E C O N O M I C S B U L L E T I N

Extend the debt as it is not deeply out-of-the-money

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Abstract

In this paper, we modify the extendible debts model proposed in Longstaff (1990) to help relieve the moral hazard problem induced in the original model. In Longstaff¡¦s model, extending the maturity of the defaulted debts gives the borrower an incentive to default even if the borrower is insolvent. In this paper, we argue that the debt should not be extended if it is defaulted severely. We have shown that the extendible debt valuation can be obtained by the compound option pricing besides the PDE approach. We also have derived the fair interest rate of the extendible debts in this paper.

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1. Introduction

In this paper, we modify the extendible debts model originally proposed by Longstaff (1990). To help relieve the moral hazard problem induced by Longstaff's model, the debt is not extended if it is defaulted severely under our specifications. And we have shown that the extendible debts valuation can be obtained by the compound option pricing technique besides the PDE approach utilized in Longstaff (1990). We also have derived the fair rate of this kind of extendible debts in our model.

Longstaff (1990) first thoroughly investigates the valuation and related issues of options with extendible maturities. He classifies options with extendible maturities as "holder-extendible options" and "writer-extendible options". The primary difference is that whether the extension of maturity is chosen by the optionholder or granted by the writer. Based on the writer-extendible puts, Longstaff continues to price the extendible bonds and show that the bondholder has an incentive to extend the debts when defaults occur if large liquidation costs incur.

Moraux and Navatte (2004) relax the assumption of Longstaff (1990) and find out that the extension solution is especially valuable when the financial distress is not severe, when contributions from the stockholders are significant, or when the realization rate is time-varing and increasing. Their motivations come from that, creditors may sometimes grant a very long time to safe only a negligible part of their claims as shown in the simulations of Longstaff (1990). One alternative resolution to the defaulted debts is the renegotiation of the loan balance. Harding and Sirmans (2002) and Miceli and Sirmans (2007) find out conditions under which the maturity extension is preferred to the renegotiation. That is, the dead weight loss from foreclosure is not too large.

One of other drawbacks of Longstaff (1990) is that the solvent debtor may have an incentive to cause the occurrence of defaults if the extension scheme is known in advance. Longstaff states that "shareholders in a nearly or slightly bankrupt firm may have an incentive to take on negative NPV projects in order to drive down the value of the firm". Technically speaking, this is because the stockholders' value is not a monotone increasing function of the firm's assets value if taking the debts extension into consideration. When defaults occur, the lender has an incentive to extend the debt if there are large liquidation costs. But the extension itself will stimulate the occurrence of defaults. In this paper, we try to mitigate this moral hazard problem by suggesting the threshold of collateral value. That is, the creditor extends the debt if the default is not too severe. But if the stockholders drive down the firm's assets value such that the value is below the threshold, stockholders will be punished by the declaration of the firm's bankruptcy.

This paper is organized as follows. Section 2 presents our model and values the extendible debts in our settings. Sections 3 proposes one of the possible extensions in this line of research. Section 4 concludes.

2. The Valuation of Extendible Debts

Consider this kind of loan arrangement: At time 0, the creditor lends De^{-rT} to the debtor (a firm) (where r denotes the loan rate), and the firm pledges its total assets (whose value is denoted as A) as the collateral for the loan. At time T, the debtor repays D to the creditor and the collateral is returned. But if at maturity the borrower is unable to meet the liability and the asset value of the firm is below D but above ϕD $(0<\phi<1)$, the creditor will not liquidate the collateral at once but extends the debt to time $T+\tau$ instead. If at time T, A is below ϕD, then the collateral is liquidated immediately. Should the debt be extended to time $T+\tau$, the firm also repays D. The interests are not charged during the maturity extension period, as in Longstaff (1990). But if the borrower defaults at time $T+\tau$, the debt owner will liquidate the collateral and absorb any losses if possible.

Assume that under the risk-neutral probability measure *Q*, the value of the firm's assets can be described by the following process :

$$dA_t = r_f A_t dt + \sigma dW_t \,, \tag{1}$$

where r_f is the risk-free rate, σ denotes the volatility of the firm's value, and W_t is the Wiener process. Under such loan contract, we can say that at time *T*, the firm is granted a portfolio (a standard put plus two kinds of compound options) if the firm defaults but is still tolerated by the creditor. And at time *T* the firm is liquidated immediately if the firm defaults severely (the debt is deeply "out-of-the-money"). The payoff function P_T of the debtor at time T is therefore as follows:

$$P_T = \begin{cases} 0 & if \quad A_T \ge D \\ F & if \quad D > A_T \ge \phi D \\ D - A_T & if \quad A_T < \phi D \end{cases}, \tag{2}$$

where $F = De^{-r_f \tau} N(-\delta) - A_T N(-\delta - \sigma\sqrt{\tau})$ ($\delta = \left(\ln \frac{A_T}{D} + \left(r_f + \frac{\sigma^2}{2}\right)\tau \right) \Big/ \sigma\sqrt{\tau}$), and $N(\cdot)$ denotes the CDF of a standard Normal distribution), which is the price of a put whose payoff at time $T+\tau$ is as follows:

$$F_{T+\tau} = \begin{cases} D - A_{T+\tau} & if \quad A_{T+\tau} \le D \\ 0 & otherwise \end{cases}. \tag{3}$$

To value this kind of debt in the start, we then need to calculate the price of P_T at time 0, and let's denote it as *P*. That is, $P = e^{-r_f T} E^Q(P_T)$. Note that P_T can be rearranged as follows:

$$P_T = \begin{cases} -F & if \quad A_T \geq D \\ F & if \quad A_T \geq \phi D \\ D - A_T & if \quad A_T < \phi D \end{cases}. \tag{4}$$

Furthermore, P_T can be divided into three payoff functions, which are

$$P_{1T} = \begin{cases} -F & if \quad A_T \geq D \\ 0 & if \quad Otherwise \end{cases}, \tag{5}$$

$$P_{2T} = \begin{cases} F & if \quad A_T \geq \phi D \\ 0 & if \quad Otherwise \end{cases}, \tag{6}$$

and

$$P_{3T} = \begin{cases} D - A_T & if \quad A_T < \phi D \\ 0 & if \quad Otherwise \end{cases}. \tag{7}$$

Let the prices of P_{1T}, P_{2T}, and P_{3T} be denoted as P_1, P_2, and P_3. P can now be derived by $P = P_1 + P_2 + P_3$.

First, it can be seen that P_{3T} is the payoff of a European put option and by the Black-Scholes formula, P_3 can be obtained as

$$P_3 = De^{-r_f T} N(-\alpha) - A_0 N\left(-\alpha - \sigma\sqrt{T}\right), \tag{8}$$

where $\alpha = \dfrac{\ln \dfrac{A_0}{\phi D} + \left(r_f + \dfrac{\sigma^2}{2} \right) T}{\sigma\sqrt{T}}$, and A_0 denotes the firm's assets value at time 0.

Second, we can compute P_2 by the compound option pricing (Geske (1979)). If at time *T*, the payoff of the debtor from P_{2T} is $F - K$ when $A_T \geq \phi D$ and nothing otherwise, then P_{2T} is indeed the payoff of a compound option (a call on a put) with strike price *K* and P_2 is of the form $De^{-r_f T} N(\cdot,\cdot,\cdot) - AN(\cdot,\cdot,\cdot) - Ke^{-rT} N(\cdot)$, where $N(\cdot,\cdot,\cdot)$ denotes the CDF of a standard bivariate Normal distribution. But now that the underlying option (whose payoff a time $T + \tau$ is $F_{T+\tau}$) is granted costlessly if the compound option is in-the-money at time *T* (a writer-extendible option in the sense of Longstaff (1990)), then letting $K = 0$, P_2 becomes

$$P_2 = De^{-r_f T} N\left(-\alpha + \sigma\sqrt{T}, -\theta + \sigma\sqrt{T+\tau}, \rho\right) - A_0 N(-\alpha, -\theta, \rho), \tag{9}$$

where $\theta = \dfrac{\ln \dfrac{A_0}{D} + \left(r_f + \dfrac{\sigma^2}{2} \right)(T + \tau)}{\sigma\sqrt{T+\tau}}$, $\rho = \sqrt{\dfrac{T}{T+\tau}}$.

Third, note that P_{1T} is the payoff of a compound option (a call on a put) with strike price zero but in a short position. P_1 can be obtained accordingly

$$P_1 = A_0 N(-\beta, -\theta, \rho) - De^{-r_f T} N\left(-\beta + \sigma\sqrt{T}, -\theta + \sigma\sqrt{T+\tau}, \rho\right), \qquad (10)$$

where $\beta = \dfrac{\ln\dfrac{A_0}{D} + \left(r_f + \dfrac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$.

Since $P = P_1 + P_2 + P_3$, P can be obtained as

$$\begin{aligned} P = {} & De^{-r_f T} N\left(-\alpha + \sigma\sqrt{T}\right) - A_0 N(-\alpha) \\ & + De^{-r_f T}\left(N\left(-\alpha + \sigma\sqrt{T}, -\theta + \sigma\sqrt{T+\tau}, \rho\right) - N\left(-\beta + \sigma\sqrt{T}, -\theta + \sigma\sqrt{T+\tau}, \rho\right)\right) \\ & - A_0\left(N(-\alpha, -\theta, \rho) - N(-\beta, -\theta, \rho)\right). \end{aligned} \qquad (11)$$

We now proceed to derive the fair interest rate of the extendible debts in our settings. Consider two kinds of risk-free investments. One is to engage in riskless lendings directly. The other is to make the extendible loans in out settings and at the same time long a portfolio whose payoff at time T is P_T. Therefore, under both investment projects, the creditor can receive D at time T. To rule out the arbitrage opportunity, the costs of the two kinds of investments at time 0 must be equal. That is, $De^{-r_f T} = De^{-rT} + P$. The fair interest rate of extendible debts in our settings is as follows:

$$r = -\ln\left(1 - P/De^{-r_f T}\right)/T + r_f, \qquad (12)$$

where P can be obtained in (11).

3. The Endogenization of the Threshold Parameter

Since we have modified Longstaff (1990) and taking all the parameters as given, we also have shown how to value the extendible loans under new model specifications. What follows is to endogenize the parameters. We can decide the threshold under which the lender will not extend the debts if the borrower defaults severely. That is, the creditor solves the following optimization problem at time 0:

$$\underset{0 \le \phi \le 1}{Max}\, H(A, D, \phi) > 0, \qquad (13)$$

where $H(\cdot)$ is the gain function defined similarly in Longstaff (1990). In Longstaff (1990), the gain function is used to decide the optimal extension period τ if the borrower defaults. In our case, we have to fix τ and then to decide the optimal threshold ϕ. If ϕ is set too low (that is, $\phi \approx 0$), then the debt is extended for almost all cases when default occurs. This is not helpful for alleviating the moral hazard incentives of borrowers. But if ϕ is set too high (that is, $\phi \approx 1$), the debt is hardly extended when default occurs. Hence, it is expected that the optimization problem in

(13) has an interior solution. To complicate the case, one can also decide the extension period τ and the threshold ϕ simultaneously.

4. Conclusion

In this paper, we modify the extendible debts model proposed in Longstaff (1990) to help relieve the moral hazard problem induced in the original model. In Longstaff (1990), extending the maturity of the defaulted debts gives the borrower an incentive to default even if the borrower is solvent. In our model, the debt is not extended if it is defaulted severely. And we have shown that the extendible debt valuation can be obtained by the compound option pricing besides the PDE approach. We also have derived the fair rate of this kind of extendible debts in this paper. For future research, we will endogenize the threshold under which the debts are not extended.

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