E C O N O M I C S   B U L L E T I N

Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result for the general case

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Abstract

It is proved that the irrelevance result of Poyago−Theotoky can be extended from the linear−quadratic case to general inverse demand and cost functions. Hence, as long as firms are profitable at the first−best, the optimal subsidy decentralizes it in mixed oligopoly irrespecitve of whether the public firm maximizes welfare or profit and moves simultaneously with private firms, or maximizes welfare and acts as a Stackelberg leader.

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1. Introduction

In a recent contribution to this journal, Poyago-Theotoky (2001) proved an irrelevance result for mixed oligopoly. This result showed that the optimal subsidy and equilibrium output levels are identical irrespective of whether (i) the game is simultaneous with the public firm choosing output to maximize welfare or (ii) the public firm is a Stackelberg leader or (iii) all firms are profit maximizers. This result extended previous work of White (1996).

The result of Poyago-Theotoky is, however, restricted to the case of linear demand and a quadratic cost function with no fixed costs. At the end of the paper she speculates that the result holds for general demand and costs. The purpose of this note is to confirm that it does.

For the linear-quadratic case, Poyago-Theotoky proves the result by explicitly calculating the equilibria. With general functions this is not possible. Instead, one is pushed to consider the general reasoning behind the result. This actually leads to a much simplified irrelevance proof.

2. The Framework

There is an industry composed of *n* private firms and one public firm producing an homogeneous good. The outputs of the private firms are denoted q_i, $i = 1,...,n$, and that of the public firm by q_0. Total output is $Q = q_0 + \sum_{i=1}^{n} q_i = \sum_{j=0}^{n} q_j$. Each firm produces with a cost function $C(q_j)$. The consumer sector is described by the inverse demand function $p(Q)$. These functions satisfy the following assumption.

Assumption 1. (i) $C(0) \geq 0$, $C'(q_j) \geq 0$, (ii) $p'(Q) < 0$.

Each private firms maximizes profit

$$\pi_i = q_i p(Q) - C(q_i) + sq_i, \quad i = 1,\dots,n. \tag{1}$$

where *s* is the subsidy paid by the government. The profit level of the public firm is

$$\pi_0 = q_0 p(Q) - C(q_0) + sq_0 . \tag{2}$$

Social welfare is defined by the sum of consumer surplus plus profits less the cost of the subsidy. Hence

$$W = CS + \pi_0 + \sum_{i=1}^{n} \pi_i - sQ \,. \tag{3}$$

For the *mixed Nash equilibrium*, each private firm chooses its output to maximize profit (1) and the public firms chooses output to maximize (3). Every firm takes the output of the others and the subsidy as given. In the *private oligopoly equilibrium*, the objective of the public firm becomes the maximization of (2). All else remains the same. The *mixed Stackelberg equilibrium* has the public firm as Stackelberg leader with (3) as its objective. For all three cases the government chooses the subsidy to maximize welfare taking into account how the subsidy affects the equilibrium choices of the firms. Formally, in stage 1 the government sets a subsidy then, in stage 2, the firms play the output game according to the rules specified.

Define the output level q^* by

$$p\left([n+1]q^*\right) = C'\left(q^*\right), \tag{4}$$

Clearly, q^* is the level of output that if produced by all firms leads to marginal cost pricing. Given the assumptions on $C'(\cdot)$ and $p(\cdot)$ it is also unique for fixed *n*. Next define the subsidy

$$s^* = -q^* p'\left([n+1]q^*\right). \tag{5}$$

This is also uniquely identified. The following profitability assumption is imposed.

Assumption 2. (Profitability) $p\left([n+1]q^*\right)q^* - C\left(q^*\right) + s^* q^* \geq 0$.

This assumption guarantees that at the subsidy s^*, the firms can be profitable when producing the marginal cost output level. As will be seen, this condition guarantees that the first-best can be decentralized.

The central result of the paper is as follows.

Theorem *Under Assumptions 1 and 2, for any demand and cost functions such that an equilibrium exists with equal positive output levels for the private firms, the equilibrium output levels and the optimal subsidy are identical for all three cases. Furthermore, the equilibrium is the first-best with price equal marginal cost.*

3. Welfare

The basis of the proof is a reduction of the welfare function (3) to its simplest form. First note that

$$CS(Q) = \int_0^Q p(X)dX - pQ\,. \tag{6}$$

Using (6), (1) and (2), welfare becomes

$$\begin{aligned} W &= CS(Q) + \pi_0 + \sum_{i=1}^{n} \pi_i - sQ \\ &= \int_0^Q p(X)dX - p(Q)Q + p(Q)q_0 - C(q_0) + sq_0 \\ &\quad + \sum_{i=1}^{n} \left[p(Q)q_i - C(q_i) + sq_i \right] - sQ \\ &= \int_0^Q p(X)dX - C(q_0) - \sum_{i=1}^{n} C(q_i)\,. \end{aligned} \tag{7}$$

Expressed in this way it can be seen that the direct effect of the subsidy nets out and its only effect is via the changes in demand that it generates. The social objective is therefore the maximization of surplus in excess of production cost. This is to be expected, since all other values are simply transfers.

4. Mixed Nash and Private Oligopoly

Using (7) and (1), the first-order conditions describing the mixed Nash equilibrium choice of outputs are

$$p\left(Q^m\right) - C'\left(q_0^m\right) = 0\,, \tag{8}$$

and

$$p\left(Q^m\right) + q_i p'\left(Q^m\right) - C'\left(q_i^m\right) + s = 0\,, \tag{9}$$

where the superscript m denotes the equilibrium concept.

Correspondingly, using (2) and (1), the private oligopoly equilibrium solves

$$p\left(Q^o\right) + q_0^o p'\left(Q^o\right) - C'\left(q_0^o\right) + s = 0\,, \tag{10}$$

and

$$p\left(Q^{o}\right)+q_{i}^{o}p'\left(Q^{o}\right)-C'\left(q_{i}^{o}\right)+s=0, \tag{11}$$

where the symmetry assumption implies $q_0^o = q_i^o$, $i = 1,\ldots,n$.

Setting $s=-q_0^o p'\left(Q^o\right)$ equates these two sets of conditions and ensures $q_j^m = q_j^o = q^*$, $j = 0,\ldots,n$. This proves the irrelevance theorem for these two cases since the first-best of $p = C'$ is assumed profitable and can therefore be decentralized..

5. Stackelberg

In the case of Stackelberg equilibrium the first-order condition for each private firm is still given by (1). For the public firm, optimization now needs to take into account the reactions, $\frac{\partial q_i}{\partial q_0}$, of the followers. Hence

$$\frac{\partial W}{\partial q_0}=p'\left(Q^s\right)-C'\left(q_0^s\right)+\sum_{i=1}^{n}\left[p'\left(Q^s\right)-C'\left(q_i^s\right)\right]\frac{\partial q_i}{\partial q_0}=0\,. \tag{12}$$

Applying the optimal subsidy, (1) reduces to $p'(Q)-C'(q_i)=0$ for all $i = 1,\ldots,n$, and hence (12) becomes

$$p'(Q)-C'(q_0)=0\,. \tag{13}$$

These are again the same equilibrium conditions as for mixed Nash and private oligopoly. This proves the theorem.

6. Conclusions

The analysis has extended the irrelevance result of Poyago-Theotoky (2001) to allow for general inverse demand and cost functions. This more general setting has actually lead to a simpler proof. The reason for this is that linearity of the demand function encourages consumer surplus to be written as the well-known $\frac{1}{2}Q^2$ (see equation (3) of Poyago-Theotoky). This obscures the fact that expenditure, pQ, can be eliminated from the expression for welfare to obtain the simpler form given in (7) from which the results follow naturally.

The assumptions used here seem to be the most general for which the irrelevance holds. The only asymmetry possible is to add a number of firms that produce zero output at the first-best equilibrium. Any other asymmetry will not allow the first-best to be sustained for all firms simultaneously. If non-uniqueness of equilibrium were introduced the theorem would have to be weakened to the claim that the first-best (plus the other equilibria) could be decentralized, though no guarantee could be offered that it would actually arise.

References

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