E C O N O M I C S B U L L E T I N

A note on output hedging with basis risk− an extension of Paroush and Wolf hedging model

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Abstract

We extend the model of Paroush and Wolf by using a general utility and general distributions.

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Several studies included basis risk [1] in hedging models, including Anderson and Danthine (1983), Newbery and Stiglitz (1981), Lapan and Moschini (1994), Li and Vukina (1998). But none of these studies investigated the impact of basis risk on the hedging and production decisions of the firm. Paroush and Wolf (1989) investigated such decisions. In their interesting paper, they presented a model of output hedging with basis risk, where both output and hedging are the decision variables. In so doing, they defined and specified the basis risk and discussed its sources. The main objective of their paper is to show the impact of the basis risk on the decision variables. They showed that the separation property does not hold in the presence of basis risk and that the basis risk adversely affects the decision variables. That is, the presence of the basis risk reduces the optimal output and hedging. They used a second-order Taylor's approximation of the utility function. It is worth noting that using such approximations is equivalent to assuming normality and constant absolute risk aversion or a quadratic utility. In their case, it is equivalent to assuming normality and constant absolute risk aversion (as they explicitly assumed constant absolute risk aversion). Their results may not be obtainable for a higher-order Taylor's expansion. Thus there is a loss of generality. Using a non-hedging model, Alghalith (2005) highlighted the limitations of the second-order Taylor's approximation of the utility function. The choice of the appropriate functional form is ultimately empirical. For example, Alghalith (2003) and Arshanapalli and Gupta (1996) empirically rejected constant absolute risk aversion and quadratic utility. Theoretically, Adam-Mueller (2003) emphasized the role of prudence; in addition, he assumed a positive prudence (decreasing absolute risk aversion) as a natural hypothesis.

Paroush and Wolf did not analyze the impact of basis risk on the hedge ratio- another relevant decision variable in addition to output and hedging. Some papers in the literature address the hedge ratio as a decision variable. Even though the impact on the hedge ratio can be determined from the production and the hedging decisions in Paroush and Wolf (1989), it is more conventional to discuss the impact on the hedge ratio as a single decision variable. Examples of using the hedge ratio as a decision variable include Lapan and Moschini (1994) and Rolfo (1980). Paroush and Wolf (1992) exhibited the same limitations in the case of input hedging. The absence of

[1] Basis risk exists if the futures price deviates from the price of the delivered commodity. The sources of basis risk are quality, timing, and location.

such results constitutes a gap in the hedging literature.

Consequently, this note extends their main results in two ways. First, it uses a general utility function and general distributions. Second, it determines the impact of basis risk on the hedge ratio. Below is a description of the model.

The risk averse firm maximizes the expected utility of the profit

$$\underset{y,h}{Max} Eu(\pi),$$

where u is a Neumann-Morgenstern utility function (continuous differentiable $u' > 0, u'' < 0$). The profit function is specified by

$$\pi = py + (b - g)h - c(y)$$

$$p = \bar{p} + \sigma\varepsilon; E\varepsilon = 0$$

$$g = p + \delta\eta; E\eta = 0; E\varepsilon\eta = 0, Var(g) = \sigma^2 + \delta^2,$$

where y is the output, h is the hedging, p is the random spot price with mean $\bar{p}$ and variance σ^2, ε is random, b is the current non-random futures price, g is the random future futures price, η is a random term representing the basis risk and independent of ε, δ is a measure of basis risk[2], and $c(y)$ is the cost function $(c'(y) > 0, c''(y) > 0)$.

The first-order conditions are

$$Eu'[p - c'(y)] = [\bar{p} - c'(y)] Eu' + Cov(u', p) = 0 \tag{1}$$

$$Eu'[b - g] = [b - \bar{p}] Eu' - \sigma Eu'\varepsilon - \delta Cov(u', \eta) = 0 \tag{2}$$

We use the superscripts $*$ and $\circ$ to denote the optimal values in the absence and the presence of basis risk, respectively.

Proposition 1. $y^* > y^\circ$

Proof. Adding (1) and (2), we obtain $c'(y^\circ) = b - \delta Cov(u', \eta)/Eu'$.In the absence of basis risk, $c'(y^*) = b$ and thus $c'(y^*) > c'(y^\circ)$ since $Cov(u', \eta) > 0$. The result is intuitive since basis risk is an additional risk, which is independent of the price risk.

Proposition 2. $h^* > h^\circ$

[2]An increase in δ means an increase in basis risk In the absence of basis risk, $\delta = 0$ and thus $\pi = py + (b - p)h - c(y)$.

Proof. Define the sets A and $\sim A$ such that

$$A = \{p | p - c'(y^\circ) \geq 0\}$$

$$\sim A = \{p | p - c'(y^\circ) \leq 0\}$$

When $h^\circ > y^\circ$, for any $p \in A$ and $p' \in \sim A$, we must have

$$\pi^\circ(p) \leq \pi^\circ(p'); p \in A, p' \in \sim A,$$

since $u'' < 0$, the inequality above implies

$$u'(\pi^\circ(p)) \geq u'(\pi^\circ(p')); p \in A, p' \in \sim A$$

Therefore, since u' is contiuous

$$I \equiv \underset{p \in A}{Inf} u'(\pi^\circ) = S \equiv \underset{p' \in \sim A}{Sup} u'(\pi^\circ).$$

Since S and I are both positive, there must exist a positive constant, t, such that

$$\frac{E_\eta S}{u'(E_\eta \pi^\circ)} = t = \frac{E_\eta I}{u'(E_\eta \pi^\circ)},$$

so that

$$tu'(E_\eta \pi^\circ) = E_\eta S \geq E_\eta u'(\pi^\circ), p \in \sim A, \tag{3}$$

since S is a maximum; multiplying both sides by $p - c'(y^\circ)$ yields

$$(p - c'(y^\circ)) tu'(E_\eta \pi^\circ) \leq (p - c'(y^\circ)) E_\eta u'(\pi^\circ), p \in \sim A, \tag{4}$$

since $p - c'(y^\circ) < 0$ for $p \in \sim A$.

Similarly,

$$tu'(E_\eta \pi^\circ) = E_\eta I \leq E_\eta u'(\pi^\circ), p \in A,$$

since I is a minimum; thus

$$(p - c'(y^\circ)) tu'(E_\eta \pi^\circ) \leq (p - c'(y^\circ)) E_\eta u'(\pi^\circ), p \in A. \tag{5}$$

Therefore,

$$(p - c'(y^\circ)) tu'(E_\eta \pi^\circ) \leq (p - c'(y^\circ)) E_\eta u'(\pi^\circ), \forall p. \tag{6}$$

Taking expectations with respect to ε, we obtain

$$tE_\varepsilon\left(p-c'\left(y^\circ\right)\right)u'\left(E_\eta\pi^\circ\right)\leq E_\varepsilon\left\{\left(p-c'\left(y^\circ\right)\right)E_\eta u'\left(\pi^\circ\right)\right\}=Eu'\left(\pi^\circ\right)\left(p-c'\left(y^\circ\right)\right)=0^3 \tag{7}$$

Now, let $\alpha\equiv E_\epsilon\left(p-c'\left(y\right)\right)u'\left(E_\eta\pi\right)$, then (7) implies that $d\alpha\leq 0$ in response to the introduction of basis risk (since, by the first-order-condition in the absence of basis risk, $\alpha\left(y^*,h^*\right)=0$ and from (7) $\alpha\left(y^\circ,h^\circ\right)\leq 0$). Totally differentiating α (and holding the parameters constant), we obtain

$$d\alpha=E_\varepsilon u''\left[p-b\right]^2\left(dy-dh\right)-c''\left(y\right)E_\varepsilon u'dy\leq 0 \tag{8}$$

and thus $dh<0$ since $dy<0$ in response to basis risk.The proof is similar when $h^\circ\leq y^\circ$. The result is also intuitive since a decline in production tends to reduce the hedging.

Proposition 3. $h^*/y^*>h^\circ/y^\circ$ if $h^*\leq y^*$

Proof. From Proposition 2, $dy>dh$ and thus $|dh|>|dy|$; therefore

$$d\left(h/y\right)=\frac{y^*dh-h^*dy}{y^{*2}}<0$$

in response to basis risk. That is, the hedge ratio falls, given the firm does not initially over-hedge. This result is expected, since the basis risk makes hedging less appealing. Hedging becomes less effective since it will not completely offset the adverse impact of the risks on output.[4]

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[3]By the independence assumption, $EX=E_\varepsilon\left(E_\eta X\right)$. The right-hand-side of (7) equals zero by the first-order condition.

[4]That is, the separation property does not hold. Separation holds if output is independent of the probability distributions and attitudes toward the risk (production decisions are separate from financial decisions). Note that, there is separation in the absence of basis risk.

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