On the robustness of concealing cost information in oligopoly

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Abstract

Competing firms are usually better informed about their own cost parameters than about those of their rivals. Therefore, it is an important issue to study the incentives of firms to exchange private cost information. We resolve and further generalize an influential model of Raith (1996) and show that, independent of the number of firms, concealing cost information is a dominant firm strategy in heterogeneous Bertrand oligopolies with substitutive as well as with complementary goods.

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1 Introduction

Competing firms are usually better informed about their own cost and demand parameters than about those of their rivals. Therefore, it is an important issue in the Industrial Organization theory to study the incentives of firms to exchange private information (see, e.g., *Vives* 1999, ch. 8). The literature on information sharing in oligopoly is vast. However, most papers study Cournot competition in homogeneous markets. Only few authors deal with price competition in heterogeneous markets, even if this kind of competition is most important in the industrial sectors of an economy. *Vives* (1984) and *Sakai* (1986) have concentrated on the exchange of demand information, while *Gal-Or* (1986) and *Sakai* (1991) have studied the expected gains of exchanging cost information. These Bertrand duopoly models yield the well known result that firms reveal their private information in Bayesian equilibrium under demand uncertainty, while under cost uncertainty they do not. In an influential paper, *Raith* (1996) has developed a unified approach in which he derives the results of these models as special cases.

While it is generally accepted in the literature that the duopoly results with demand uncertainty generalize to oligopolistic market structures, *Raith* (1996, p. 279) has argued that with cost uncertainty "*... results obtained for duopolies do not extend to larger markets*". Instead, for a large number of rivals in the market, unilateral revelation of private cost information should be a dominant strategy. This far-reaching conclusion indicates that the concealing strategy of firms is not very robust. As has been pointed out by *Jin* (2000), however, this surprising implication is solely based on an algebraic error in the original model.

The present paper therefore presents a general solution of *Raith*'s (1996) model for the theoretically and empirically most interesting case of cost uncertainty and derives the robust result that concealing cost information is an unambiguously dominant strategy in price competition with substitutive as well as with complementary goods.

2 The Model

We consider a market consisting of $n \geq 2$ firms, each producing a differentiated good. According to most models in the information sharing literature, we assume a quasi-linear quadratic utility function

$$U\left(q_0, \mathbf{q}\right) = q_0 + \sum_{i=1}^{n} \alpha_i q_i - \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} q_i q_j \tag{1}$$

of a representative consumer. Each consumer demands q_0 units of the numéraire good and q_i units of the differentiated goods $i = 1, \ldots, n$, represented by the vector $\mathbf{q} = (q_1, \ldots, q_n)'$. We impose the usual parameter restrictions $\alpha_i > 0$, $\beta_{ij} = \beta > 0\ \forall\ i = j$ and $\beta_{ij} = \gamma \in \left(-\frac{1}{n-1}\beta; \beta\right)\ \forall\ i \neq j$. Consumers maximize utility subject to the budget constraint

$$q_0 + \sum_{i=1}^{n} p_i q_i \leq I, \tag{2}$$

where I denotes income and $\mathbf{p} = (p_1, \ldots, p_i, \ldots, p_n)'$ is the price vector of goods $i = 1, \ldots, n$. If income is large enough, the first-order conditions determining the optimal consumption levels of all goods lead to the linear demand functions

$$D_i\left(\mathbf{p}\right) = a_i - b p_i + d \sum_{j \neq i} p_j \quad , \qquad i, j = 1, \ldots, n \tag{3}$$

where $a_i := \frac{[\beta+(n-2)\gamma]\alpha_i - \gamma \sum_{j\neq i} \alpha_j}{(\beta-\gamma)[\beta+(n-1)\gamma]}$, $b := \frac{\beta+(n-2)\gamma}{(\beta-\gamma)[\beta+(n-1)\gamma]}$ and $d := \frac{\gamma}{(\beta-\gamma)[\beta+(n-1)\gamma]}$, which implies $d \in \left(-b; \frac{1}{n-1}b\right)$. In this general model, corresponding to the demand parameters γ and d, the goods are substitutes $(\gamma, d > 0)$, complements $(\gamma, d < 0)$ or independent $(\gamma = d = 0)$.

We assume the same type of information structure as in *Gal-Or* (1986), i.e., firms know the distribution function of their constant unit cost, but are only imperfectly informed about the realizations. The deviations τ_i from the expected values c_i are independently and identically normally distributed with means zero and variances $t \geq 0$, i.e. $\tau_i \sim N(0,t)$.[1]

If firms knew the realization of their respective deviation parameters τ_i, but were uncertain about the rivals' cost parameters, the underlying information structure would be a standard one of asymmetric information. However, the assessment of the advantage or disadvantage of information exchange becomes more complicated if firms have to decide about their information revelation behavior at a point in time when information about their own unit cost is also uncertain. As in the *Gal-Or* (1986) model, we simultaneously analyze both stochastic uncertainty of firms about their own unit cost as well as asymmetric information between the competitors.

Firm i's ex ante observed signal for the deviation parameter τ_i is $\varphi_i = \tau_i + \psi_i$, where the signal errors ψ_i are also assumed to be independently and identically normally distributed with means zero and variances $u \geq 0$, i.e. $\psi_i \sim N(0,u)$. Thus, firms can make no inferences about the unit cost of their rivals based on their private cost information. However, each firm has the option to signal its perception of unit cost to the rivals. In a very general way we may account for the precision of a strategic information revelation by specifying the signal as $\hat{\varphi}_i = \varphi_i + \xi_i$. The strategic revelation deviations ξ_i are also assumed to be independently and identically normally distributed random variables with means zero and variances $r_i \geq 0$, i.e. $\xi_i \sim N(0,r_i)$. If the signal is sent with zero variance $(r_i = 0)$, firm i perfectly reveals its cost information. In the case of an infinitely high variance $(r_i \to \infty)$, it conceals its private information.[2] All firms have to make their pricing decisions using all available cost information, represented by their respective information sets $\mathbf{z_i} = (\varphi_i, \hat{\boldsymbol{\varphi}})'$ with the vector $\hat{\boldsymbol{\varphi}}' = (\hat{\varphi}_1, \ldots, \hat{\varphi}_n)$ of revealed information by all firms.

[1] There may be parameter constellations where the nonnegativity constraint of unit cost is not fulfilled if the random variables are assumed to be normally distributed. However, this distribution function which is usually applied in the information exchange literature can be interpreted as an approximation of any specific distribution function or as the result of the additive clustering of several independently distributed singular random variables.

[2] Under these circumstances, strategic lying in the revelation process as modeled by *Ziv* (1993) is excluded. Firms only have the option to reveal their cost information with an arbitrarily large noise. This means that concealing occurs by announcing and sending worthless signals.

Given the information set $\mathbf{z_i}$, the ex ante expected profit function of firm i is

$$\mathrm{E}^i\left[\pi^i(\mathbf{p})\middle|\,\mathbf{z_i}\right] \;=\; \mathrm{E}^i\left\{\left\{[p_i-(c_i+\tau_i)]\left(a_i-bp_i+d\sum_{j\neq i}p_j\right)\right\}\middle|\,\mathbf{z_i}\right\} \tag{4}$$

where E is the expected value operator. The necessary first-order conditions[3] lead to the reaction functions

$$p_i(\mathbf{z_i}) \;=\; \frac{a_i+b\left[c_i+\mathrm{E}^i(\tau_i|\,\mathbf{z_i})\right]+d\sum_{j\neq i}\mathrm{E}^i\left[p_j(\mathbf{z_j})|\,\mathbf{z_i}\right]}{2b} \quad . \tag{5}$$

Since the resulting equilibrium strategies are affine in the information sets $\mathbf{z_i}$, the proposed solution equations take the form

$$p_i(\mathbf{z_i}) \;=\; \eta_{0i}+\eta_{1i}\varphi_i+\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}} \quad . \tag{6}$$

In order to solve for prices in Bayesian equilibrium, we have to determine the coefficients $\eta_{0i},\eta_{1i}\in\mathbb{R}$ and $\boldsymbol{\eta_{2i}}\in\mathbb{R}^n$. For the expected price decisions of the competitors we obtain

$$\mathrm{E}^i\left[p_j(\mathbf{z_j})|\,\mathbf{z_i}\right] \;=\; \eta_{0j}+\eta_{1j}\mathrm{E}^i(\varphi_j|\,\mathbf{z_i})+\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\,, \quad i,j=1,\ldots,n, \quad i\neq j \quad . \tag{7}$$

Due to the assumptions of the normal distributions, the conditional means $\mathrm{E}^i(\varphi_j|\,\mathbf{z_i})$ solve as $\mathrm{E}^i(\varphi_j|\,\mathbf{z_i})=\frac{t+u}{t+u+r_j}\mathbf{e_j}'\hat{\boldsymbol{\varphi}}$, where $\mathbf{e_j}$ is the j-th unit vector. In an analogous way, the expected deviations of unit cost from their mean are $\mathrm{E}^i(\tau_i|\,\mathbf{z_i})=\frac{t}{t+u}\varphi_i$. By inserting these conditional means together with equation (7) into the reaction functions (5) and equating the resulting expressions with the proposed solution equations (6), the coefficients can be identified as

$$\eta_{0i} \;=\; \frac{[2b-(n-2)\,d]\,(a_i+bc_i)+d\sum_{j\neq i}(a_j+bc_j)}{(2b+d)\,[2b-(n-1)\,d]} \tag{8}$$

$$\eta_{1i} \;=\; \frac{t}{2\,(t+u)} \tag{9}$$

[3] The sufficient conditions for a profit maximum are globally met. In order to simplify the analysis, we generally assume parameter values that guarantee Bayesian equilibria with positive quantities for all firms.

$$\boldsymbol{\eta_{2i}} = t\left\{\frac{bd}{(2b+d)\left[2b-(n-1)\,d\right]}\mathbf{v} - \frac{d}{2\,(2b+d)\,(t+u+r_i)}\mathbf{e_i}\right\} \tag{10}$$

with the uncertainty vector $\mathbf{v} := \left(\frac{1}{t+u+r_1}, \ldots, \frac{1}{t+u+r_n}\right)'$. Consequently, $\boldsymbol{\eta_{2i}}$ contains the elements

$$\eta_{2ii} = \frac{(n-1)\,d^2 t}{2\,(2b+d)\left[2b-(n-1)\,d\right](t+u+r_i)} \tag{11}$$

$$\eta_{2ij} = \frac{bdt}{(2b+d)\left[2b-(n-1)\,d\right](t+u+r_j)} \quad \forall\, i \neq j \quad . \tag{12}$$

Inserting these expressions into the solution equations (6), we obtain the optimal pricing strategy

$$\begin{aligned} p_i\left(\mathbf{z_i}\right) = {} & \frac{\left[2b-(n-2)\,d\right](a_i+bc_i)+d\sum_{j\neq i}(a_j+bc_j)}{(2b+d)\left[2b-(n-1)\,d\right]} \\ & +t\left\{\frac{1}{2\,(t+u)}\varphi_i + \frac{(n-1)\,d^2}{2\,(2b+d)\left[2b-(n-1)\,d\right]}\frac{1}{t+u+r_i}\hat{\varphi}_i\right. \\ & \left. +\frac{bd}{(2b+d)\left[2b-(n-1)\,d\right]}\sum_{i\neq j}\frac{1}{t+u+r_j}\hat{\varphi}_j\right\} \quad . \end{aligned} \tag{13}$$

In order to calculate the ex ante expected equilibrium profits, we substitute the optimal pricing strategy (13) into (4). Making use of the distributional properties of the deviation parameters τ_i and taking the expected value over all possible information sets $\mathbf{z_i}$ yields the ex ante expected equilibrium profits (see the Appendix A.1):

$$\begin{aligned} \mathrm{E}_{\mathbf{z_i}}\left[\pi_i\left(\mathbf{p}\right)\right] = {} & \left(a_i - b\eta_{0i} + d\sum_{j\neq i}\eta_{0j}\right)(\eta_{0i}-c_i) + bt^2\left\{\frac{1}{4\,(t+u)}\right. \\ & +\frac{(n-1)\,d^2\left[3\,(n-1)\,d^2+4\,(n-2)\,bd-8b^2\right]}{4\,(2b+d)^2\left[2b-(n-1)\,d\right]^2}\frac{1}{t+u+r_i} \\ & \left. +\frac{b^2d^2}{(2b+d)^2\left[2b-(n-1)\,d\right]^2}\sum_{j\neq i}\frac{1}{t+u+r_j}\right\} \end{aligned} \tag{14}$$

Differentiating these reduced-form profit functions with respect to the revelation variances r_i yields

$$\frac{\partial \mathrm{E}_{\mathbf{z_i}}\left[\pi_i\left(\mathbf{p}\right)\right]}{\partial r_i} = -\frac{(n-1)\,bd^2\left[3\,(n-1)\,d^2+4\,(n-2)\,bd-8b^2\right]}{4\,(2b+d)^2\left[2b-(n-1)\,d\right]^2}\frac{t^2}{(t+u+r_i)^2} \,. \tag{15}$$

As is shown in Appendix A.2, this derivative equals zero for $d = 0$, but has a positive sign for all $d \neq 0$. Figure 1 illustrates this result for the case of substitutive goods ($0 < (n-1)d < b$), i.e. for a fixed parameter d which implies that the Figure 1 depends only on the variables b and n. Therefore, in contrast to the corresponding

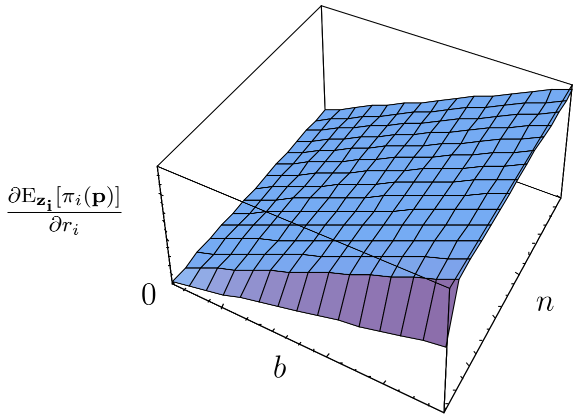

Figure 1: Derivative (15) in the case of substitutive goods.

result derived by *Raith* (1996), firms producing substitutive goods will generally choose an infinite variance ($r_i \to \infty$) in information transmission which is equivalent to concealing their private cost information. Using this conceiling strategy, firms weaken the price competition and, hence, increase their expected profits. Thus, *Gal-Or*'s (1986) result for a duopolistic market indeed generalizes to oligopolistic market structures. Even for a large number of firms, unilateral revelation of private cost information never constitutes a Bayesian equilibrium strategy.

As is shown in Figure 2, with complementary goods (i.e. d assumed to be within the range $-b < d < 0$), firms also gain by choosing infinitely high variances r_i because there is again a positive effect on the expected profit level. Thus, independent of the level of the demand parameter b and independent of the number of firms in the market, also in the case of complementary goods firms always conceal their private cost information.

In the special case of independent goods ($d = 0$) the derivative simplifies to $\frac{\partial E_{\mathbf{z_i}}[\pi_i(\mathbf{p})]}{\partial r_i} = 0$. As in such an industry firms behave as monopolists in their respective markets, there will be no effect of better or worse information about the unit costs of the other firms on their own expected profit.

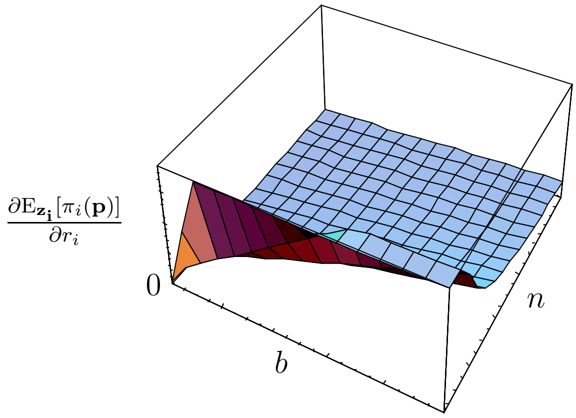

Figure 2: Derivative (15) in the case of complementary goods.

Therefore, our generalized model setup shows that Bertrand oligopolists always conceal their private cost information. Silence is the golden strategy in all the cases covered by our general model. This information policy does not depend on the number of firms in the market and holds for substitutive as well as for complementary goods. Cost information exchange never constitutes a Bayesian equilibrium strategy.

Consequently, using the solution equations (6) with the coefficients defined in equations (8) to (10), we obtain the equilibrium prices

$$p_i\left(\mathbf{z_i}\right) = \frac{1}{2b}\left[\left(a_i + d\sum_{j\neq i}\eta_{0j}\right) + bc_i + b\frac{t}{t+u}\varphi_i\right] \tag{16}$$

and from (14) the expected equilibrium profits

$$\mathrm{E}_{\mathbf{z_i}}\left[\pi_i\left(\mathbf{p}\right)\right] = \left(a_i - b\eta_{0i} + d\sum_{j\neq i}\eta_{0j}\right)\left(\eta_{0i} - c_i\right) + b\frac{t^2}{4\left(t+u\right)} \quad . \tag{17}$$

Obviously, cost uncertainty softens competition and rises expected profits, i.e. firms gain from a higher cost uncertainty.

3 Conclusion

By resolving and further generalizing an influential model of *Raith* (1996) we showed by theoretical analysis that, independent of the number of firms, concealing cost information is a dominant strategy in heterogeneous Bertrand oligopolies with substitutive as well as with complementary goods.

Furthermore, the presented model explains and supports the empirical observation that firms generally refuse to reveal private cost information. As our analysis has shown, concealing successful efforts in obtaining process innovations should be an important firm strategy, especially if patent protection is not perfect.

Appendix

A.1 Ex Ante Expected Equilibrium Profits

Substituting the optimal price strategies (13) into (4) and taking the expected value over all possible information sets $\mathbf{z_i}$, in a first step leads to the following expression of the ex ante expected equilibrium profits:

$$\begin{aligned}
\mathrm{E}_{\mathbf{z_i}}\left[\pi_i\left(\mathbf{p}\right)\right] \quad = \quad & \left(a_i - b\eta_{0i} + d\sum_{j\neq i}\eta_{0j}\right)\left(\eta_{0i} - c_i\right) - b\eta_{1i}^2\mathrm{E}\left(\varphi_i^2\right) - b\eta_{1i}\mathrm{E}\left(\varphi_i \boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) \\
& + d\eta_{1i}\sum_{j\neq i}\eta_{1j}\mathrm{E}\left(\varphi_i\varphi_j\right) + d\eta_{1i}\sum_{j\neq i}\mathrm{E}\left(\varphi_i\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\right) - b\eta_{1i}\mathrm{E}\left(\varphi_i\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) \\
& - b\mathrm{E}\left(\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) + d\sum_{j\neq i}\eta_{1j}\mathrm{E}\left(\varphi_j\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) + d\sum_{j\neq i}\mathrm{E}\left(\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\right) \\
& + b\eta_{1i}\mathrm{E}\left(\tau_i\varphi_i\right) + b\mathrm{E}\left(\tau_i\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) - d\sum_{j\neq i}\eta_{1j}\mathrm{E}\left(\tau_i\varphi_j\right) - d\sum_{j\neq i}\mathrm{E}\left(\tau_i\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\right)
\end{aligned} \tag{18}$$

Making use of the distributional properties of the deviation parameters τ_i, the private signals φ_i and the signals $\hat{\varphi}_i$, $i = 1, \ldots, n$, we obtain:

$$\mathrm{E}\left(\varphi_i^2\right) = t + u \tag{19}$$
$$\mathrm{E}\left(\tau_i\varphi_i\right) = t \tag{20}$$
$$\mathrm{E}\left(\tau_i\varphi_j\right) = 0 \tag{21}$$
$$\mathrm{E}\left(\varphi_i\varphi_j\right) = 0 \tag{22}$$
$$\mathrm{E}\left(\tau_i\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) = t\eta_{2ii} \tag{23}$$
$$\mathrm{E}\left(\tau_i\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\right) = t\eta_{2ji} \tag{24}$$
$$\mathrm{E}\left(\varphi_i\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) = \left(t+u\right)\eta_{2ii} \tag{25}$$
$$\mathrm{E}\left(\varphi_i\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\right) = \left(t+u\right)\eta_{2ji} \tag{26}$$
$$\mathrm{E}\left(\varphi_j\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) = \left(t+u\right)\eta_{2ij} \tag{27}$$
$$\mathrm{E}\left(\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\right) = \left(t+u+r_i\right)\eta_{2ii}^2 + \sum_{j\neq i}\left(t+u+r_j\right)\eta_{2ij}^2 \tag{28}$$
$$\begin{aligned}
\mathrm{E}\left(\boldsymbol{\eta_{2i}}'\hat{\boldsymbol{\varphi}}\boldsymbol{\eta_{2j}}'\hat{\boldsymbol{\varphi}}\right) = & \left(t+u+r_i\right)\eta_{2ii}\eta_{2ji} + \left(t+u+r_j\right)\eta_{2ij}\eta_{2jj} \\
& + \sum_{\substack{k\neq i \\ k\neq j}}\left(t+u+r_k\right)\eta_{2ik}\eta_{2jk}
\end{aligned} \tag{29}$$

Inserting these expressions (19) - (29) into (18) and rearranging yields the ex ante expected equilibrium profits (14).

A.2 Impact of Revelation Variances on Ex Ante Equilibrium Profits

For $d \neq 0$, $-\frac{(n-1)bd^2t^2}{4(2b+d)^2[2b-(n-1)d]^2(t+u+r_i)^2} < 0$, so that the sign of (15) only depends on the sign of:

$$3(n-1)d^2 + 4(n-2)bd - 8b^2 \tag{30}$$

As this is a convex quadratic function of the demand parameter d there exist up to two values of d (furtheron defined as d_1 and d_2) which imply (30) to be zero. In the case of independent goods (i.e. $d = 0$) we obtain:

$$3(n-1)d^2 + 4(n-2)bd - 8b^2 \;\; = \;\; -8b^2 \; < \; 0 \tag{31}$$

Whether this negative sign holds for the entire range of substitutability of goods $d \in \left(-b; \frac{1}{n-1}b\right)$ can be proofed by an analysis of d_1 and d_2: If neither d_1 nor d_2 is located inside the range of d there is no change of the sign, wehereas if d_1 and/or d_2 are located inside there will happen a change of the sign of (15). The values of d_1 and d_2 can be derived as:

$$\begin{aligned} d_{1,2} &= \frac{-4(n-2)b \pm \sqrt{16(n-2)^2 b^2 + 96(n-1)b^2}}{6(n-1)} \\ &= \frac{-2(n-2)b \pm 2b\sqrt{n^2+2n-2}}{3(n-1)} \end{aligned} \tag{32}$$

If

$$d_1 = \frac{-2(n-2)b + 2b\sqrt{n^2+2n-2}}{3(n-1)} \tag{33}$$

is located inside the range of $d \in \left(-b; \frac{1}{n-1}b\right)$, the condition $-b < d_1 < \frac{1}{n-1}b$ must hold. This implies on the one hand $d_1 > -b$, i.e.:

$$\begin{aligned} \frac{-2\,(n-2)\,b+2b\sqrt{n^2+2n-2}}{3\,(n-1)} &> -b \\ \Leftrightarrow \qquad n+1+2\sqrt{n^2+2n-2} &> 0 \end{aligned} \tag{34}$$

This condition holds for any number of firms $n \geq 2$. The condition $-b < d_1 < \frac{1}{n-1}b$ on the other hand implies that $d_1 < \frac{1}{n-1}b$, i.e.:

$$\begin{aligned} \frac{-2\,(n-2)\,b+2b\sqrt{n^2+2n-2}}{3\,(n-1)} &< \frac{1}{n-1}b \\ \Leftrightarrow \qquad -2n+1+2\sqrt{n^2+2n-2} &< 0 \end{aligned} \tag{35}$$

This condition holds for no number of firms $n \geq 2$. From (34) and (35) we therefore conclude that $d_1 \geq \frac{1}{n-1}b$ which means that it is generally located outside of $d \in \left(-b; \frac{1}{n-1}b\right)$.

In the following, an analogous analysis is presented for d_2. If

$$d_2 = \frac{-2\,(n-2)\,b-2b\sqrt{n^2+2n-2}}{3\,(n-1)} \tag{36}$$

is located inside the range of $d \in \left(-b; \frac{1}{n-1}b\right)$, the condition $-b < d_2 < \frac{1}{n-1}b$ must hold. This implies on the one hand $d_2 > -b$, i.e.:

$$\begin{aligned} \frac{-2\,(n-2)\,b-2b\sqrt{n^2+2n-2}}{3\,(n-1)} &> -b \\ \Leftrightarrow \qquad n+1-2\sqrt{n^2+2n-2} &> 0 \end{aligned} \tag{37}$$

This condition holds for no number of firms $n \geq 2$. The condition $-b < d_2 < \frac{1}{n-1}b$ on the other hand implies that $d_2 < \frac{1}{n-1}b$, i.e.:

$$\begin{aligned} \frac{-2\,(n-2)\,b-2b\sqrt{n^2+2n-2}}{3\,(n-1)} &< \frac{1}{n-1}b \\ \Leftrightarrow \qquad -2n+1-2\sqrt{n^2+2n-2} &< 0 \end{aligned} \tag{38}$$

This condition holds for any number of firms $n \geq 2$. From (37) and (38) we therefore conclude that $d_2 \leq -b$ which means that it is generally located outside of $d \in \left(-b; \frac{1}{n-1}b\right)$.

Therefore, both values d_1 and d_2 are located outside range of substitutability of goods $d \in \left(-b; \frac{1}{n-1}b\right)$. Combining this knowledge with the result (31) for independent goods we can conclude that the expression (30) is always negative. Considering this in the analysis of (15) we can proof that $\frac{\partial \mathrm{E}_{\mathbf{z_i}}[\pi_i(\mathbf{p})]}{\partial r_i} > 0$ for substitutes ($d > 0$) and complements ($d < 0$), while for independent goods $\frac{\partial \mathrm{E}_{\mathbf{z_i}}[\pi_i(\mathbf{p})]}{\partial r_i} = 0$.

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