E C O N O M I C S   B U L L E T I N

On KPSS with GARCH errors

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Abstract

In this paper we discuss the finite sample behavior of the KPSS test in the presence of conditionally heteroskedastic errors. We confirm that under stationary GARCH errors the asymptotics of the KPSS remains valid. However, in finite samples we observe a slight size distortion and a power distortion. Interestingly, IGARCH errors do not seem to affect the size of the test, however they may often cause a substantial loss of power.

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1 Introduction

Recently, mostly motivated by practical applications, there has been an increasing interest in analyzing the effects that errors with time varying variance have on unit root tests. Kim and Schmidt (1993) found that Dickey-Fuller (1979,1981) tests tend to overreject in the presence of GARCH errors, with the problem becoming most severe in the case of near integrated variance. Nelson *et al.* (2001) and Cavaliere (2003) examine the asymptotic properties of unit root tests with Markov switching variances. Seo (1999) and Ling *et al.* (2003) investigate the asymptotic distribution theory of unit root tests with GARCH errors and propose likelihood-based unit root tests. Cavaliere and Taylor (2004) provide a new approach to unit root testing in the presence of a general class of permanent variance changes. Cavaliere (2004) investigates the effects of a permanent variance shift on the stationarity test of Kwiatkowski *et al.* (1992), KPSS hereafter, and Cavaliere and Taylor (2005a) analyze the effects of a general class of time varying variances (including GARCH) on the KPSS test.

This paper extends the literature on stationarity tests by examining the case where the errors are conditionally heteroskedastic. Specifically, we study the finite sample behavior of KPSS tests in the presence of conditionally heteroskedastic errors of the form GARCH(1,1) as proposed in Bollerslev(1986). Since many economic and financial series display conditional heteroskedasticity we believe that it is interesting to assess whether and to what extent the presence of GARCH errors affects the performance of the KPSS test.

The paper is organized as follows. Section 2 gives the background to the KPSS test, section 3 discusses the asymptotic properties of the test under GARCH errors. In section 4 we examine the finite sample performance of the KPSS test when the error term follows various ARMA-GARCH processes. A brief summary concludes.

2 The KPSS Test

The KPSS tests the null hypothesis that a series is I(0) against the alternative that the series is I(1). This is done in the context of the unobserved component model:

$$y_t = d_t + \mu_t + u_t \tag{1}$$
$$\mu_t = \mu_{t-1} + \epsilon_t \tag{2}$$

where $y_t, \quad t = 1 \dots T$ are the observed data, d_t is a deterministic component, u_t satisfies the strong mixing conditions of Phillips and Perron (1988) with long run variance σ_u^2, $\epsilon \sim$ i.i.d.$(0, \sigma_\epsilon^2)$, and the initial value μ_0 is treated as fixed and serves the role of an intercept. The null hypothesis is therefore $H_0 : \sigma_\epsilon^2 = 0$ and the alternative is $H_1 : \sigma_\epsilon^2 > 0$.

The KPSS test is constructed using the residuals $\{\hat{u}_t\}_{t=1}^T$ from the regression of y_t on d_t. As in KPSS, we focus on the two cases: (i) $d_t = \mu$ a constant; (ii) $d_t = \mu + \tau t$ a constant plus a time trend. The KPSS test rejects H_0 in favor of H_1 for large values of the statistic

$$\hat{\eta} = \frac{T^{-2} \sum_{t=1}^T (\sum_{i=1}^t \hat{u}_i)^2}{\hat{\sigma}_u^2} \tag{3}$$

where $\hat{\sigma}_u^2$ is a consistent estimate of σ_u^2 (see Kwiatkowski *et al.* 1992 p. 164 for details). In the constant case we identify the statistic as $\hat{\eta}_\mu$, while in the linear trend case it will be $\hat{\eta}_\tau$. Representations for the limit null distribution of the test statistic in the two cases and the relative critical values are found in Kwiatkowski *et al.*, (1992) pp. 164-167.

3 Properties of KPSS under GARCH errors

Here we investigate the behavior of the KPSS statistics when y_t is generated by:

$$y_t = d_t + \mu_t + u_t \tag{4}$$
$$\mu_t = \mu_{t-1} + \epsilon_t \tag{5}$$
$$u_t = \nu_t \sqrt{h_t}, \quad h_t = \omega + \alpha u_{t-1}^2 + \beta h_{t-1} \tag{6}$$

where y_t and d_t are as above, $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\{\nu_t\}$ is a sequence of i.i.d. real-valued random variables with continuous density (with respect to a Lebesgue measure on the real line) which is positive on $(-\infty, +\infty)$, and is independent of h_0, and has $E(\nu_t) = 0$, $E(\nu_t^2) = 1$, and $E(\nu_t^4) < \infty$. Furthermore $\epsilon_t \sim i.i.d.(0, 1)$ and ν_t and ϵ_t are independent of each other.

In order for the KPSS test to be robust to the presence of conditional heteroskedasticity it is required that u_t satisfies the regularity conditions of Phillips and Perron (1988) or alternatively, either the linear process conditions of Phillips and Solo (1989) or the martingale central limit theorem as in Hall and Heyde (1980).

As demonstrated in Carrasco and Chen (2002) (pp 24-25), under suitable moment conditions, the necessary and sufficient condition for covariance stationarity of GARCH sequences $\alpha + \beta < 1$ (see Bollerslev (1986)) is the sufficient condition for strict stationarity and exponential β-mixing of u_t. Therefore, under these conditions the limit distribution of the KPSS statistic under stable GARCH(1,1) errors remains valid [1]. However, this condition excludes cases where $\alpha + \beta = 1$, the IGARCH case (see Nelson 1990). In such a case, for $\omega > 0$, the unconditional variance of u_t diverges, and u_t does not satisfy the definition of a covariance stationary process (nor does u_t^2).

In the next section we report the finite sample performance of the KPSS test under different ARMA-GARCH specifications of the residuals.

4 Finite Sample Results

To investigate the finite sample accuracy of the KPSS statistics under conditionally heteroskedastic errors, we produce 50000 samples of sizes $T = 50, 100, 200$ using the following DPG:

$$y_t = d_t + \mu_t + u_t \tag{7}$$
$$\mu_t = \mu_{t-1} + \epsilon_t \tag{8}$$
$$(1 - \phi L) u_t = (1 + \theta L) e_t \tag{9}$$
$$e_t = \nu_t \sqrt{h_t}, \quad h_t = \omega + \alpha e_{t-1}^2 + \beta h_{t-1} \tag{10}$$

where as before $\nu_t \sim iid(0, 1)$, We study the finite (small) sample properties of $\hat{\eta}_\mu$ and $\hat{\eta}_\tau$ for $\phi = 0, 0.5, 0.8, 0.9$, $\theta = 0, 0.4, -0.4$, $\omega = 0.1$, $\alpha = 0.2$ $\beta = 0.7, 0.8$.

[1] Note also that stable GARCH processes with finite fourth moments are martingale difference sequences, and therefore the martingale central limit theorem in Hall and Heyde (1980) may also be used to establish convergence of the partial sums of u_t. Furthermore, GARCH models are believed to satisfy the conditions for linear precesses of Phillips and Solo 1989 (see also Stock, 1994).

Note that (see Kwiatkowski *et al.* 1992), as for the i.i.d. case the size of the test depends only on the sample size (T), and the number of lags (l) used to calculate $\hat{\sigma}_u^2$. Here, we are interested on the effect of introducing both an ARMA structure (defined by ϕ and θ), and a stationary GARCH(1,1) or an integrated GARCH effects. As in the original work from Kwiatkowski *et al.* (1992), we consider three values of l as a function of T: $l0 = 0$, $l4 = integer[4(T/100)^{1/4}]$, and $l12 = integer[12(T/100)^{1/4}]$. Tables 1-3 display the size properties of the KPSS for T=50, 100, 200 respectively, offering a comparison between the accuracy of the test with i.i.d., ARMA, ARMA-GARCH, and ARMA-IGARCH errors.

Examining the i.i.d. case first, we can see that the test is correctly sized for $l0$, regardless of the sample size T considered. However, as l increases the test rejects the null too seldom, and this under-rejection improves only very slowly with larger samples. Applying stationary GARCH (1,1) errors seems to add something to these effects. For the $l0$ case, the test rejects more often than in the pure i.i.d. case, and this effect decreases slowly for larger T. When considering the case of $l4$ and $l12$, the effect of GARCH (1,1) errors is to cause KPSS tests to reject even less often than when the errors are white noise.

Going on to the statistical properties of the KPSS with ARMA errors, the results obtained confirm, for a given value of θ, the expected over-rejection for larger $\phi + \theta$ and under-rejection when $\phi + \theta < 0$. it seems that the severity of the over-rejection (under-rejection) depends on how large and positive (negative) $\phi + \theta$ is. The results for the ARMA-GARCH (1,1) cases, are not too different from the simple ARMA ones. However, an exception is provided by the case where there is near cancellation with $\phi = 0.5$ and $\theta = -0.4$. In this case, we observe that in presence of slightly autocorrelated and conditionally heteroskedastic errors the KPSS test tends to reject less often than it should as compared with the case where errors are slightly autocorrelated. It is important to notice that this effect persists for all the sample sizes considered.

An interesting result is that the KPSS test does not seem to be greatly affected by the presence of IGARCH errors. In this case, Cavaliere and Taylor (2004), show that the scaled partial sums of u_t weakly converge to a non-standard Brownian motion whose variance depends on the underlying volatility process (rather than a standard Brownian motion), and consequently the distribution of the numerator of the KPSS statistic (3) is not of the form given in Kwiatkowski *et al.* (1992) pp164-167. Moreover, it is not clear what the denominator of (3) will converge to, or indeed if it does converge at all, given that the long run variance is diverging. Notice that, Cavaliere and Taylor (2005a) also find little size distortion in unit root tests under IGARCH innovations. This issue is interesting and merits further investigation, although this is beyond the scope of the present paper.

Lastly, in tables 4, 5 and 6, we present simulation results giving the size adjusted power of the KPSS test in the presence of i.i.d. and GARCH(1,1) errors as a function of only two relevant parameters, namely T and $\lambda = \sigma_\epsilon^2/\sigma_u^2$. This is done for T=50, 100, 200, and λ between 0.001 and 1000. In the stationary GARCH case, the different values of λ are obtained by simulating the process in (7-10) holding $\sigma_\epsilon^2 = 1$ and, recalling that the long run variance of u_t, $\sigma_u^2 = \omega/1 - (\alpha + \beta)$, keeping $\alpha = 0.2$ and $\beta = 0.7$, and allowing only ω to vary in order to achieve the desired values of σ_u^2 and consequently of λ.

In general, for a given sample size T, the power of the KPSS test increases with λ, even though it does not necessarily reach unity as $\lambda \to \infty$. Also, as expected, for fixed λ, power increases with T, reflecting the consistency of the test. However the rate at which this happens depends strongly on l, so that choosing a larger l will cost power. Examining the simulation results in greater detail, we note that for a given T the presence of stationary GARCH(1,1) errors seems to 'improve' the power of the KPSS test as compared with the case of simple i.i.d. errors, especially when λ is smaller. However, this is true only when the GARCH process is stationary. In the case where errors are IGARCH and λ is small the power of KPSS tests is very low. Indeed, it needs to be highlighted that in this case the ratio λ is difficult to computate as $\sigma_u \to \infty$ and therefore the values for λ in that case are not going to be exactly the ones displayed on the table. Yet, it is again interesting that for larger values of λ this lack of power disappears. It is not straightforward to provide an explanation for what happens, mainly because it is not clear what really happens to the quantity λ. Again, this is another interesting issue, and as such will constitute object of further research. As a final note, it is worth highlighting that, unlike other studies on unit root testing with stationary GARCH errors (Kim and Schmidt, 1993) changing the values of α and β does not provide different results from the one presented, nor does the presence of near integrated GARCH errors.

5 Summary

In this paper, we have discussed the finite sample properties of the KPSS test in the presence of conditionally heteroskedastic errors, and analyzed by means of Monte Carlo methods its finite sample performance under various ARMA-GARCH specifications. To summarize, while the presence of stationary GARCH errors which satisfy certain moments conditions are known not to alter the asymptotic distribution of the KPSS test, in finite samples we observe a slight size distortion when we apply a stationary GARCH effect to i.i.d. or slightly autocorrelated innovations. Also we observe that presence of stationary GARCH(1,1) errors seems to distort the power of the KPSS tests as compared with the case of simple i.i.d. errors. Finally, while they do not satisfy the standard regularity conditions of the Functional Central Limit Theorem, IGARCH errors do not seem to significantly alter the size properties of the test; however, they cause a great loss of power especially in relatively small samples.

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Table 1: Size of η_μ and η_τ at 5% level with ARMA, ARMA-GARCH, and ARMA-IGARCH errors

				T=50					
					η_μ			η_τ	
α	β	ϕ	θ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
0	0	0	0	0.0483	0.0390	0.0099	0.0543	0.0415	0.0650
0.2	0.7	0	0	0.0614	0.0371	0.0087	0.0759	0.0426	0.0579
0.2	0.8	0	0	0.0543	0.0414	0.0094	0.0611	0.0448	0.0634
0	0	0	0.4	0.1549	0.0527	0.0122	0.2133	0.0582	0.0688
0.2	0.7	0	0.4	0.1575	0.0513	0.0105	0.2216	0.0598	0.0598
0.2	0.8	0	0.4	0.1615	0.0580	0.0135	0.2235	0.0654	0.0721
0	0	0	-0.4	0.0006	0.0100	0.0050	0.0004	0.0091	0.0560
0.2	0.7	0	-0.4	0.0042	0.0101	0.0044	0.0044	0.0116	0.0489
0.2	0.8	0	-0.4	0.0014	0.0122	0.0041	0.0010	0.0096	0.0583
0	0	0.5	0	0.3418	0.1000	0.0182	0.4926	0.1176	0.0649
0.2	0.7	0.5	0	0.3360	0.1021	0.0168	0.4746	0.1162	0.0646
0.2	0.8	0.5	0	0.3351	0.0989	0.0194	0.4728	0.1173	0.0671
0	0	0.5	0.4	0.4470	0.1088	0.0182	0.6557	0.1320	0.0685
0.2	0.7	0.5	0.4	0.4414	0.1110	0.0165	0.6385	0.1331	0.0623
0.2	0.8	0.5	0.4	0.4416	0.1142	0.0181	0.6429	0.1338	0.0663
0	0	0.5	-0.4	0.1059	0.0583	0.0128	0.1293	0.0648	0.0654
0.2	0.7	0.5	-0.4	0.0621	0.0383	0.0086	0.0742	0.0423	0.0543
0.2	0.8	0.5	-0.4	0.1064	0.0613	0.0120	0.1275	0.0619	0.6540
0	0	0.8	0	0.7421	0.2861	0.0360	0.8909	0.3353	0.0646
0.2	0.7	0.8	0	0.7421	0.2940	0.0382	0.8703	0.3371	0.0681
0.2	0.8	0.8	0	0.7317	0.2840	0.0378	0.8728	0.3365	0.0700
0	0	0.8	0.4	0.7881	0.2914	0.0378	0.9333	0.3523	0.0708
0.2	0.7	0.8	0.4	0.7871	0.2992	0.0379	0.9221	0.3613	0.0707
0.2	0.8	0.8	0.4	0.7847	0.2995	0.0392	0.9223	0.3532	0.0696
0	0	0.8	-0.4	0.5575	0.2386	0.0336	0.6744	0.2646	0.0656
0.2	0.7	0.8	-0.4	0.5431	0.2404	0.0321	0.6454	0.2733	0.0671
0.2	0.8	0.8	-0.4	0.5570	0.2446	0.0347	0.6506	0.2739	0.0690
0	0	0.9	0	0.8750	0.4666	0.0829	0.9403	0.5050	0.0898
0.2	0.7	0.9	0	0.8750	0.4666	0.0829	0.9403	0.5050	0.0898
0.2	0.8	0.9	0	0.8733	0.4484	0.0704	0.9454	0.4961	0.0937
0	0	0.9	0.4	0.9046	0.4577	0.0761	0.9720	0.5210	0.0899
0.2	0.7	0.9	0.4	0.9029	0.4744	0.0811	0.9661	0.5251	0.0956
0.2	0.8	0.9	0.4	0.8986	0.4480	0.0736	0.9696	0.5112	0.0923
0	0	0.9	-0.4	0.7821	0.4173	0.0662	0.8388	0.4389	0.0913
0.2	0.7	0.9	-0.4	0.7718	0.4256	0.0734	0.8211	0.4480	0.0922
0.2	0.8	0.9	-0.4	0.7679	0.4210	0.0671	0.8226	0.4324	0.0879

Table 2: Size of η_μ and η_τ at 5% level with ARMA, ARMA-GARCH, and ARMA-IGARCH errors

				T=100					
					η_μ			η_τ	
α	β	ϕ	θ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
0	0	0	0	0.0495	0.0441	0.0311	0.0528	0.0473	0.0375
0.2	0.7	0	0	0.0636	0.0434	0.0265	0.0731	0.0482	0.0335
0.2	0.8	0	0	0.0595	0.0511	0.0355	0.0601	0.0526	0.0382
0	0	0	0.4	0.1568	0.0615	0.0372	0.2179	0.0658	0.0410
0.2	0.7	0	0.4	0.1605	0.0593	0.0316	0.2260	0.0684	0.0390
0.2	0.8	0	0.4	0.1569	0.0641	0.0371	0.2320	0.0753	0.0432
0	0	0	-0.4	0.0004	0.0108	0.0175	0.0001	0.0093	0.0202
0.2	0.7	0	-0.4	0.0029	0.0103	0.0126	0.0031	0.0112	0.0168
0.2	0.8	0	-0.4	0.0015	0.0128	0.0174	0.0008	0.0095	0.0201
0	0	0.5	0	0.3578	0.1085	0.0463	0.5451	0.1362	0.0540
0.2	0.7	0.5	0	0.3524	0.1143	0.0436	0.5096	0.1373	0.0509
0.2	0.8	0.5	0	0.3540	0.1191	0.0550	0.5187	0.1430	0.0523
0	0	0.5	0.4	0.4637	0.1208	0.0504	0.6970	0.1558	0.0548
0.2	0.7	0.5	0.4	0.4572	0.1208	0.0432	0.6721	0.1566	0.0521
0.2	0.8	0.5	0.4	0.4553	0.1256	0.0528	0.6818	0.1633	0.0598
0	0	0.5	-0.4	0.1107	0.0652	0.0382	0.1396	0.0746	0.0431
0.2	0.7	0.5	-0.4	0.0592	0.0426	0.0254	0.0731	0.0468	0.0321
0.2	0.8	0.5	-0.4	0.1152	0.0720	0.0404	0.1412	0.0788	0.0438
0	0	0.8	0	0.7981	0.3115	0.0982	0.9552	0.4407	0.1090
0.2	0.7	0.8	0	0.7926	0.3151	0.0921	0.9400	0.4509	0.1106
0.2	0.8	0.8	0	0.7881	0.3123	0.1026	0.9342	0.4441	0.1094
0	0	0.8	0.4	0.8394	0.3275	0.1025	0.9792	0.4606	0.1147
0.2	0.7	0.8	0.4	0.8318	0.3310	0.0945	0.9626	0.4626	0.1115
0.2	0.8	0.8	0.4	0.8281	0.3316	0.1006	0.9624	0.4623	0.1149
0	0	0.8	-0.4	0.6317	0.2715	0.0901	0.8132	0.3725	0.1020
0.2	0.7	0.8	-0.4	0.6173	0.2742	0.0868	0.7759	0.3734	0.1003
0.2	0.8	0.8	-0.4	0.6143	0.2745	0.0933	0.7936	0.3748	0.1094
0	0	0.9	0	0.9434	0.5297	0.1867	0.9927	0.6978	0.2090
0.2	0.7	0.9	0	0.9380	0.5425	0.1969	0.9882	0.6978	0.2135
0.2	0.8	0.9	0	0.9329	0.5330	0.1947	0.9894	0.6816	0.2147
0	0	0.9	0.4	0.9530	0.5427	0.1938	0.9953	0.6976	0.2162
0.2	0.7	0.9	0.4	0.9537	0.5536	0.1995	0.9935	0.7135	0.2262
0.2	0.8	0.9	0.4	0.9440	0.5359	0.1917	0.9915	0.6858	0.2157
0	0	0.9	-0.4	0.8834	0.5024	0.1863	0.9600	0.6462	0.2040
0.2	0.7	0.9	-0.4	0.8679	0.5056	0.1872	0.9446	0.6480	0.2044
0.2	0.8	0.9	-0.4	0.8710	0.4950	0.1863	0.9510	0.6354	0.2049

Table 3: Size of η_μ and η_τ at 5% level with ARMA, ARMA-GARCH, and ARMA-IGARCH errors

				T=200					
					η_μ			η_τ	
α	β	ϕ	θ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
0	0	0	0	0.0496	0.0464	0.0406	0.0523	0.0489	0.0426
0.2	0.7	0	0	0.0589	0.0453	0.0356	0.0701	0.0507	0.0385
0.2	0.8	0	0	0.0603	0.0549	0.0436	0.0666	0.0610	0.0489
0	0	0	0.4	0.1600	0.0626	0.0458	0.2232	0.0703	0.0475
0.2	0.7	0	0.4	0.1602	0.0614	0.0403	0.2297	0.0717	0.0447
0.2	0.8	0	0.4	0.1577	0.0683	0.0483	0.2265	0.0761	0.0500
0	0	0	-0.4	0.0002	0.0110	0.0247	0.0001	0.0096	0.0250
0.2	0.7	0	-0.4	0.0015	0.0108	0.0185	0.0017	0.0106	0.0194
0.2	0.8	0	-0.4	0.0020	0.0181	0.0293	0.0005	0.0135	0.0248
0	0	0.5	0	0.3720	0.1174	0.0593	0.5646	0.1466	0.0651
0.2	0.7	0.5	0	0.3578	0.1124	0.0527	0.5347	0.1449	0.0585
0.2	0.8	0.5	0	0.3597	0.1229	0.0622	0.5415	0.1573	0.0708
0	0	0.5	0.4	0.4741	0.1273	0.0604	0.7192	0.1688	0.0673
0.2	0.7	0.5	0.4	0.4680	0.1261	0.0550	0.6834	0.1703	0.0644
0.2	0.8	0.5	0.4	0.4521	0.1292	0.0670	0.6824	0.1727	0.0727
0	0	0.5	-0.4	0.1130	0.0705	0.0503	0.1442	0.0786	0.0510
0.2	0.7	0.5	-0.4	0.0568	0.0452	0.0354	0.0698	0.0497	0.0386
0.2	0.8	0.5	-0.4	0.1196	0.0785	0.0557	0.1513	0.0847	0.0573
0	0	0.8	0	0.8389	0.3367	0.1178	0.9778	0.5077	0.1451
0.2	0.7	0.8	0	0.8227	0.3423	0.1108	0.9630	0.5087	0.1437
0.2	0.7	0.8	0	0.8060	0.3265	0.1135	0.9642	0.4913	0.1434
0	0	0.8	0.4	0.8663	0.3477	0.1179	0.9867	0.5172	0.1433
0.2	0.7	0.8	0.4	0.8578	0.3500	0.1160	0.9794	0.5233	0.1413
0.2	0.8	0.8	0.4	0.8439	0.3439	0.1195	0.9766	0.5079	0.1491
0	0	0.8	-0.4	0.6764	0.2965	0.1103	0.8869	0.4339	0.1315
0.2	0.7	0.8	-0.4	0.6633	0.2999	0.1081	0.8477	0.4286	0.1268
0.2	0.8	0.8	-0.4	0.6516	0.2931	0.1130	0.8556	0.4281	0.1404
0	0	0.9	0	0.9708	0.5860	0.2264	0.9992	0.7990	0.2969
0.2	0.7	0.9	0	0.9661	0.5974	0.2352	0.9980	0.8024	0.3120
0.2	0.8	0.9	0	0.9555	0.5691	0.2232	0.9980	0.7827	0.2999
0	0	0.9	0.4	0.9751	0.6007	0.2341	0.9993	0.8022	0.3068
0.2	0.7	0.9	0.4	0.9708	0.6049	0.2281	0.9986	0.8134	0.3120
0.2	0.8	0.9	0.4	0.9709	0.5887	0.2289	0.9991	0.7930	0.3070
0	0	0.9	-0.4	0.9324	0.5675	0.2248	0.9912	0.7632	0.2948
0.2	0.7	0.9	-0.4	0.9217	0.5645	0.2251	0.9830	0.7663	0.2947
0.2	0.8	0.9	-0.4	0.9160	0.5483	0.2159	0.9851	0.7514	0.2933

Table 4: Power of η_μ and η_τ at 5% level

				η_μ			η_τ		
T	α	β	λ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
50	0	0	0.001	0.0741	0.706	0.0642	0.0570	0.0548	0.0505
	0.2	0.7	0.001	0.1043	0.1087	0.0907	0.0603	0.0615	0.0515
	0.2	0.8	0.001	0.0450	0.0480	0.0503	0.0467	0.0491	0.0468
	0	0	0.01	0.2879	0.2466	0.1771	0.1260	0.1067	0.0500
	0.2	0.7	0.01	0.3960	0.3591	0.2569	0.1819	0.1602	0.1130
	0.2	0.8	0.01	0.0529	0.0558	0.0479	0.0538	0.0525	0.0479
	0	0	0.1	0.7280	0.5643	0.3770	0.5731	0.3449	0.438
	0.2	0.7	0.1	0.7751	0.6023	0.4102	0.6214	0.4052	0.1817
	0.2	0.8	0.1	0.1313	0.1215	0.1000	0.0746	0.0735	0.0470
	0	0	1	0.9240	0.6616	0.4305	0.9088	0.5231	0.2791
	0.2	0.7	1	0.9233	0.6725	0.4509	0.9030	0.5280	0.2905
	0.2	0.8	1	0.4518	0.3746	0.2656	0.2533	0.1899	0.1465
	0	0	10	0.9568	0.6787	0.4411	0.9639	0.5544	0.3318
	0.2	0.7	10	0.9497	0.6839	0.4540	0.9524	0.5492	0.3500
	0.2	0.8	10	0.8102	0.6039	0.4109	0.6929	0.4142	0.2409
	0	0	100	0.9584	0.6772	0.4398	0.9703	0.5572	0.3395
	0.2	0.7	100	0.9522	0.6857	0.4587	0.9585	0.5547	0.3605
	0.2	0.8	100	0.9310	0.6578	0.4356	0.9242	0.5273	0.3393
	0	0	1000	0.9603	0.6806	0.4444	0.9700	0.5614	0.3398
	0.2	0.7	1000	0.9524	0.6803	0.4542	0.9630	0.5504	0.3651
	0.2	0.8	1000	0.9495	0.6670	0.4402	0.9632	0.5505	0.3390

Table 5: Power of η_μ and η_τ at 5% level

					η_μ			η_τ	
T	α	β	λ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
100	0	0	0.001	0.1738	0.1623	0.1445	0.0854	0.0793	0.0730
	0.2	0.7	0.001	0.2455	0.2545	0.2296	0.1048	0.1153	0.1014
	0.2	0.8	0.001	0.0494	0.0501	0.0501	0.0521	0.0503	0.0489
	0	0	0.01	0.5899	0.5238	0.4332	0.3517	0.2924	0.2051
	0.2	0.7	0.01	0.6769	0.6130	0.5043	0.4541	0.4149	0.2829
	0.2	0.8	0.01	0.0822	0.0817	0.0768	0.0559	0.0571	0.0540
	0	0	0.1	0.9232	0.7735	0.5990	0.8779	0.6964	0.4037
	0.2	0.7	0.1	0.9367	0.7967	0.6167	0.8983	0.7380	0.4331
	0.2	0.8	0.1	0.2727	0.2505	0.2228	0.1272	0.1226	0.1033
	0	0	1	0.9892	0.8288	0.6213	0.9928	0.8156	0.4531
	0.2	0.7	1	0.9881	0.8413	0.6416	0.9884	0.8256	0.4687
	0.2	0.8	1	0.6873	0.5966	0.4858	0.4989	0.4078	0.2679
	0	0	10	0.9943	0.8378	0.6285	0.9980	0.8297	0.4530
	0.2	0.7	10	0.9926	0.8426	0.6413	0.9965	0.8370	0.4737
	0.2	0.8	10	0.9294	0.7753	0.5966	0.9008	0.7173	0.4110
	0	0	100	0.9948	0.8398	0.6279	0.9988	0.8369	0.4611
	0.2	0.7	100	0.9927	0.8448	0.6400	0.9972	0.8359	0.4742
	0.2	0.8	100	0.9858	0.8169	0.6143	0.9895	0.8085	0.4476
	0	0	1000	0.9948	0.8349	0.6262	0.9986	0.8355	0.4587
	0.2	0.7	1000	0.9932	0.8415	0.6349	0.9977	0.8369	0.4776
	0.2	0.8	1000	0.9948	0.8269	0.6178	0.9981	0.8236	0.4531

Table 6: Power of η_μ and η_τ at 5% level

					η_μ			η_τ	
T	α	β	λ	$l0$	$l4$	$l12$	$l0$	$l4$	$l12$
200	0	0	0.001	0.4026	0.3849	0.3507	0.1855	0.1743	0.1523
	0.2	0.7	0.001	0.4613	0.4608	0.4258	0.2262	0.2414	0.2194
	0.2	0.8	0.001	0.0596	0.0619	0.0607	0.0491	0.0489	0.0490
	0	0	0.01	0.8482	0.7845	0.6677	0.7228	0.6456	0.5039
	0.2	0.7	0.01	0.8730	0.8208	0.7014	0.7631	0.7141	0.5750
	0.2	0.8	0.01	0.1380	0.1366	0.1365	0.0734	0.0749	0.0694
	0	0	0.1	0.9904	0.9274	0.7605	0.9901	0.9230	0.7148
	0.2	0.7	0.1	0.9890	0.9304	0.7674	0.9852	0.9299	0.7343
	0.2	0.8	0.1	0.4848	0.4577	0.4189	0.2703	0.2507	0.2180
	0	0	1	0.9993	0.9476	0.7701	0.9999	0.9610	0.7471
	0.2	0.7	1	0.9987	0.9491	0.7816	0.9993	0.9630	0.7688
	0.2	0.8	1	0.8716	0.8120	0.6915	0.7728	0.7030	0.5465
	0	0	10	0.9997	0.9493	0.7742	1.0000	0.9645	0.7486
	0.2	0.7	10	0.9995	0.9504	0.7864	0.9999	0.9658	0.7648
	0.2	0.8	10	0.9837	0.9242	0.7573	0.9789	0.9186	0.7063
	0	0	100	0.9997	0.9503	0.7773	1.0000	0.9653	0.7535
	0.2	0.7	100	0.9995	0.9483	0.7820	1.0000	0.9654	0.7678
	0.2	0.8	100	0.9979	0.9422	0.7732	0.9988	0.9529	0.7548
	0	0	1000	0.9998	0.9489	0.7745	1.0000	0.9665	0.7529
	0.2	0.7	1000	0.9995	0.9513	0.7861	1.0000	0.9640	0.7664
	0.2	0.8	1000	0.9993	0.9471	0.7838	0.9998	0.9614	0.7395