JGTRRA, which was intended to accelerate the EGTRRA tax cuts. The average two-year interval (or equivalently, eight quarters) pins down the symmetric transition matrix for policy regime  $z$ . The unemployment rates in the different policy regimes,  $u_0$  and  $u_1$ , are set so that the impact of the exogenous policy shock is comparable with that of JGTRRA. House and Shapiro (2006) argue that both the production and employment levels recovered sharply in response to JGTRRA, and they estimate that the tax cuts raised the employment rate above the trend by about 1.25%. We calibrate the unemployment rate in the passive policy regime  $u_0$  at 6%, which matches the unemployment rate before mid-2003, according to the Labor Force Statistics from the Current Population Survey. Thus, the unemployment rate in the active policy regime is set as  $u_1 = 1 - (1 - 0.06) \times 1.0125 \simeq 0.0483$ .

The transition matrix  $\Pi$  must satisfy

$$u_z(\pi_{00zz'}/\pi_{zz'}) + (1 - u_z)(\pi_{10zz'}/\pi_{zz'}) = u_{z'}, \quad z, z' \in \{0, 1\} \quad (1)$$

to be compatible with the exogenous aggregate labor employed by the government or firms,  $1 - u_z$ .  $\Pi$  is also restricted by the mean duration of unemployment for each state, which we calibrate as 2.5 quarters for state 0 and 1.5 quarters for state 1 following KS. This calibration is compatible with the average duration of unemployment reported by the Current Population Survey from 1995 to 2010. We divide the sample years according to whether the duration exceeded or fell short of the total average. The averages of the sub-sample are 22.7 and 15.4 weeks, respectively, whereas the total average is 17.8 weeks. These values are comparable to the KS calibration.

We also follow the KS calibrations,  $\pi_{0001} = 0.75\pi_{0011}$  and  $\pi_{0010} = 1.25\pi_{0011}$ . This implies that the job-finding rate when the policy switches from 0 to 1 overshoots the rate when the policy stays active in state 1, while it drops when the policy switches back to a passive state. These restrictions fully determine  $\Pi$ :

$$\Pi = \begin{bmatrix} 0.5250 & 0.3500 & 0.0313 & 0.0938 \\ 0.0223 & 0.8527 & 0.0044 & 0.1206 \\ 0.0938 & 0.0313 & 0.2917 & 0.5833 \\ 0.0031 & 0.1219 & 0.0296 & 0.8454 \end{bmatrix}. \quad (2)$$

The debt limit  $\phi$  is set at 3, which is roughly equal to three months' average income. This value is chosen so that the gap between the consumption growth rates of the low and high asset holders roughly matches Zeldes' estimate (Zeldes (1989); Nirei (2006)). The other parameters are set to match the quarterly U.S. statistics on the share of capital in production, the rate of depreciation, and the steady-state annual real interest rate (KS and Hansen (1985)). The risk-aversion parameter is set at  $\sigma = 1$  and put to a robustness check in Appendix . Table 1 summarizes the parameter values.

| Description                             | Symbol   | Value |
|---|----------|-------|
| Capital share                           | $\alpha$ | 0.36  |
| Discount factor                         | $\beta$  | 0.99  |
| Depreciation rate                       | $\delta$ | 0.025 |
| Risk aversion                           | $\sigma$ | 1     |
| Debt limit                              | $\phi$   | 3     |
| Unemployment rate in the passive regime | $u_0$    | 6%    |
| Unemployment rate in the active regime  | $u_1$    | 4.83% |

Table 1: Parameter values

## C Other simulated moments of interest

Table 2 lists the other estimates.  $C^e$  and  $C^u$  denote the consumption per worker for the employed and unemployed households, respectively, that is time-averaged for all periods through policy transitions. Column  $C^e/C^u$  gives the ratio of the average consumption of the employed and unemployed. Although the households partially hedge their unemployment risk by accumulating wealth, this shows that a substantial gap (4.12%) remains uninsured. Table 3 shows the approximated law of motion for the aggregate capital. The high  $R^2$  shows that the approximation is accurate.

|        | $C^e/C^u$          | $C$                | $I/Y$              | $\bar{K}$           |
|--------|--------------------|--------------------|--------------------|---------------------|
| GE I   | 1.0412<br>(0.0010) | 2.5899<br>(0.0001) | 0.2569<br>(0.0000) | 35.8107<br>(0.0040) |
| GE II  | 1.0708<br>(0.0034) | 2.5591<br>(0.0009) | 0.2647<br>(0.0002) | 35.7585<br>(0.0072) |
| Tax I  | 1.0468<br>(0.0011) | 2.5936<br>(0.0008) | 0.2521<br>(0.0001) | 34.9805<br>(0.0150) |
| Tax II | 1.0469<br>(0.0021) | 2.5242<br>(0.0015) | 0.2719<br>(0.0001) | 34.9874<br>(0.0207) |

Table 2: Other estimates 1

|        | $\hat{R}_0^2$      | $\hat{a}_0$        | $\hat{b}_0$        | $\hat{R}_1^2$      | $\hat{a}_1$        | $\hat{b}_1$        |
|--------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| GE I   | 0.9988<br>(0.0003) | 0.0208<br>(0.0055) | 0.9942<br>(0.0015) | 0.9997<br>(0.0001) | 0.0548<br>(0.0036) | 0.9847<br>(0.0010) |
| GE II  | 0.9999<br>(0.0000) | 0.1653<br>(0.0032) | 0.9537<br>(0.0009) | 0.9999<br>(0.0000) | 0.1540<br>(0.0022) | 0.9570<br>(0.0006) |
| Tax I  | 1.0000<br>(0.0000) | 0.1402<br>(0.0010) | 0.9605<br>(0.0003) | 1.0000<br>(0.0000) | 0.1378<br>(0.0008) | 0.9613<br>(0.0002) |
| Tax II | 1.0000<br>(0.0000) | 0.1358<br>(0.0010) | 0.9616<br>(0.0003) | 1.0000<br>(0.0000) | 0.1353<br>(0.0006) | 0.9621<br>(0.0002) |

Table 3: Other estimates 2

## D Sensitivity analysis

In this section, we check the robustness of our outcomes by conducting three types of sensitivity analysis in terms of the risk aversion, debt limits, and endogenous labor supply. In all of these dimensions, we find our computation results to be robust.

### D.1 Risk aversion

First, we change the risk-aversion parameter  $\sigma$  from 1 to 2 and 5 for GE I. We find an increase in the mean capital level as the risk aversion rises, which is consistent with the theoretical prediction that risk aversion implies more precautionary savings and a lower consumption demand. Since a higher level of capital contributes to a positive income effect, the aggregate consumption response toward various risk aversions depends on the relative strength of these two opposing forces: a lower consumption demand and a positive income effect. In addition, we confirm a stronger nonlinearity in the consumption function as the households become more risk-averse.

|     | $\sigma = 1$ |         |        | $\sigma = 2$ |         |        | $\sigma = 5$ |         |        |
|-----|--------------|---------|--------|--------------|---------|--------|--------------|---------|--------|
| $z$ | $C_z^e$      | $C_z^u$ | $C_z$  | $C_z^e$      | $C_z^u$ | $C_z$  | $C_z^e$      | $C_z^u$ | $C_z$  |
| 0   | 2.5974       | 2.4682  | 2.5896 | 2.5971       | 2.4856  | 2.5912 | 2.6002       | 2.4994  | 2.5942 |
| 1   | 2.5942       | 2.5188  | 2.5905 | 2.5943       | 2.5295  | 2.5904 | 2.5979       | 2.5393  | 2.5951 |

Table 4: Same as Table 1

|              | $\bar{K}$ | Log diff | $K$ effect | Risk effect                    |                          |                                  |
|--------------|-----------|----------|------------|--------------------------------|--------------------------|----------------------------------|
|              |           |          |            | $(1 - u_0) \log c_1^e / c_0^e$ | $u_1 \log c_1^u / c_0^u$ | $(u_0 - u_1) \log c_1^e / c_0^u$ |
| $\sigma = 1$ | 35.8084   | 0.0004   | 0.0000     | -0.0005                        | 0.0002                   | 0.0005                           |
| $\sigma = 2$ | 35.8841   | 0.0003   | 0.0000     | -0.0004                        | 0.0002                   | 0.0004                           |
| $\sigma = 5$ | 36.2699   | 0.0004   | 0.0000     | -0.0004                        | 0.0002                   | 0.0004                           |

Table 5: Same as Table 2

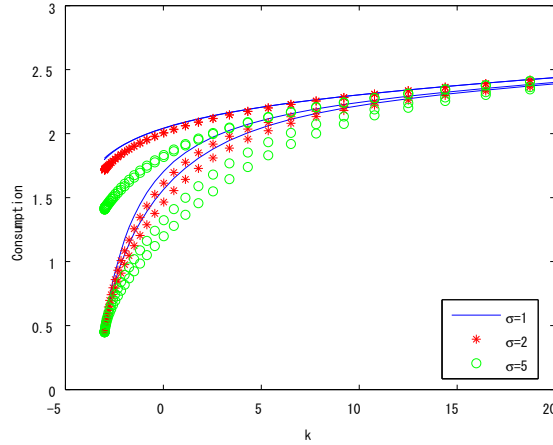


Figure 1: Policy functions with different risk aversions

The policy functions (Figure 2) show that higher risk aversion results in lower consumption levels and stronger nonlinearity (at the consumption levels not influenced by minimum transfer  $\iota(0)$ ). This is because the higher risk aversion induces more precautionary savings and less consumption.

## D.2 Debt limits

In the second sensitivity analysis, we change the level of a debt limit. In the benchmark case,  $\phi$  is set at three months' worth of wage income; that is,  $\phi = 3$ . We change this to  $\phi = 0$ ; that is, no debt limit at all. We note that the aggregate consumption level decreases as the debt limit is relaxed. When the borrowing constraint is relaxed, the households save less owing to diminished precautionary motives, and therefore the aggregate capital level decreases. This leads to a decrease in the production level and hence to further decreases in the aggregate consumption level.

In every simulation, we find no agents who are bound by debt limits. This does not imply that the borrowing constraint has no effect on household behavior. Since the households are highly concerned with the possibility of a binding debt limit and zero consumption, they begin to severely reduce their consumption level when their wealth is well above the debt limit. Thus, the effect of a debt limit manifests itself in the form of nonlinear consumption functions rather than constrained agents.

|            | $\bar{K}$ | $Y$    | $C$    |
|------------|-----------|--------|--------|
| $\phi = 0$ | 35.8112   | 3.4854 | 2.5901 |
| $\phi = 3$ | 35.8093   | 3.4853 | 2.5901 |

Table 6: Mean capital, aggregate production, and consumption

|     | $\phi = 0$ |         |        | $\phi = 3$ |         |        |
|-----|------------|---------|--------|------------|---------|--------|
| $z$ | $C_z^e$    | $C_z^u$ | $C_z$  | $C_z^e$    | $C_z^u$ | $C_z$  |
| 0   | 2.5969     | 2.4752  | 2.5896 | 2.5974     | 2.4682  | 2.5896 |
| 1   | 2.5941     | 2.5218  | 2.5906 | 2.5942     | 2.5188  | 2.5905 |

Table 7: Same as Table 1

|            | Log diff | $K$ effect | Risk effect                    |                          |                                  |
|------------|----------|------------|--------------------------------|--------------------------|----------------------------------|
|            |          |            | $(1 - u_0) \log c_1^e / c_0^e$ | $u_1 \log c_1^u / c_0^u$ | $(u_0 - u_1) \log c_1^e / c_0^u$ |
| $\phi = 0$ | 0.0004   | 0.0000     | -0.0004                        | 0.0002                   | 0.0004                           |
| $\phi = 3$ | 0.0004   | 0.0000     | -0.0005                        | 0.0002                   | 0.0005                           |

Table 8: Same as Table 2

The policy function (Figure 3) shows that the aggregate consumption decreases as the borrowing constraint is relaxed (greater  $\phi$ ). This is because a looser credit constraint makes the households less motivated to retain precautionary savings and thus the aggregate capital decreases. The lower aggregate capital results in lower output and the consumption level decreases.

## D.3 Disutility from the labor supply

In the third sensitivity analysis, we generalize the preference specification to incorporate the utility from leisure. The utility function is generalized, where the Frisch elasticity varies with the new parameter  $\psi$ . The benchmark specification correspond to the case where  $\psi = 0$ . If the labor supply is exogenous, the inclusion of the disutility of labor does not change the equilibrium outcome under the log utility setup where  $\sigma = 1$  as in the benchmark models. Thus, we focus on the case of an endogenous labor supply, where households choose the hours

that they work when they are employed. The simulation results when  $\psi = 0.1$  show that the contribution of leisure lowers the consumption level, because the precautionary motive is weakened by increased leisure when people are unemployed. However, the qualitative pattern of the consumption response to the regime switch is unchanged from the benchmark model.

In order to incorporate the disutility from labor in our analysis, we modify the momentary utility function as  $\left((c_t^{1-\psi}(1-h_t)^\psi)^{(1-\sigma)} - 1\right)/(1-\sigma)$ . Households decide the hours worked  $h_t$  when they are employed. The aggregate hours also become endogenous, and hence, households need to forecast the evolution of the aggregate hours to form expectations on future prices. We approximate the expected aggregate hours as a log-linear function of the contemporaneous mean capital level. In the GE I model, we obtain regression outcomes for  $\psi = 0.1$  as:

$$\log L_0 = -0.0765 - 0.0289 \log \bar{K}_0 \quad \bar{R}_0^2 = 0.2447$$

$$\log L_1 = -0.0888 - 0.0253 \log \bar{K}_1 \quad \bar{R}_1^2 = 0.2176.$$

$\bar{R}^2$  is low because the aggregate employment in the productive sector is constant across policies in GE I. Thus, to improve the regression accuracy, we choose to work in TAX I, where the employment in the productive sector changes across policies. The regression results in TAX I are as follows:

$$\log L_0 = -0.0773 - 0.0301 \log \bar{K}_0 \quad \bar{R}_0^2 = 0.9050$$

$$\log L_1 = -0.0763 - 0.0303 \log \bar{K}_1 \quad \bar{R}_1^2 = 0.9149.$$

The inclusion of leisure implies a relatively high utility for the unemployed. This lowers the precautionary savings and aggregate capital leading to a lower consumption level.

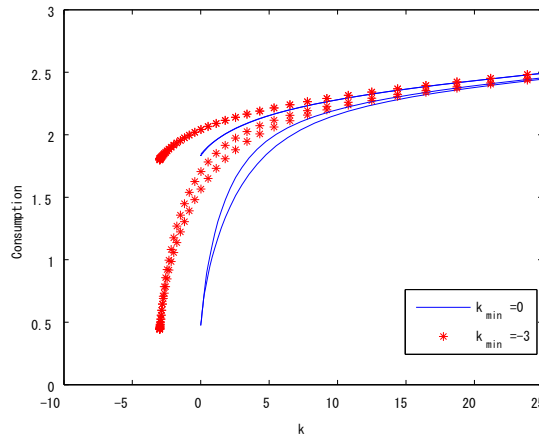


Figure 2: Policy functions with different borrowing constraints

|     | $\psi = 0$ |         |        | $\psi = 0.1$ |         |        |
|-----|------------|---------|--------|--------------|---------|--------|
| $z$ | $C_z^e$    | $C_z^u$ | $C_z$  | $C_z^e$      | $C_z^u$ | $C_z$  |
| 0   | 2.6010     | 2.4552  | 2.5923 | 2.3034       | 2.2293  | 2.2989 |
| 1   | 2.6021     | 2.5161  | 2.5980 | 2.3069       | 2.2629  | 2.3048 |

Table 9: Same as Table 1

|              | Log diff | $K$ effect | Risk effect                    |                          |                                  |
|--------------|----------|------------|--------------------------------|--------------------------|----------------------------------|
|              |          |            | $(1 - u_0) \log c_1^e / c_0^e$ | $u_1 \log c_1^u / c_0^u$ | $(u_0 - u_1) \log c_1^e / c_0^u$ |
| $\psi = 0$   | 0.0004   | 0.0000     | -0.0005                        | 0.0002                   | 0.0005                           |
| $\psi = 0.1$ | 0.0003   | 0.0000     | -0.0005                        | 0.0002                   | 0.0005                           |

Table 10: Same as Table 2



## References

- Hansen, Gary D. (1985) 'Indivisible labor and the business cycle.' *Journal of Monetary Economics* 16, 309–327
- House, Christopher L., and Matthew D. Shapiro (2006) 'Phased-in tax cuts and economic activity.' *American Economic Review* 96, 1835–1849
- Maliar, Lilia, Serguei Maliar, and Fernando Valli (2010) 'Solving the incomplete markets model with aggregate uncertainty using the Krusell-Smith algorithm.' *Journal of Economic Dynamics and Control* 34, 42–49
- Nirei, Makoto (2006) 'Quantifying borrowing constraints and precautionary savings.' *Review of Economic Dynamics* 9, 353–363
- Zeldes, Stephen P. (1989) 'Consumption and liquidity constraints: An empirical investigation.' *Journal of Political Economy* 97, 305–346