E C O N O M I C S B U L L E T I N

Noise Trader Risk and the Welfare Effects of Privatization

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Abstract

Excessive volatility of asset prices like that generated in the 'noise trader' model of De Long et al. is one factor that plausibly might contribute to an explanation of the equity premium. We extend the De Long et al. model to allow for privatization of publicly−owned assets and assess the welfare effects of such privatization in the presence of excess volatility arising from noise traders' mistaken beliefs.

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1 Introduction

Privatization of public assets has been a widely-adopted policy in recent years. However, the sale proceeds realized through privatization are less than the expected earnings of the enterprise under continued public ownership, discounted at the real bond rate (Vickers and Yarrow 1988, Quiggin 1995). The fundamental reason for the divergence between sale prices and the present value future earnings is the risk premium demanded by equity investors, which is typically around six percentage points. As was first observed by Mehra and Prescott (1985), the magnitude of the equity premium is a puzzle, since application of the standard consumption capital asset pricing model (CCAPM) with plausible parameters suggests that the premium should be less than one percentage point.

The work of Shiller (1989) suggests an alternative approach to the equity premium puzzle. If, as Shiller argues, financial markets display excess volatility, then returns to holdings of equity are riskier than are the associated streams of corporate profits. Shiller's insight has been formalized in the 'noise trader' model of De Long et al. (1990). In this model, risk over and above that due to the dividend-generating process is introduced into the economy by the distorted and stochastic, beliefs of misinformed investors referred to as 'noise traders'.

In this paper, we consider the implications of volatility generated by noise traders for the welfare and distributional effects of privatization. We modify the De Long et al. model to allow for the existence of an asset that is initially publicly owned, but is otherwise similar to the private asset considered by De Long et al. We then examine the consequences of privatization for asset prices and demands, and for the welfare of different groups. We show that if the equity premium arises from the mistaken beliefs of noise traders, privatization may reduce public sector net worth. Moreover, evaluated in terms of the correct beliefs of sophisticated investors, there is a reduction in social welfare associated with privatization.

2 The Analysis

Following De Long et al. (1990) we introduce a stripped-down overlapping generations model with two-period lived agents. There is a single consumption good but there is no consumption when young, no labor supply decision and no bequest motive. The only decision an agent makes is her choice of portfolio when young to finance her consumption when old.

There is no fundamental risk and all assets pay a fixed real dividend r. One asset, the *safe* asset, is in perfectly elastic supply. Any unit of the safe asset can be converted into one unit of the consumption good, and vice versa. As De Long et al. note, the safe asset is formally equivalent to a storage technology that pays a real net return of r. Furthermore, if we take the consumption good in each period as the numeraire, the price of the safe asset is always one. A second asset, that we shall interpret as the *pre-privatization* economy-wide portfolio of equity, is in fixed supply, normalized to one. The price of this equity asset in period t is denoted by p_t^e. De Long et al. point out that if the price of the equity asset were simply the net present value of its future dividends, then its price in every period would also be 1. But, in the presence of noise traders, De Long et al. show that this is not the case.

Extending De Long et al., we introduce a third asset that is also in fixed supply, x, and that generates a real dividend r. Initially, this asset is owned by the government and financed entirely through short-term (one-period) government debt. Government debt, the fourth asset in our model, pays a guaranteed fixed real interest r. As government debt is a perfect substitute for the safe asset, its price in every period is always one.

Every generation is the same size and can be divided into two classes: a proportion λ who are noise traders (denoted N) whose behavior is described in more detail in subsequent subsections below, and a proportion $1-\lambda$ who are sophisticated investors (denoted I). In each period, the representative sophisticated investor has rational expectations about the distribution of returns from holding a portfolio with risky assets, and so maximizes her expected utility given the distribution of her wealth implied by her portfolio choice.

2.1 The Pre-Privatization Equilibrium

For any period t in which the third asset remains in government ownership, the government issues x units of new debt (of one-period maturation) which is purchased by individuals who are young in period t. Using the proceeds of this bond sale together with the real dividend generated by the government-owned asset, the government pays out the amount $(1+r)\,x$ to the holders of the x units of government debt that was issued in period $t-1$ and that has matured in period t.

In each period t, the representative noise trader who is young in that period misperceives the expected price of the asset in period $t+1$ by an independent and identically normally-distributed random variable

$$d_t^e \sim N\left(d^*, \sigma_d^2\right).$$

We assume that both sophisticated investors and noise traders are expected utility maximizers characterized by a constant coefficient of absolute risk aversion equal to γ. Thus, an agent who is young in period t, chooses her portfolio to maximize her *certainty equivalent consumption* in period $t+1$. That is, she maximizes

$$CE = \overline{w} - \gamma\sigma_w^2/2,$$

where $\overline{w}$ is the expected final wealth in period $t+1$, and σ_w^2 is the variance of her period $t+1$ wealth.

De Long et al. (1990) show that, in a stationary equilibrium, that is, where one imposes the requirement that the unconditional distribution of p_{t+1}^e be identical to the distribution of p_t^e, the pricing rule for equity takes the form

$$p_t^e = 1 + \frac{\lambda\left(d_t^e - d^*\right)}{1+r} + \frac{\lambda d^*}{r} - \frac{\gamma\lambda^2\sigma_d^2}{r\left(1+r\right)^2}, \qquad (1)$$

where $\lambda^2\sigma_d^2/(1+r)^2$ is the constant one-period-ahead variance of p_t^e (see De Long et al. (1990) pp. 708–11 for the derivation.)

We shall assume that noise traders are bullish, $d^* > 0$, and that equities are underpriced, that is, $\mathrm{E}[p_t^e] < 1$, or equivalently,

$$\frac{\gamma\lambda\sigma_d^2}{(1+r)^2} - d^* > 0. \tag{2}$$

2.2 The Post-Privatization Equilibrium

We shall now consider the situation where the government announces at the beginning of period 0 that it is privatizing the third asset which it has held in government ownership up to that date. The amount $(1+r)x$ owing on the outstanding stock of government bonds which are held by the current old generation, will be paid out of the dividend xr generated by the asset and the revenue $p_0^{ne}x$ generated by the sell-off of the x units of supply of this asset at the price p_0^{ne}. Any shortfall (respectively, windfall) will be met by a stream of higher (respectively, lower) taxes on the young in each subsequent period that has the same net present value as the shortfall (respectively, windfall).

As a natural generalization of the De Long et al. model, we assume that the misperceptions for each period t (for $t \geq 0$) of the expected price of equity and the expected price of the privatized asset (the "new equity") in period $t+1$ are independently and identically distributed as bivariate normal

$$\begin{pmatrix} d_t^e \\ d_t^{ne} \end{pmatrix} \sim N\left(d^*\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \sigma_d^2\begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix}\right).$$

We assume that the government has to make its decision to privatize the asset *before* d_0^e and d_0^{ne} are realized.

The term β represents the correlation between misperceptions of the prices of existing private equity and of the newly privatized asset. The explanation of the equity premium provided by De Long et al. requires substantial misperception of systematic risk for private equity, suggesting that β should be near 1.

To aid the exposition let us introduce the following notations:

$$\begin{aligned} \mathbf{d}_t &= \begin{pmatrix} d_t^e \\ d_t^{ne} \end{pmatrix}, \mathbf{p}_t = \begin{pmatrix} p_t^e \\ p_t^{ne} \end{pmatrix}, \boldsymbol{\mu}_t = \begin{pmatrix} \mu_t^e \\ \mu_t^{ne} \end{pmatrix} = \mathrm{E}_t\left[\begin{pmatrix} p_{t+1}^e \\ p_{t+1}^{ne} \end{pmatrix}\right], \\ \boldsymbol{\Sigma}_t &= \left[\mathrm{E}_t\left[\left(p_{t+1}^i - \mu_t^i\right)\left(p_{t+1}^j - \mu_t^j\right)\right]_{i,j=e,ne}\right] \text{ and } \mathbf{1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \end{aligned}$$

Consider a sophisticated investor with a pre-tax amount of labor income y_t facing a tax liability of τ_t. Her objective is to choose a portfolio $\mathbf{q}_I^{\mathrm{T}} = (q_I^e, q_I^{ne})$, where q_I^e is her holding of equity and

q_I^{ne} is her holding of the privatized asset, and the remainder of her post-tax income $(y_t - \tau_t - \mathbf{p}_t^{\mathrm{T}}\mathbf{q}_I)$ is invested in the safe asset and/or government bonds. Her optimal portfolio choice maximizes:

$$(y_t - \tau_t)(1+r) + [r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t]^{\mathrm{T}}\mathbf{q}_I - \frac{\gamma}{2}\mathbf{q}_I^{\mathrm{T}}\boldsymbol{\Sigma}_t\mathbf{q}_I. \tag{3}$$

Similarly, the representative noise trader with an amount $y_t - \tau_t$ to invest in period t, chooses a portfolio $\mathbf{q}_N^{\mathrm{T}} = (q_N^e, q_N^{ne})$ that maximizes

$$(y_t - \tau_t)(1+r) + [r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t]^{\mathrm{T}}\mathbf{q}_N - \frac{\gamma}{2}\mathbf{q}_N^{\mathrm{T}}\boldsymbol{\Sigma}_t\mathbf{q}_N + \mathbf{d}_t^{\mathrm{T}}\mathbf{q}_N. \tag{4}$$

The only difference between (3) and (4) is the last term of (4) which reflects the noise traders' misperceptions of the expected returns from holding equity and from holding the privatized asset.

The corresponding first order conditions derived from (3) and (4) yield the asset demands

$$\mathbf{q}_I = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}[r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t] \tag{5}$$

$$\mathbf{q}_N = \frac{1}{\gamma}\boldsymbol{\Sigma}_t^{-1}[r\mathbf{1} + \boldsymbol{\mu}_t - (1+r)\mathbf{p}_t + \mathbf{d}_t]. \tag{6}$$

Solving for the market-clearing prices we obtain

$$\mathbf{p}_t = \frac{1}{1+r}[r\mathbf{1} + \boldsymbol{\mu}_t + \lambda\mathbf{d}_t] - \frac{\gamma}{1+r}\Sigma_t\begin{pmatrix} 1 \\ x \end{pmatrix}. \tag{7}$$

We confine attention to steady-state equilibria by imposing the requirement that the unconditional distribution of $\mathbf{p}_{t+1}$ be identical to the distribution of $\mathbf{p}_t$. Analogous to the method used by De Long et al (see pp710-11), we can solve (7) recursively to obtain

$$\mathbf{p}_t = \mathbf{1} + \frac{\lambda}{1+r}(\mathbf{d}_t - d^*\mathbf{1}) + \frac{\lambda d^*}{r}\mathbf{1} - \frac{\gamma}{r}\Sigma_t\begin{pmatrix} 1 \\ x \end{pmatrix}. \tag{8}$$

Inspection of (8) reveals a time-invariant variance–covariance matrix for $\mathbf{p}_t$ of

$$\Sigma = \frac{\lambda^2\sigma_d^2}{(1+r)^2}\begin{pmatrix} 1 & \beta \\ \beta & 1 \end{pmatrix}$$

That is, we have

$$p_t^e = 1 + \frac{\lambda(d_t^e - d^*)}{1+r} + \frac{\lambda d^*}{r} - \frac{\gamma\lambda^2\sigma_d^2}{r(1+r)^2}(1+\beta x), \tag{9}$$

$$p_t^{ne} = 1 + \frac{\lambda(d_t^{ne} - d^*)}{1+r} + \frac{\lambda d^*}{r} - \frac{\gamma\lambda^2\sigma_d^2}{r(1+r)^2}(\beta + x). \tag{10}$$

Notice that for small x, the correlation coefficient β is approximately equal to the 'beta' coefficient that would come out of the standard CAPM. More significantly, since preferences display constant absolute risk aversion, and since the sequence of taxes τ_t is determined in period 0, it follows that the level of labor income in any period and the distribution of labor-income taxes between taxpayers in future periods have no impact on asset demands or equilibrium prices.

2.3 Welfare Effects of the Privatization

The intertemporal production technology for the consumption good implicitly embodies a real net return of r. Hence, welfare changes across generations in this economy can be characterized in terms of the net present value of the changes in consumption streams using a discount rate of r.

For $t \geq 0$, let $\Delta\,\mathrm{CE}_t^I$ (respectively, $\Delta\,\mathrm{CE}_t^N$) denote for the representative sophisticated investor (respectively, noise-trader) who is young in period $t-1$, the change that results from the privatization in his or her certainty equivalent consumption in period t. If we let ΔW denote the net present value of the changes in the certainty equivalent consumption of every generation, then the *ex ante* change is given by

$$\mathrm{E}\left[\Delta W\right] = \sum_{t=0}^{\infty} \frac{(1-\lambda)\,\mathrm{E}\left[\Delta\,\mathrm{CE}_t^I\right] + \lambda\,\mathrm{E}\left[\Delta\,\mathrm{CE}_t^N\right]}{(1+r)^t} \tag{11}$$

Let $\overline{z}$ denote the value the variable z would have taken if the government had not privatized the asset in period 0. We consider the changes to present and future consumers.

Consumers in period 0.

These consumers have already made their portfolio choice in the previous period. The only action they undertake in period 0 is to sell their portfolio on the market to finance their consumption. Inspection of (1) and (9) reveals that $\overline{p}_0^e$ and p_0^e have the same variance. Hence it follows that:

$$(1-\lambda)\,\mathrm{E}\left[\Delta\,\mathrm{CE}_0^I\right] + \lambda\,\mathrm{E}\left[\Delta\,\mathrm{CE}_0^N\right] = \mathrm{E}\left[p_0^e - \overline{p}_0^e\right] = -\frac{\gamma\lambda^2\sigma_d^2}{r\,(1+r)^2}\beta x,$$

which means that consumers in period 0 will lose (respectively, gain) if the noise traders' misperception of the expected price of the newly privatized asset in the next period is positively (respectively, negatively) correlated with their misperception of the expected price of the old equity in the next period. As a referee has pointed out, for any financial innovation, holders of existing assets will gain if the new asset's return is negatively correlated with the existing stock market as the price of the existing stocks increases since investors can better diversify their portfolio. Here the only source of uncertainty is the misperception of the noise traders.

Sophisticated consumers in period $t \geq 1$.

For the sophisticated investor who will consume in period t we have

$$\begin{aligned}\Delta\,\mathrm{CE}_t^I &= \left[r\mathbf{1} + \boldsymbol{\mu}_{t-1} - (1+r)\,\mathbf{p}_{t-1}\right]^{\mathrm{T}} \mathbf{q}_I - \frac{\gamma}{2}\mathbf{q}_I^{\mathrm{T}}\boldsymbol{\Sigma}\mathbf{q}_I \\ &- \left(r + \mathrm{E}_{t-1}\left[\overline{p}_t^e\right] - (1+r)\,\overline{p}_{t-1}^e\right)\overline{q}_I^e + \frac{\gamma}{2}\frac{\lambda^2\sigma_d^2}{(1+r)^2}\left(\overline{q}_I^e\right)^2 - (1+r)\,E\left[\Delta\tau_{t-1}\right]\end{aligned}$$

where $\Delta\tau_t$ denotes the change in taxation of labor income of the young in period $t \geq 1$ that arises as a result of the privatization and where $\Delta\tau_0 = 0$.

As is shown in an appendix available from the authors, this may be expressed as

$$\Delta\,\mathrm{CE}_t^I = \frac{(1+r)^2}{2\gamma\sigma_d^2} \times \frac{\left(\beta d_{t-1}^e - d_{t-1}^{ne}\right)^2}{\left(1-\beta^2\right)} + \lambda x\left(\frac{\gamma\lambda\sigma_d^2}{(1+r)^2}\left(\beta + \frac{x}{2}\right) - d_{t-1}^{ne}\right) - (1+r)\,E\left[\Delta\tau_{t-1}\right].$$

Thus,

$$\mathrm{E}\left[\Delta\,\mathrm{CE}_t^I\right]=\frac{(1+r)^2\left((1-\beta)(d^*)^2+(1+\beta)\,\sigma_d^2\right)}{2\gamma(1+\beta)\,\sigma_d^2}+\lambda x\left(\frac{\gamma\lambda\sigma_d^2}{(1+r)^2}\left(\beta+\frac{x}{2}\right)-d^*\right)-(1+r)\,E\left[\Delta\tau_{t-1}\right]. \tag{12}$$

Noise consumers' expected utility with respect to true distribution

For a noise trader who will consume in period t we have two possible measures of the impact on welfare. Expected utility may be calculated either with respect to the true distribution or with respect to the distorted distribution that forms the basis of asset trading decisions. In this paper, welfare will be measured with respect to the *true* distribution of consumption.[1] The only agents this affects are the noise traders who will be consuming in periods $1, 2, \ldots$. Let $\Delta\widehat{\mathrm{CE}}_t^N$ denote this value.

$$\Delta\widehat{\mathrm{CE}}_t^N=\Delta\,\mathrm{CE}_t^N-\left(\mathbf{d}_{t-1}^{\mathrm{T}}\mathbf{q}_N-d_{t-1}^e\overline{q}_N^e\right)$$

Recall

$$\mathbf{q}_N=\frac{1}{\gamma}\Sigma^{-1}\left[r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}+\mathbf{d}_{t-1}\right].$$

From (7) we have

$$r\mathbf{1}+\boldsymbol{\mu}_{t-1}-(1+r)\,\mathbf{p}_{t-1}+\mathbf{d}_{t-1}=\gamma\Sigma\begin{bmatrix}1\\x\end{bmatrix}+(1-\lambda)\,\mathbf{d}_{t-1},$$

and hence

$$\mathbf{d}_{t-1}^{\mathrm{T}}\mathbf{q}_N-d_{t-1}^e\overline{q}_N^e=xd_{t-1}^{ne}+\frac{(1-\lambda)(1+r)^2}{\gamma\lambda^2\sigma_d^2\left(1-\beta^2\right)}\left(\beta^2\left(d_{t-1}^e\right)^2-2\beta d_{t-1}^e d_{t-1}^{ne}+\left(d_{t-1}^{ne}\right)^2\right).$$

This gives us a corrected measure of the change in *ex ante* welfare of

$$\mathrm{E}\left[\Delta\widehat{W}\right]=-\frac{\lambda x}{r}\left(d^*-\frac{\gamma\lambda\sigma_d^2 x}{2(1+r)^2}\right)-\frac{(1-\lambda)(1+r)^2\left((1-\beta)(d^*)^2+(1+\beta)\,\sigma_d^2\right)}{2r\lambda\gamma(1+\beta)\,\sigma_d^2}-\sum_{t=1}^{\infty}\frac{E\left[\Delta\tau_t\right]}{(1+r)^t} \tag{13}$$

Notice that the difference between (**??**) and (13) is

$$\frac{\lambda x}{r}d^*+\frac{(1-\lambda)(1+r)^2\left((1-\beta)(d^*)^2+(1+\beta)\,\sigma_d^2\right)}{r\lambda\gamma(1+\beta)\,\sigma_d^2}.$$

The first term reflects the extra consumption that noise traders' misperceptions lead them to expect, on average, to receive by holding the privatized asset while the second term reflects the certainty-equivalent consumption cost of the risk associated with taking bets based on misperceived expected future prices of the privatized asset.

Finally, we may evaluate the expected net present value of the tax change as

$$\begin{aligned}\sum_{t=1}^{\infty}\frac{E\left[\Delta\tau_t\right]}{(1+r)^t} &= \left(x(1+r)-xr-\mathrm{E}\left[p_0^{ne}\right]x\right)\\ &= x\left(1-\mathrm{E}\left[p_0^{ne}\right]\right)\\ &= \frac{\lambda x}{r}\left(d^*-\frac{\gamma\lambda\sigma_d^2}{(1+r)^2}(\beta+x)\right).\end{aligned} \tag{14}$$

[1] Analysis with respect to the perceived distribution is undertaken in an Appendix available from the authors.

Assuming that condition (2) is satisfied, so that there is a positive equity premium, the government will be worse off whenever $\beta + x$ is sufficiently close to or greater than 1.

And by substituting (14) into (13) we see that overall ex ante welfare (with respect to the true distribution) is necessarily lower whenever $\beta \geq 0$ since

$$\mathrm{E}\left[\Delta\widehat{W}\right] = -\frac{\gamma\lambda^2\sigma_d^2 x\left(2\beta + x\right)}{2\left(1+r\right)^2} - \frac{\left(1-\lambda\right)\left(1+r\right)^2\left(\left(1-\beta\right)\left(d^*\right)^2 + \left(1+\beta\right)\sigma_d^2\right)}{2r\lambda\gamma\left(1+\beta\right)\sigma_d^2}$$

The converse holds if $\beta < 0$, x is small enough that the correlation of the new asset with market returns remains negative after privatization and λ, the proportion of noise traders, is sufficiently close to 1. In these circumstances, partial privatization, combined with appropriate lump-sum transfers, will yield an ex ante Pareto-improvement.

3 Concluding comments

In this paper, we have considered the implications of the 'noise trader risk' model of De Long et al for the analysis of privatisation. In this model, the misperceptions held by noise traders create risk which has real social costs, reflected in the risk premium for equity.

As we have shown, the creation of additional equity through privatisation may either exacerbate or mitigate the effects of noise trader risk. The crucial issue is the correlation between noise trader misperceptions of existing private equity and of the equity newly created by privatisation. If this correlation is low, privatisation will create opportunities for diversification and therefore reduce risk. If it is positive, privatisation will increase total risk and reduce welfare.

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