E C O N O M I C S B U L L E T I N

An extended 'Feder' model of dualistic growth

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Abstract

Feder's (1982) model of dualistic growth is derived in levels, suitable for time–series analysis; and (i) extended to contexts where aggregate input data are unavailable; (ii) sectoral externalities and productivity differentials are generalised in a two– and three–sector (agriculture–manufacturing–services) context.

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1. Introduction

In a much-cited contribution to the growth literature, Feder (1982) proposed a model of growth for developing countries (LDCs) that recognised the importance of dualism – in his case, technology differences between sectors. Feder incorporated sectoral disequilibrium in the form of a productivity differential, and externality spillovers between two sectors, into a neoclassical growth model, using an export/non-export distinction. This approach underlies most subsequent investigations of dualistic growth, though an agriculture-manufacturing distinction has more commonly been adopted (see Feder 1986, Hwa 1989, Dowrick 1990 and Dowrick and Gemmell 1991).[1] Below we use this agriculture-manufacturing distinction but the principles can be applied to any dualistic structure.

Feder specifies two sectoral production functions:

$$A = F(K_a, L_a; M) \tag{1}$$
$$M = G(K_m, L_m) \tag{2}$$

where A (M) is agricultural (manufacturing) output, K_i (L_i) is capital (labor) in sector i (= a, m denoting agriculture and manufacturing respectively), and the term in M in (1) captures externalities.[2] Lacking data on sectoral inputs (for a cross-section of countries), Feder (1982) shows that the two production functions can be solved in first differences to express aggregate (A+M) output growth in terms of *aggregate* input growth and the growth rate of the externality-originating sector (M). For Feder's two-sector case and cross-country data this provides a clever solution to his data deficiencies.

Given the weaknesses of cross-section growth regression methods and the growing availability of time-series data for LDCs, an interesting issue is whether the Feder model can be applied in a time-series context. As Engle and Granger (1987) showed, long-run output relationships are revealed through data in *levels*, while short-run behavior is captured by relationships in *first differences*. In a time-series context therefore, Feder's method of first differencing equations (1) and (2) and applying the resulting growth equation to period-averaged data would be inappropriate since it cannot capture the equilibrium relationship embodied in non-stationary data.

This paper shows how the dualistic model can be adapted to a time-series context by solving the model in levels. We also extend the model in three ways. Firstly, we generalise externalities to allow spillovers between *both* sectors. Indeed, for many countries a large fraction of output is generated in the service sector, so that significant interactions may be missed if this sector is ignored. We therefore extend the Feder model to three sectors in an appendix (since this is analytically straightforward and yields empirical rather than analytical insights). Thirdly, as we show, the model in levels requires data on the (aggregate) *stock*, rather than the *flows,* of inputs. Since suitable

[1] Matsuyama (1992) provides an alternative endogenous dualistic growth model based on learning-by-doing in manufacturing. Dowrick (1990) also includes a service sector and Ram (1986) replicates the Feder model for government-private sectors.

[2] M represents externalities here rather than an input since firms in agriculture are assumed to ignore manufacturing outputs in their profit maximising decisions; i.e. 'they are not reflected in market prices' (Feder, 1982, p.61).

capital stock data are often unavailable in LDCs[3], we demonstrate that one additional assumption allows aggregate inputs to be eliminated, which reduces data requirements and overall complexity. Though this combines 'fixed resource' and 'resource growth' effects, we argue that empirically short-run dynamics are likely to approximate the 'fixed resource' case.

2.Adapting and Extending the 'Feder' Model

To solve the Feder model in levels consider a linear approximation of (1) and (2) such that[4]:

$$A = \varphi_a + \alpha_a L_a + \beta_a K_a + \gamma_a M \tag{1'}$$

$$M = \varphi_m + \alpha_m L_m + \beta_m K_m + \gamma_m A \tag{2'}$$

where $0 < \alpha_i, \beta_i < 1; \gamma_i > 0$ ($i = a, m$). Here, two-way spillovers (externalities), γ_i, are possible, marginal products are measured by α_i and β_i, and φ_i translates inputs into outputs. Adopting Feder's (1982, p.61) assumption of marginal productivity differences between sectors:

$$\alpha_m / \alpha_a = \beta_m / \beta_a = 1 + \delta \qquad \delta > 0 \tag{3}$$

and adding (1') and (2') yields:

$$Y = A + M = \varphi + \alpha_a L + \beta_a K + (\delta / 1 + \delta)\{\alpha_m L_m + \beta_m K_m\} + \gamma_a M + \gamma_m A \tag{4}$$

where $L = L_a + L_m$, and $K = K_a + K_m$, ($\varphi = \varphi_a + \varphi_m$). Substituting for $\alpha_m L_m + \beta_m K_m$ from (2'), it can be shown that:

$$A = \varphi' + \left(\frac{1+\delta}{1+\delta-\gamma_m}\right)\{\alpha_a L + \beta_a K\} + \left\{\frac{(1+\delta)\gamma_a - 1}{1+\delta-\gamma_m}\right\}M \tag{5}$$

where $\varphi' = \left(\frac{\varphi(1+\delta)}{1+\delta-\gamma_m}\right)$. Equation (5) is the equivalent in levels of the first difference expression derived by Feder, though in his case *aggregate* output growth appears on the left-hand-side (LHS). Writing *A*, rather than *Y* on the LHS of (5) avoids collinearity problems in empirical tests of Feder's model for which Ram's (1986) application was

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[3] Such constraints can sometimes be overcome by summing (and depreciating) data on aggregate investment over a number of years prior to the period of investigation. Unfortunately such data are unavailable and/or unsuitable in many LDCs, though Nehru and Dhareshwar (1993) have produced some capital stock estimates to 1990.

[4] The model can also be solved for the Cobb-Douglas case, but the resulting expressions for sector outputs (equivalent to (5) and (6) below) are not as directly comparable with Feder's (1982) results. For time-series analysis of a growing economy, the linear approximation is likely to be less restrictive than it would be in cross-section since, empirically, production function shifts (due to technical progress) can be expected to dominate movements along production functions.

criticised.[5] The parameters in (5) reduce to those in Feder's (1982), equation (9), for $\gamma_m = 0$, and with Y on the LHS.

The term $\left\{\frac{(1+\delta)\gamma_a - 1}{1+\delta-\gamma_m}\right\}$ captures the effects on agriculture of manufacturing expansion, for given factor endowments, and may be positive or negative. Externalities have positive effects on agriculture; productivity differentials reduce agricultural output for $\gamma_a < 1$,[6] as expansion of manufacturing, conditional on fixed (*total*) endowments, competes inputs away from agriculture.

The terms $\left(\frac{\alpha_a(1+\delta)}{1+\delta-\gamma_m}\right)$ and $\left(\frac{\beta_a(1+\delta)}{1+\delta-\gamma_m}\right)$ capture the effects of expansion in *L* and *K* respectively. Since $0 < \alpha_a, \beta_a < 1$, and $\delta > 0$, larger values of δ imply lower agricultural outputs - a larger productivity differential implies more of any resource increase is drawn into manufacturing. The externality, γ_m, also affects agriculture positively via factor accumulation, which boosts manufacturing output.

Given data on aggregate capital and labor stocks and sector outputs, equation (5) may be interpreted as capturing the long-run or steady-state relationship and is readily estimated using standard time-series methods. With cointegration between *A* and *M*, for example, an error correction representation identifies short and long-run effects of expansion of aggregate inputs and sectoral productivity. Without such data, the model can be solved to eliminate aggregate inputs. This requires two further assumptions. Firstly, the Feder (1982, 1986) assumption: that sectoral marginal products of labor are proportional to average economy-wide labor productivity: $\alpha_a = \alpha(Y/L)$. Secondly, we make a symmetric assumption for capital productivity: $\beta_a = \beta(Y/K)$. This allows substitution for α_a and β_a in (5), giving:

$$A = \varphi'' + \left\{\frac{(1+\delta)(\gamma_a + \alpha + \beta) - 1}{(1+\delta)(1-(\alpha+\beta)) - \gamma_m}\right\} M \tag{6}$$

where $\varphi'' = \phi \Big/ \left[(1-(\alpha+\beta)) - \frac{\gamma_m}{1+\delta}\right]$. In addition to the effects identified earlier, larger values of α or β imply larger agricultural output since a larger fraction of any endowment expansion goes to agriculture if the sector's capital or labor productivity is relatively high. An additional spillover from manufacturing arises here because manufacturing output expansion raises *Y/L* or *Y/K*, thereby raising agricultural marginal products, α_a, β_a.

Like equation (5), (6) cannot be regarded as a conventional reduced form relationship since both *A* and *M* are potentially endogenous.[7] This suggests that vector autoregressive

[5] See the 'comments' on Ram (1986) in *American Economic Review*, 79 (1989).

[6] If there are no externalities $(\gamma_a = \gamma_m = 0)$, (5) shows that manufacturing competes resources away from agriculture due to the productivity differentials, captured by the term $-1/(1+\delta)$ in (5).

[7] This problem also afflicted Feder's applications, even though endowments were included, since one sector's output is a RHS variable.

(VAR) methods - which treat all variables as potentially endogenous - may provide a valuable model testing tool, especially when aggregate input level data are unavailable. Of course, if (6) is estimated empirically the parameter on M captures the combined effects of externalities and productivity differentials for given resources, and the effects of resource expansion. This necessarily limits the conclusions that can be drawn from any observed inter-sectoral linkages. The VAR approach however allows separate identification of short- and long-run effects. Since in the short-run factor endowments are likely to be relatively fixed, short-run time-series evidence from regressions on (6) should approximate the effects of expansion in M identified in (5). The difference between short- and long-run parameters will then mainly capture the effects of resource growth. If inter-sectoral externalities take some time to come through, short-run effects of manufacturing expansion will primarily reflect resource competition as captured by the parameter element $-1/(1+\delta)$ in (5) and (6).

Finally, an appendix available from the authors presents three-sector equivalents of equations (5) and (6) above, which reveal analogous relationships between agriculture, manufacturing *and* services. Though interactions become more complex, as with two sectors, expansion of either manufacturing or services can have positive or negative net effects on agriculture depending on the size of sector (marginal) productivity differentials, inter-sectoral externalities and sectoral competition for resources.

3. Conclusions

This paper has derived Feder's (1982) model of dualistic growth in levels rather than first differences suitable for time-series applications. We incorporated multiple inter-sectoral externalities, and demonstrated that, with one additional assumption, aggregate inputs can be eliminated. In this latter case, though effects associated with given resource endowments cannot be separated from endowment expansion effects, short-run estimates from time-series regressions are likely to approximate the fixed endowment case.

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Appendix

In a more detailed appendix we derive a three-sector version of the model in Section 2. Here we present the main results. Let *A, M* and *S* represent output in agriculture, manufacturing and services respectively; $L = L_a + L_m + L_s$ and $K = K_a + K_m + K_s$ represent inputs of labor and capital respectively. Sector production functions are given by:

$$A = \alpha_a L_a + \beta_a K_a + \gamma_a^m M + \gamma_a^s S \tag{A1}$$

$$M = \alpha_m L_m + \beta_m K_m + \gamma_m^a A + \gamma_m^s S \tag{A2}$$

$$S = \alpha_s L_s + \beta_s K_s + \gamma_s^m M + \gamma_s^a A \tag{A3}$$

Now, let $\frac{\alpha_i}{\alpha_a} = \frac{\beta_i}{\beta_a} = 1 + \delta_i \qquad i = m, s$ (A4)

Following the procedure of Section 2, a three-sector version of equation (5) is obtained as:

$$A = \left[1 - \frac{\gamma_m^a}{1+\delta_m} - \frac{\gamma_s^a}{1+\delta_s}\right]^{-1} \left(\alpha_a L + \beta_a K\right) + \left[\frac{\gamma_a^m - \frac{1}{1+\delta_m} + \frac{\gamma_s^m}{1+\delta_s}}{1 - \frac{\gamma_m^a}{1+\delta_m} - \frac{\gamma_s^a}{1+\delta_s}}\right] M + \left[\frac{\gamma_a^s - \frac{1}{1+\delta_s} + \frac{\gamma_m^s}{1+\delta_m}}{1 - \frac{\gamma_m^a}{1+\delta_m} - \frac{\gamma_s^a}{1+\delta_s}}\right] S \tag{A5}$$

Eliminating inputs is again achieved by assuming that the marginal productivity of labor and capital in agriculture is proportional to average productivity in the economy as a whole, such that, $\alpha_a = \alpha(Y/L)$ and $\beta_a = \beta(Y/K)$. The three-sector equivalent to equation (6) is then derived as:

$$A = \left[\frac{(1+\delta_m)\left[\alpha + \beta + \gamma_a^m + \frac{\gamma_s^m}{1+\delta_s}\right] - 1}{(1+\delta_m)\left[1 - (\alpha+\beta) - \frac{\gamma_s^a}{1+\delta_s}\right] - \gamma_m^a}\right] M + \left[\frac{(1+\delta_m)\left[\alpha + \beta + \gamma_a^s - \frac{1}{1+\delta_s}\right] + \gamma_m^s}{(1+\delta_m)\left[1 - (\alpha+\beta) - \frac{\gamma_s^a}{1+\delta_s}\right] - \gamma_m^a}\right] S \tag{A6}$$