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Frictional unemployment, labor market institutions, and endogenous economic growth

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Abstract

For a given set of labor market institutions, the rate of frictional unemployment depends on the evolution of the pool of job-seekers. Unemployment rises with the growth rate of labor supply that is proportionate to the rate of population growth. If economic growth is semi-endogenous, the steady-state growth rate depends positively on the rate of population growth. This suggests a trade-off between growth and unemployment: a faster growing economy has a higher unemployment rate. As a consequence, faster growth may not be desirable from a welfare point of view. We make this point in a parsimonious setting where semi-endogenous growth derives from the division of labor and the associated gains from specialization.

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1 Introduction

Frictional unemployment involves people being temporarily between jobs. The duration of such spells of unemployment depends on the institutional framework of the labor market. Examples of such institutions abound. Agencies providing information to job-seekers and employers tend to reduce the length of unemployment spells. The way the unemployment insurance pays its benefits may affect workers' effort devoted to job search. Job protection laws impinge on the choosiness of employers (see, e. g., Layard, Nickell, and Jackman (2005), Chapters 4-6, for details).

This paper argues that the quality of these institutions may determine whether an economy faces a trade-off between unemployment and economic growth. We make this point in a parsimonious semi-endogenous growth model, where unemployment and economic growth are endogenous. Hence, factors that affect economic growth may also affect the rate of unemployment.

The notion of semi-endogenous growth refers to a class of endogenous growth models with the property that the "long-run growth rate is not endogenous, (...), in the sense that traditional policy changes have long-run growth effects" (Jones (1995), p. 760-761). Besides technological parameters it is the rate of population growth that determines the steady-state growth rate of an economy. In the economy under scrutiny here economic growth is not R&D-based as in Jones (1995) but due to gains from specialization associated with the division of labor in a growing economy.

In the labor market, it is the duration of an average unemployment spell that reflects the quality of the institutional framework in which the labor market operates. The evolution of the pool of job-seekers determines the rate of frictional unemployment. Let this pool consist of the currently unemployed, i. e., the difference between the aggregate labor supply and the currently employed workforce. The growth rates of aggregate labor supply and of employment determine whether the unemployment rate increases or decreases. In a steady state these growth rates coincide. The steady-state unemployment rate is constant and higher the faster labor supply grows.

Both concepts suggest a trade-off between growth and unemployment. On the one hand, frictional unemployment increases if population, thus labor supply, grows faster. On the other hand, economic growth increases in the population growth rate. Thus, a faster growing economy has a higher unemployment rate. This trade-off hinges on the presence of labor market frictions. In a perfectly competitive labor market, it can, by definition, not arise. However, consistent with the findings described by Nickell and Layard (1999), these institutional frictions increase unemployment but leave the economy's growth rate unaffected.

Our results are based on the characterization of the global dynamics of the economy. This allows us to establish that the trade-off between growth and unemployment arises not only in the steady state but also along the transition towards it. Moreover, we analyze the implications of this trade-off for welfare. We find that faster economic growth reduces welfare if the effect of labor market frictions is sufficiently pronounced.

The literature on the relationship between economic growth and unemployment dates at least back to the works of Riccardo (1821) and Marx (1867). The more recent contributions have used elements of the so-called endogenous growth theory to shed new light on this relationship. For instance, Pissarides (1990), Aghion and Howitt (1994), or Mortensen and Pissarides (1998) emphasize that a trade-off between growth and unemployment may result from opposing incentives faced by firms to create and destroy jobs. Daveri and Tabelini (2000) or Irmen and Wigger (2003) stress the role of unemployment for aggregate savings and capital accumulation for such a trade-off. Irmen and Wigger (2006) extends this line of reasoning to an open economy. Labor market frictions may also give rise to a growth-unemployment trade-off if international trade requires the reallocation of workers. This has been shown by Arnold (2002) for an economy facing North-South trade in the spirit of Helpman (1993).

Unlike these studies, the present paper considers an environment in which the labor force is allowed to grow. Then, the growth-unemployment trade-off arises since a semi-endogenous increase in the growth rate also increases the unemployment rate in the presence of labor market frictions.

The remainder of this paper is organized as follows. We present the model in Section 2. Section 3 studies the equilibrium and derives our main result: the trade-off between growth and unemployment is not restricted to the steady state but occurs also along the transition. Section 4 derives the welfare implications. Section 5 concludes. Proofs are relegated to the Appendix.

2 The Model

Consider a closed economy comprising a final-good sector, an intermediate-good sector, and a household sector. Time is continuous, i. e., $t \in [0, \infty)$. In all periods there is a market for the consumption good, for all available varieties of the intermediate good, and for labor. The final good serves as numéraire. To simplify the notation we omit the time argument except where it is needed for clarity.

The Final-Good Sector There are many competitive firms producing a homogeneous good under constant returns to scale. We may therefore describe the final-good sector in terms of the actions of a single, aggregate firm. Its production function is

$$Y = \left[A^{(\sigma-1)\,(1-\alpha)} \int_0^A x\,(j)^{\alpha}\, dj \right]^{\frac{1}{\alpha}}, \quad 0 < \alpha < 1,, \tag{1}$$

where Y is output at t, $x(j)$ is the amount of intermediate j used at t, and $A \in \mathbb{R}_+$ is the measure of varieties of intermediates available at t. The parameter α determines the elasticity of substitution between any pair of intermediates, $\varepsilon \equiv 1/(1-\alpha)$. Following Ethier (1982), the term in front of the integral introduces $\sigma \in (0, \infty)$ as a measure of gains from specialization. If $\sigma = 0$, these gains vanish.

Denote $p(j)$ the price of intermediate-good j, and

$$P \equiv \left[A^{\sigma-1} \int_0^A p\,(j')^{1-\varepsilon}\, dj' \right]^{\frac{1}{1-\varepsilon}} \tag{2}$$

the minimum cost of one unit of Y. Then, the cost function is PY and with (2) we obtain the conditional factor demands for each intermediate good from an application of Shephard's lemma. This delivers

$$x(j) = \frac{Y}{\left[A^{(\sigma-1)(1-\alpha)} \int_0^A p\,(j')^{1-\varepsilon}\, dj' \right]^{\frac{1}{\alpha}}}\, p(j)^{-\varepsilon} \quad \text{for all } j \in [0, A]. \tag{3}$$

The Intermediate-Good Sector Each variant of the intermediate good is produced according to the production function

$$x(j) = \max\left\{0, l(j) - \bar{l}\,\right\}; \tag{4}$$

here $l(j)$ is the total amount of labor employed by firm j, and $\bar{l} > 0$ denotes the amount of quasi-fixed labor.

There is free entry into the production of each variety. The quasi-fixed wage cost implies that the equilibrium has only one manufacturer of each variety earning zero profits. With w denoting the real wage, the per-period profit of the manufacturer of variety j is

$$\pi\left(j\right) = \left(p\left(j\right) - w\right) x\left(j\right) - w\,\bar{l}. \tag{5}$$

In view of (3), the profit-maximizing price is

$$p\left(j\right) = p = \frac{w}{\alpha} \tag{6}$$

and implies that $x(j) = x$. The zero-profit condition

$$\left(p - w\right) x = w\,\bar{l} \tag{7}$$

in conjunction with (6) determines the equilibrium output of each intermediate $j \in [0, A]$ as

$$x^* = \frac{\alpha}{1-\alpha}\,\bar{l}. \tag{8}$$

The Household Sector The household sector at t comprises N identical workers/consumers. Each worker has a per-period labor endowment equal to one. Labor is inelastically supplied in exchange for wages. Aggregate labor supply at t is

$$N(t) = N_0 e^{g_N t}, \tag{9}$$

where $N_0 > 0$ is the initial value and $g_N > 0$ denotes the population growth rate.

The labor market is not *infinitely* flexible. While workers offer their time endowment they may not *immediately* find firms willing to hire them at the equilibrium real wage. We refer to L as the level of employment at t. There is a probability, $\beta\,dt$, for an unemployed to find work in the short time interval dt. The parameter $\beta \in (0, \infty)$ subsumes the characteristics of the labor market alluded to in the introduction. The expected duration of unemployment is $1/\beta$ and 'infinite flexibility' corresponds to the limit $\beta \to \infty$. In other words, the assumption is that

$$dL = \beta\,dt\left(N - L\right). \tag{10}$$

Dividing by dt, the evolution of employment is given by the two differential equations

$$\dot{L}(t) = \beta\,\left(N(t) - L(t)\right) \quad \text{and} \quad \dot{N}(t) = g_N\,N(t) \tag{11}$$

with $0 < L_0 < N_0$ as initial values.

There is a continuum $[0,1]$ of identical, infinitely-lived households to which workers belong. The representative household maximizes intertemporal preferences

$$U = \int_0^\infty \frac{c(t)^{1-\theta} - 1}{1-\theta}\, e^{-(\rho - g_N)t} dt, \tag{12}$$

where $c(t)$ is per-capita consumption, $\theta > 0$ is the inverse of the intertemporal elasticity of substitution, and $\rho > g_N$ is the instantaneous rate of time preference.

We assume that each household comprises many members such that the fraction of unemployed household members coincides with the deterministic aggregate unemployment rate. Hence, there is no uncertainty about household income.

We abstract from means to transfer resources between periods.[1] Moreover, there is no storage technology. Then, in each period the representative household finds it optimal to choose

$$c = \frac{w\,L}{N}. \tag{13}$$

3 Equilibrium Analysis

Given L_0, N_0, and the evolution of labor supply (9), the equilibrium determines an allocation, i. e., a sequence $\{Y(t), A(t), x(j,t), l(j,t), c(t), L(t)\}_{t=0}^{\infty}$, and a price system, i. e., a sequence $\{p(j,t), w(j,t)\}_{t=0}^{\infty}$, that satisfy for all t conditions (3), (6), and (7) for the production sector, condition (13) for the household sector, and the equilibrium conditions of all markets, where (11) governs the evolution of the level of employment.

Consider the labor market. The aggregate demand for labor $A\,(x^* + \bar{l})$ adjusts to L, i. e., $A\,(x^* + \bar{l}) = L$. With (8) this gives the equilibrium measure of varieties

$$A^* = \frac{1-\alpha}{\bar{l}}\, L. \tag{14}$$

Thus, the division of labor is determined by the degree of substitutability of intermediates, α, the quasi-fixed costs, $\bar{l}$, and the level of employment, L, which represents the extend of the market.

Using (8) and (14) in (1), we find that aggregate output of the final good is

$$Y^* = \Lambda\, L^{1+\sigma\left(\frac{1}{\alpha}-1\right)}, \tag{15}$$

where $\Lambda \equiv \alpha\left((1-\alpha)/\bar{l}\right)^{\sigma((1/\alpha)-1)} > 0$ is a time-invariant parameter. From Euler's Law we have $Y = A\,p\,x$. Moreover, using (6), (8), and (14) reveals that $A^* p\, x^* = w\,L$. Hence, at all t,

$$c = \frac{\Lambda\, L^{1+\sigma\left(\frac{1}{\alpha}-1\right)}}{N} \tag{16}$$

[1] This assumption is innocuous for the existence of a growth-unemployment trade-off in equilibrium. One way to introduce productive capital into aggregate production is to replace (1) by $\bar{Y} = \Delta\, K^\gamma\, Y^{1-\gamma}$, $\Delta > 0$, $0 < \gamma < 1$. Details are available from the author upon request.

such that

$$\frac{\dot{c}}{c} = \left(1 + \sigma\left(\frac{1}{\alpha} - 1\right)\right) g_L - g_N, \quad c(0) = \Lambda\, L_0^{1+\sigma\left(\frac{1}{\alpha}-1\right)}. \tag{17}$$

Before we turn to the characterization of the evolution of per-capita consumption we need to study the evolution of unemployment.

Proposition 1 The unemployment rate, $u(t) \equiv 1 - L(t)/N(t)$, evolves according to

$$u(t) = u^* - (u^* - u_0)\, e^{-(\beta+g_N)\,t}, \tag{18}$$

where

$$u^* = \lim_{t\to\infty} u(t) = \frac{g_N}{\beta + g_N}. \tag{19}$$

According to Proposition 1, the employment rate converges to a steady-state value for all initial values. Moreover, the steady-state unemployment rate satisfies

$$\frac{\partial u^*}{\partial \beta} < 0, \quad \text{and} \quad \frac{\partial u^*}{\partial g_N} > 0. \tag{20}$$

Intuitively, a rise in β reduces the duration of unemployment and leads to a lower steady-state unemployment rate. Faster population growth accentuates existing frictions and implies a higher u^*.

Proposition 2 Let

$$\rho > g_N \left[1 + \sigma\left(\frac{1}{\alpha} - 1\right)(1-\theta)\right]. \tag{21}$$

1. There is a unique equilibrium path of per-capita consumption given by

   $$c(t) = c_0\, e^{\int_0^t \zeta(\tau)\,d\tau}, \tag{22}$$

   where

   $$\zeta(\tau) \equiv \left[1 + \sigma\left(\frac{1}{\alpha} - 1\right)\right] \beta \left(\frac{1}{1-u(\tau)} - 1\right) - g_N. \tag{23}$$

2. In the limit $t \to \infty$, all per-capita magnitudes grow at rate

   $$g^* = \sigma\left(\frac{1}{\alpha} - 1\right) g_N. \tag{24}$$

According to Proposition 2, the equilibrium path of per-capita consumption converges for all admissible initial conditions. Moreover, the economy exhibits semi-endogenous growth in the sense that steady-state growth depends on three exogenous parameters: gains from specialization since $\sigma > 0$, the degree of product differentiation $\alpha \in (0,1)$, and the population growth rate $g_N > 0$. Intuitively, $\sigma > 0$ and $\alpha \in (0,1)$ imply that the reduced form production function (15) has "increasing returns to scale", i. e., it is strictly convex in L. Therefore, growth becomes feasible under a strictly positive population growth rate.

A comparison of (20) and (24) reveals a trade-off between the growth rate of per-capita magnitudes and the rate of unemployment in the steady state. A higher population growth rate means faster economic growth and a higher unemployment rate. The following proposition strengthens this result. It shows that the trade-off between economic growth and unemployment is not confined to the steady-state.

Proposition 3 Denote $g(t)$ the equilibrium path of the growth rate of per-capita magnitudes. There is a critical value $t_c < \infty$ such that

$$\frac{\partial u(t)}{\partial g_N} > 0 \quad \text{and} \quad \frac{\partial g(t)}{\partial g_N} > 0 \qquad \text{for all } t \geq t_c. \tag{25}$$

Hence, along the transition towards the steady state there is a finite critical t_c such that the economy faces a trade-off between growth and unemployment once this period is reached: a permanent rise in the population growth rate that occurs at some $t \geq t_c$ accelerates economic growth at the cost of a higher unemployment rate in all later periods.

To understand the intuition for this finding consider first the path of the unemployment rate as given by (18). A rise in g_N affects $u(t)$ in three ways. First, the unemployment rate rises since a rise in the growth rate of the labor supply increases the steady-state unemployment rate u^*. Second and related, a higher u^* affects the distance between the initial level and the steady state $u^* - u_0$. Third, the (asymptotic) speed of convergence increases.[2] The point of Proposition 3 is that the first effect dominates if the economy is sufficiently far away from its starting point.[3]

To grasp the effect of a rise in g_N on per-capita consumption growth consider the instantaneous growth rate $\zeta(t)$ of (23) and (17). Then, there are two effects to be considered. First, there is a negative direct effect through population growth. Second, there is an indirect effect on the growth rate of aggregate output that depends on the growth rate of the employed work force. In accordance with (11), g_L is larger when the unemployment rate is high. Moreover, from the preceding paragraph we know that an increase in g_N must eventually augment the unemployment rate. Then, the point of Proposition 3 is that along its transition the economy reaches a critical period t_c^g in finite time such that the indirect effect dominates the direct effect for all $t \geq t_c^g < \infty$. Observe that this finding needs growth through gains from specialization. Otherwise the trade-off between growth and unemployment vanishes in the steady state.

[2]The speed of convergence is defined as $-\partial(\dot{u}/u)/\partial \ln u$. Evaluated at u^* gives the asymptotic speed of convergence equal to $\beta + g_N$.

[3]From the proof of Proposition 3 we also learn that $\partial u(t)/\partial g_N > 0$ holds for all $t > 0$ if $u^* > u_0$. On the other hand, for $u_0 > u^*$ and some finite period t_c^u, it is possible to have $\partial u(t)/\partial g_N < 0$ whenever $u_0 - u^* > \left[\partial u^*/\partial g_N \left(e^{(\beta+g_N)t} - 1\right)\right]/t$ and $t < t_c^u$.

4 Welfare Considerations

Since faster growth per se is not an end in itself, the question arises whether the trade-off between unemployment and growth has implications for welfare. To address this question, we focus on the steady state, i. e., all per-capita magnitudes grow at rate g^* of (24) for $t \geq 0$. Then, the representative household's utility integral (12) becomes

$$U^* = \frac{1}{\rho - g_N - (1-\theta)g^*}\left[\frac{c(0)^{1-\theta}-1}{1-\theta} + \frac{g^*}{\rho - g_N}\right]. \tag{26}$$

Since the unemployment rate is pegged at u^* of (19), the steady-state initial values L_0 and N_0 are no longer independent but satisfy $L_0 = \beta N_0/(\beta + g_N)$. As a consequence, we obtain period-zero consumption from (16) as

$$c(0) = \Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)} \left(\frac{\beta}{\beta + g_N}\right)^{1+\sigma\left(\frac{1}{\alpha}-1\right)}. \tag{27}$$

Then, a rise in g_N has the following effects on U^*. First, there is a trade-off inside the brackets reflecting a level effect on $c(0)$ and a growth effect through g^*. Faster population growth means a rise in the unemployment rate and $c(0)$ declines; it also implies a rise in the steady-state growth rate. Second, a trade-off may or may not arise if we consider the effect on the effective discount rate in front of the bracketed term. The following proposition shows that clear-cut results are available for a neighborhood where $\theta = 1$ and $g_N = 0$.

Proposition 4 Let $\lim_{\beta\to\infty} c(0) \geq 1$. Then, there is

$$\bar{\beta} \equiv \rho\left(\frac{1+\sigma\left(\frac{1}{\alpha}-1\right)}{\ln\Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)} + \sigma\left(\frac{1}{\alpha}-1\right)}\right) \in (0,\infty). \tag{28}$$

Moreover, it holds that

$$\left.\frac{\partial U^*}{\partial g_N}\right|_{\theta=1, g_N=0} \gtreqless 0 \quad \Leftrightarrow \quad \beta \gtreqless \bar{\beta}. \tag{29}$$

Roughly speaking, Proposition 4 establishes that the overall effect of faster population growth on welfare depends on the labor market's ability to accommodate change. If this ability is high such that $\beta > \bar{\beta}$ then faster economic growth increases welfare. Intuitively, the labor market integrates a faster growing labor supply without a significant increase in the unemployment rate. As a result the level effect on $c(0)$ is dominated by the effects on g^* and on the effective discount rate.[4] Since the latter two effects are positive and independent of labor market frictions whereas the level effect on $c(0)$ vanishes in the limit $\beta \to \infty$, the economy under full employment unambiguously benefits from faster population growth.

Observe that the results of Proposition 4 apply to a sufficiently small neighborhood of $(\theta, g_N) = (1, 0)$. However, the focus on this neighborhood is less restrictive than it might appear at first sight. For instance, Browning, Hansen, and Heckman (1999) argue that there is no strong evidence against θ close to one.[5] Moreover, the vicinity of $g_N = 0$ is consistent with recent evidence suggesting that the current and expected population growth rates in many industrialized countries rates are close to zero (see, e. g., Krüger and Ludwig (2007) for a discussion).

[4]The condition $\lim_{\beta\to\infty} c(0) = \Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)} \geq 1$ assures that the effect of g_N on U^* through the effective discount rate does not apply to a negative number which would be the case if we allowed for $\ln\Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)} < 0$.

[5]Many calibration studies stipulate a utility function with $\theta = 1$. See, e. g., Heer and Maussner (2008) for an elaborate discussion of computable general equilibrium models and their calibration.

5 Concluding Remarks

A trade-off between economic growth and unemployment may arise when labor-market frictions are present and gains from specialization give rise to semi-endogenous economic growth. Intuitively, labor market frictions reduce the ability of the labor market to absorb a growing supply of labor and give rise to a higher unemployment rate. However, semi-endogenous growth is higher the faster the supply of labor grows.

We also establish that the trade-off between unemployment and growth has implications for the desirability of faster growth from a welfare point of view. Intuitively, since faster economic growth increases the unemployment rate, a larger fraction of the population does not benefit from economic growth. In the presence of a representative household, this reduces welfare if labor market frictions are strong.

These results restate that faster economic growth generates winners and losers. To identify them, one would need to extend our parsimonious analytical framework. For instance, one could replace the assumption of identical households with household heterogeneity in the spirit of Caselli and Ventura (2000). As long as economic growth remains semi-endogenous traditional policies will not have long-run growth effects. However, the question arises whether government interventions can be justified aiming at an enlargement of the fraction of the population that benefits from economic growth. We leave such extensions for future research.

6 Appendix

6.1 Proof of Proposition 1

Denote $\lambda \equiv L/N$ the employment rate at t. From the definition of λ and the evolution of the level of employment (11) it follows that $\dot{\lambda} = \beta - (\beta + g_N)\ \lambda$. This is a linear, first-order differential equation with constant coefficient that can be solved. The particular solution for some $\lambda_0 < 1$ is $\lambda(t) = \lambda^* + (\lambda_0 - \lambda^*)\, e^{-(\beta+g_N)\, t}$, where $\lambda^* = \lim_{t\to\infty} \lambda(t) = \beta/(\beta + g_N)$. Proposition 1 follows from the definition of u. ■

6.2 Proof of Proposition 2

As to the first statement, observe that (11) and the definition of u imply

$$g_L(t) = \beta \left(\frac{1}{\lambda(t)} - 1 \right) = \beta \left(\frac{1}{1 - u(t)} - 1 \right). \tag{30}$$

Then, with (17), the evolution of $c(t)$ can be written as

$$\dot{c}(t) = \zeta(t)\, c(t), \tag{31}$$

where

$$\zeta(t) \equiv \left(1 + \sigma \left(\frac{1}{\alpha} - 1 \right) \right) \beta \left(\frac{1}{1 - u(t)} - 1 \right) - g_N. \tag{32}$$

Equation (31) is a linear, first-order differential equation with variable coefficient that can be solved. Equation (22) is the particular solution for a given initial condition $c(0) = c_0$. In accordance with (16), we have $c_0 = \Lambda\, L_0^{1+\sigma(1/\alpha-1)}/N_0$.

As to the second statement, we invoke u^* of Proposition 1 and find that

$$
\begin{aligned}
\lim_{t\to\infty} \zeta(t) &= \lim_{t\to\infty} \left[\left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \beta \left(\frac{1}{1-u(t)}-1\right) - g_N \right] \\
&= \left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \beta \left(\frac{1}{1-\lim_{t\to\infty} u(t)}-1\right) - g_N \\
&= \left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \beta \left(\frac{\beta+g_N}{\beta}-1\right) - g_N \\
&= \sigma\left(\frac{1}{\alpha}-1\right) g_N .
\end{aligned}
$$

Hence, in the limit $t \to \infty$, per-capita consumption grows at the rate given in (24). From (15), it is obvious that this growth rate also applies to per-capita output.

Condition (21) assures that the utility, U of (12), is finite in the limit. ■

6.3 Proof of Proposition 3

Consider $u(t)$ of (18). It follows that

$$
\frac{\partial u(t)}{\partial g_N} = \frac{\partial u^*}{\partial g_N} - \left[\frac{\partial u^*}{\partial g_N} - t\,(u^* - u_0)\right] e^{-(\beta+g_N)\,t}. \tag{33}
$$

Since $\lim_{t\to\infty} \partial u(t)/\partial g_N = \partial u^*/\partial g_N > 0$, there must be $t_c^u < \infty$ with the property that $\partial u(t)/\partial g_N > 0$ for all $t \geq t_c^u$.

Next, consider $g(t) = \dot{c}(t)/c(t)$. From (31) and (32) we have

$$
\frac{\partial g(t)}{\partial g_N} = \left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \frac{\beta\, \frac{\partial u(t)}{\partial g_N}}{[1-u(t)]^2} - 1. \tag{34}
$$

Moreover,

$$
\begin{aligned}
\lim_{t\to\infty} \frac{\partial g(t)}{\partial g_N} &= \left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \beta \lim_{t\to\infty} \frac{\frac{\partial u(t)}{\partial g_N}}{[1-u(t)]^2} - 1 \\
&= \left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \frac{\beta\, \frac{\partial u^*}{\partial g_N}}{[1-u^*]^2} - 1 \\
&= \left(1+\sigma\left(\frac{1}{\alpha}-1\right)\right) \frac{\frac{\beta^2}{(\beta+g_N)^2}}{\left(\frac{\beta}{\beta+g_N}\right)^2} - 1 \\
&= \sigma\left(\frac{1}{\alpha}-1\right) > 0.
\end{aligned} \tag{35}
$$

Hence, there must be $t_c^g < \infty$ with the property that $\partial g(t)/\partial g_N > 0$ for all $t \geq t_c^g$. With $t_c = \max\{t_c^u, t_c^g\}$ Proposition 3 holds. ■

6.4 Proof of Proposition 4

Consider (26) and (27). Then,

$$\begin{aligned}\frac{\partial U^*}{\partial g_N} &= \frac{1+(1-\theta)\frac{\partial g^*}{\partial g_N}}{(\rho-g_N-(1-\theta)g^*)^2}\left[\frac{c(0)^{1-\theta}-1}{1-\theta}+\frac{g^*}{\rho-g_N}\right] \\ &- \frac{c(0)^{1-\theta}\left[1+\sigma\left(\frac{1}{\alpha}-1\right)\right]}{(\beta+g_N)\left(\rho-g_N-(1-\theta)g^*\right)} \\ &+ \frac{\frac{\partial g^*}{\partial g_N}(\rho-g_N)+g^*}{(\rho-g_N)^2\left(\rho-g_N-(1-\theta)g^*\right)}. \qquad (36)\end{aligned}$$

Evaluated at $\theta=1$, the latter becomes

$$\left.\frac{\partial U^*}{\partial g_N}\right|_{\theta=1} = \frac{\ln c(0)+\frac{g^*}{\rho-g_N}}{(\rho-g_N)^2} - \frac{1+\sigma\left(\frac{1}{\alpha}-1\right)}{(\beta+g_N)(\rho-g_N)} + \frac{\frac{\partial g^*}{\partial g_N}(\rho-g_N)+g^*}{(\rho-g_N)^3}. \qquad (37)$$

It follows that

$$\left(\left.\frac{\partial U^*}{\partial g_N}\right|_{\theta=1}\right)\Bigg|_{g_N=0} = \frac{\ln\Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)}}{\rho^2} - \frac{1+\sigma\left(\frac{1}{\alpha}-1\right)}{\beta\rho} + \frac{\sigma\left(\frac{1}{\alpha}-1\right)}{\rho^2}. \qquad (38)$$

Observe, that

$$\left(\left.\frac{\partial U^*}{\partial g_N}\right|_{\theta=1}\right)\Bigg|_{g_N=0} = \left(\left.\frac{\partial U^*}{\partial g_N}\right|_{g_N=0}\right)\Bigg|_{\theta=1} \equiv \left.\frac{\partial U^*}{\partial g_N}\right|_{\theta=1,g_N=0}. \qquad (39)$$

Therefore,

$$\left.\frac{\partial U^*}{\partial g_N}\right|_{\theta=1,g_N=0} \gtreqless 0 \quad \Leftrightarrow \quad \beta \gtreqless \rho\left(\frac{1+\sigma\left(\frac{1}{\alpha}-1\right)}{\ln\Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)}+\sigma\left(\frac{1}{\alpha}-1\right)}\right) \equiv \bar{\beta}. \qquad (40)$$

From (27) it follows that the assumption $\lim_{\beta\to\infty} c(0) \geq 1$ implies $\ln\Lambda\, N_0^{\sigma\left(\frac{1}{\alpha}-1\right)} \geq 0$. Hence, $\bar{\beta} \in (0,\infty)$. ■

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