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Allocation of costs to clean up a polluted river: an axiomatic approach

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Abstract

This paper proposes a method to share the costs of cleaning up a polluted river among the agents located along it. This method is based on a model of pollutant transport and the polluter pays principle. We provide an axiomatic characterization for this method and investigate its relationship with both the (weighted) Shapley value and the tau-value of the corresponding cost game generated from the problem. We show that the solution of the proposed method coincides with both the weighted Shapley value and the tau-value.

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1. Introduction

Water pollution is a serious problem faced by many countries, especially developing countries. In this paper we investigate the following question: how to divide fairly the costs of reducing pollution in rivers among the agents located along it? The answer to this question is not trivial since water pollutants are transported with the water and some of them are biodegradable. To answer this question we resort to a model of transport of pollutant and the polluter pays principle. This principle is widely accepted and assigns the responsibility for the pollution to the polluters. Thus, they would be responsible, for example, in the context of our analysis, for the costs of cleaning up a polluted river.

The pollution pays principle has received strong support from the Organization for Economic Co-operation and Development (OECD) and the European Union (EU). Since the 1972 Recommendation by the OECD Council on Guiding Principles concerning International Economic Aspects of Environmental Policies this principle has been recommend as the principle to be used for allocating costs related to pollution. The scope of this principle has evolved over time to include also accidental pollution and clean-up costs and, in its new version, is often referred to as *extended Polluter Pays principle*.

There are various definitions, interpretations or versions of the polluter pays principle in the literature.[1] The definition that best fits the method proposed in this paper is given in the Glossary of Statistical Terms of the OECD (2012): "The polluter pays principle is the principle according to which the polluter should bear the cost of measures to reduce pollution according to the extent of either the damage done to society or the exceeding of an acceptable level (standard) of pollution". Based in this principle, we propose a method to divide fairly the costs of cleaning up river pollutants among the polluters and provide an axiomatic characterization for this method.

This paper is closely related to the paper by Ni and Wang (2007). These authors propose two methods to split the costs of cleaning up a polluted river among the agents located along it. They based their methods, called the *Local Responsibility Sharing* (LRS) method and the *Upstream Equal Sharing* (UES) method, respectively, on the two main advocated doctrines in international disputes: the *Absolute Territorial Sovereignty* (ATS) and the *Unlimited Territory Integrity* (UTI).[2] These two doctrines are interpreted in terms of responsibilities in the pollution cost allocation problem. Under the ATS doctrine, for example, the costs to clean up pollutants in a given segment of the river should be assigned only to polluters located in that segment. However, under the UTI doctrine, polluting-cleaning costs in a given segment should be assigned to polluters located in that segment as well as all upstream polluters.

Our approach differs from the approach of Ni and Wang (2007) on two key points. First, the method we propose is based on a model of pollutant transport and the polluter pays principle. Second, to share downstream costs, we seek to establish the responsibility of each upstream polluter according to its contribution to the pollution level in each segment of the river, that is, upstream polluters are not treated symmetrically as do Ni and Wang (2007). As these authors, we also investigate the relationship of the proposed method and its solution with the (weighted) Shapley value of the associated cost game generated from the problem. In addition, we also investigate the relation between the τ - value of the corresponding cost game and the solution of the method proposed in this paper.

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[1] For alternative versions or interpretations of the polluter pays principle see, for example, Bugge (1996).

[2] For more details on the ATS and the UTI doctrines see, for example, Kilgour and Dinar (1996).

Gómez-Rúa (2013) also investigates the allocation of costs to clean up a polluted river and takes as a starting point the work of Ni and Wang (2007). She replaces the axiom that treats upstream polluters symmetrically by three alternative axioms and accordingly considers three different systems of weights to charge polluters. Regarding the method proposed in this paper, the main difference is that our system of weights is different from those considered by Gómez-Rúa (2013). We base our system of weights on transfer coefficients of a model of pollutant transport while she specifies how the weights are determined in only one of her systems of weights and, in this case, they depend on exogenous factors, such as city population, water consumption, etc. Her analysis also does not include a comparison with the τ - value of the cost game generated from the problem. Gómez-Rúa (2012) extends her previous work to the context of a river network.

The rest of this paper is structured as follows. The next section presents the model used to formalize the *Polluter Pays* method of allocating the total cost to clean up pollutants in a river. The third section provides an axiomatic characterization of the *Polluter Pays* method. We used four axioms to provide this characterization. In this section we also show that the *Polluter Pays* method is the only method that satisfies the four axioms presented and that its solution coincides with both the weighted Shapley value and the τ - value and is in the core of the corresponding cost game generated from the problem. Finally, in the fourth and final section, we present some concluding comments.

2. The model

As in Ni and Wang (2007), consider a river divided into n segments that are indexed in a given order $j = 1,...,n$ from upstream to downstream. We assume that each segment is inhabited by people who may be adversely affected by water pollution and that at the beginning of each segment, according to the above order, there is a firm or industry that discharges pollutants of some kind into the river. In every segment j, an environmental authority sets the maximum level of pollution that is allowed for that segment. If the pollution level exceeds that limit, the polluters are required to spend c_j so that the quality of the water body meets the environmental standard set for the segment j. The total costs to clean pollutants across the river is equal to $\sum_{j=1}^{n} c_j = c_1 + c_2 + \cdots + c_n$. We assume that these costs are at their lowest levels and that the technology used in cleaning the river is the most efficient. We want to find a method to allocate to total cost of cleaning up river pollutants fairly among the n polluting firms located along the river.

To establish the method to allocate those costs we need to introduce some additional notations. We recall that a key feature of water pollution is that the pollutant is transported with the water from an upstream point (source) to a downstream point (receptor). Some river pollutants are biodegradable and the concentration of pollution they cause is reduced along the river. To describe the transport of the water pollutant and the level of pollution it causes along the river we use transfer coefficients.[3] To define these coefficients, suppose there is a clean river and that a certain amount of pollutant e_i is discharged at an upstream point i (source) and that, after some time, we have the level (concentration) of pollution p_j at the downstream point j (receptor). To establish the relationship between the emission level at

[3] Several authors have used transfer coefficients to analyze the spatial aspects associated with pollution. Hung and Shaw (2005), for example, use transfer coefficients in the analysis of a system for trading water pollution discharge permits.

the point (segment) i and the pollution concentration at the point (segment) j, we can use the following expression (see, for example, Kolstad, 2000, p. 155-157):

$$p_j = a_{ij} e_i \quad (1)$$

where a_{ij} is the transfer coefficient. Typically these coefficients take strictly positive values and $a_{ij} \geq a_{ik}$, $k > j$. Note that if the emissions at the point i change by a little (Δe_i), according to the expression (1), the level of pollution at the point j will change by $a_{ij}\Delta e_i$, so we can define the transfer coefficient as the ratio of the change in pollution concentration at point j to the change in emissions at point i, i.e., $a_{ij} = \Delta p_j / \Delta e_i$. Basically, this coefficient gives the conversion rate for emissions to pollution level. If, for example, $a_{ij} = 1.2$, this means that a unit of emissions at the point i generates 1.2 unit of pollution concentration at point j.

The expression (1) can be generalized to consider, for example, emissions at various upstream points (segments), $e_1, e_2, \ldots, e_I$ (where I represents the number of sources), and the river is not completely clean. In this case, the expression (1) changes to

$$p_j = \sum_{i=1}^{I} a_{ij} e_i + B_j$$

where p_j represents the level of pollution at the downstream point j ($j \geq I$) and B_j is the background level of pollution at the point (segment) j.[4] Note that emissions from firm 1 (located in segment 1 of the river) can increase pollution levels in all downstream segments. However, pollution levels in the segment 2 are due only to the emissions of firm 1 (located in segment 1) and firm 2 (located in segment 2).

We assume that all the n segments of the river represent both sources and receptors of pollutants. Thus, the matrix of transfer coefficients of the n segments is represented by the following upper triangular matrix.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ & a_{22} & a_{23} & \cdots & a_{2n} \\ & & a_{33} & \cdots & a_{3n} \\ & & & \cdots & \cdots \\ & & & & a_{nn} \end{bmatrix}$$

In this paper we also assume that the transfer coefficients are determined experimentally for each river and type of water pollutant. For a given amount of pollutant, the values of these coefficients may vary according, for example, to the volume of water flowing in a river and/or water temperature. Once determined the transfer coefficients, the environmental authority need only measure the level of pollution in each river segment. Based on the levels of pollution in each segment and transfer coefficients, the levels of emissions in each segment can be calculated indirectly. Therefore, it is not necessary to measure directly the levels of emissions of each firm in each segment.

Now we can define the four axioms that support the cost allocation method proposed in this paper. The formalization of these axioms is made in the next section. The fact that downstream polluters are not responsible (or do not have control) for the costs incurred in upstream segments is translated into an axiom called *Independence of Upstream Costs*.

[4] Note that we are assuming that the relationship between emissions and level of pollution is linear. This assumption is very important in the context of our analysis.

Basically this axiom said that the costs incurred in the segment j shall be allocated only to the polluters located in this segment as well as all the upstream polluters. The next axiom, called *Upstream Asymmetry*, is based on the assumption that the contributions of upstream polluters to the level of pollution in a given downstream segment are not necessarily equal. In this case, according to the polluter pays principle, the responsibility of each upstream polluting firm must be in accordance with the proportion of its contribution to the concentration of pollution in each segment of the river. The third axiom is called *Efficiency*. It simply states that the total costs of cleaning up pollutants should be fully allocated among the polluters. The last axiom is called *Additivity*. It is used as a requirement of consistency in order to obtain a unique value allocation.

Ni and Wang (2007) establish and use the axioms *Independence of Upstream Costs*, *Upstream Symmetry*, *Efficiency* and *Additivity* to characterize their UES method. In this paper, we replace the axiom *Upstream Symmetry* by the axiom *Upstream Asymmetry*. This change reflects the way upstream polluters are treated in the cost allocation problem to reduce pollution in a given downstream segment. While Ni and Wang (2007) treat upstream polluters symmetrically, we treat them asymmetrically. This change is possible with the help of the polluter pays principle and the previously defined transfer coefficients and extends the scope of the UES method.

Gómez-Rúa (2013) uses also the same set of axioms suggested by Ni and Wang (2007), except *Upstream Symmetry*. This axiom is replaced by three alternative axioms: *Upstream Monotonicity*, *δ- Biodegradation Rate*, and *Proportional Tax*. For the first, it is assumed that, for biodegradable pollutants, the further away the segment of the river is from polluter i, the smaller the part of the cost this polluter should pay for cleaning that segment. For the second, it is assumed that the biodegradation rate of a pollutant is known and equal to δ. For the last alternative axiom, it is assumed that the cost assigned to polluter i for cleaning a given segment should be calculated based on exogenous factors, such as city population, water consumption, pollution load, etc.

Now suppose that the total polluting-cleaning cost $\sum_{j=1}^{n} c_j$ is represented by the vector $c = (c_1,...,c_n) \in R_+^n$, where c_j denotes the costs required to reduce pollution in the jth segment, and that the *pollution cost allocation problem* is represented by the pair $C = (N,c)$, where $N = \{1,...,n\}$ is a finite number of agents (polluters). A solution to this problem is a vector $x = (x_1,...,x_n) \in R_+^n$ such that $\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} c_j$, where x_j is the cost share assigned to the jth polluter. A method is a rule that assigns to each problem (N,c) a solution $x(N,c)$. Applying the polluter pays principle and based on the model described above, we propose for any $c \in R_+^n$ the following method, called *Polluter Pays* method.

$$x_j^{PP}(c) = \sum_{k=j}^{n} \frac{a_{jk}}{\sum_{i=1}^{k} a_{ik}} \cdot c_k, \quad j = 1,...,n \tag{2}$$

where a_{ij} are the transfer coefficients. To illustrate the use of this method, consider the following simple example: a polluted river is divided into three segments, i.e., $N = \{1,2,3\}$, and the cost to reduce pollution set by the environmental authority in the three segments is given by the vector $c = (c_1, c_2, c_3)$. According to the expression (2), the cost share assigned to polluter $j \in N$ is given by

$$x_1^{PP}(c) = \frac{a_{11}}{a_{11}} \cdot c_1 + \frac{a_{12}}{a_{12} + a_{22}} \cdot c_2 + \frac{a_{13}}{a_{13} + a_{23} + a_{33}} \cdot c_3$$

$$x_2^{PP}(c) = \frac{a_{22}}{a_{12} + a_{22}} \cdot c_2 + \frac{a_{23}}{a_{13} + a_{23} + a_{33}} \cdot c_3$$

$$x_3^{PP}(c) = \frac{a_{33}}{a_{13} + a_{23} + a_{33}} \cdot c_3$$

Note that clearly, $\sum_{j=1}^{3} x_j = \sum_{j=1}^{3} c_j$. If we assume that the transfer coefficients are given by $a_{13} = 1$, $a_{23} = 2$ and $a_{33} = 3$, then the cost share assigned to the three polluters in the third segment are $x_1^{PP}(c) = \frac{1}{6} c_3$, $x_2^{PP}(c) = \frac{2}{6} c_3$, and $x_3^{PP}(c) = \frac{3}{6} c_3$, i.e., the upstream polluters are treated asymmetrically. Note also that if we assume that all transfer coefficients are equal, i.e., if $a_{ij} = \bar{a}$, for $i \leq j$; $i = 1,...,n$; $j = 1,...,n$, in expression (2), then we obtain the UES method proposed by Ni and Wang (2007). In this sense, the *Polluter Pays* method is more general and includes the UES method as a special case.

Regarding the systems of weights, Gómez-Rúa (2013) shows that a method $x(N,c)$ satisfies the axioms *Independence of Upstream Costs*, *Efficiency* and *Additivity* if and only if for every segment $j = 1,...,n$, there exists a weight system $\left(s_i^j\right)_{i \in N} \in R_+^n$ such that $s_i^j = 0$ for polluter $i > j$ and $\sum_{i=1}^{n} s_i^j = 1$. If we add *Upstream Monotonicity* to these three axioms, the weight system must satisfy additionally $s_i^j \leq s_k^j$ for every $i < k \leq j$, or $s_i^j = \delta^{k-i} s_k^j$ for every $i < k \leq j$, if, instead, we had added the δ - *Biodegradation Rate* axiom. In both cases, she does not specify how the weights are obtained. Only when the *Proportional Tax* is added to the three previous axioms, she assumes that the weights are given by $s_i^j = w_i / \sum_{l=1}^{j} w_l$ for all $i \leq j$, where w is a weight vector based on exogenous factors. In our approach, the weights are determined endogenously and come from a pollutant transport model.

3. Results

In this section we present four propositions. The first proposition shows that the *Polluter Pays* method is the only method that satisfies the four axioms mentioned earlier. The second and third propositions show, respectively, that the allocation of the total costs of cleaning river pollutants suggested by the *Polluter Pays* method coincides with both the weighted Shapley value and the τ - value of the cost game generated from the problem. The fourth proposition shows that both the weighted Shapley value and the τ - value, and therefore the solution of the *Polluter Pays* method, are in the core of the corresponding cost game generated from the problem.

We now formalize the four axioms that characterize the *Polluter Pays* method.

Independence of Upstream Costs. For any $c^1, c^2 \in R_+^n$ and $i \in N$ such that $c_l^1 = c_l^2$, $l > i$, for all $j > i$, we have $x_j(c^1) = x_j(c^2)$.

This formalization of the axiom follows Ni and Wang (2007) and simply states that the cost share of a given firm corresponds to its own pollution cost as well as all downstream costs, but does not include upstream costs.

Upstream Asymmetry. For any $c \in R^n_+$ and $i \in N$, if $c_i \neq 0$, then we have the following proportion of this cost to be paid by firm $j \leq i$

$$x_j(c) = \frac{a_{ji}}{\sum_{k=1}^{i} a_{ki}}, \qquad j = 1,\ldots,n,$$

where a_{ji} ($i = 1,\ldots,n$; $j = 1,\ldots,n$) are transfer coefficients as defined in the previous section.

Efficiency. $\sum_{j=1}^{n} x_j = \sum_{j=1}^{n} c_j$.

Additivity. For $c^1 = (c_1^1,\ldots,c_n^1) \in R^n_+$ and $c^2 = (c_1^2,\ldots,c_n^2) \in R^n_+$, we have $x_j(c^1 + c^2) = x_j(c^1) + x_j(c^2)$ for all $j = 1,\ldots,n$.

Proposition 1. *The Polluter Pays method is the only method satisfying Independence of Upstream Costs, Upstream Asymmetry, Efficiency and Additivity.*

Proof. It is easy to verify that the *Polluter Pays* method satisfies the four axioms above. Now we show that the *Polluter Pay* method is the only method that satisfies these axioms.

Consider the n-dimensional cost vector $c^k = (0,\ldots,0,1,0,\ldots,0)$, $k = 1,2,\ldots,n$, where 1 is the kth component of c^k. By *Independence of Upstream Costs*, $x_j(c^k) = 0$ for all $j > k$. Now suppose that for all $j \leq k$, $x_j(c^k) = \beta$, where β is some nonnegative real number. By *Upstream Asymmetry* and by *Efficiency*, we must have

$$x_j(c^k) = \frac{a_{jk}}{\sum_{l=1}^{k} a_{lk}} \text{ if } j \leq k \text{ and } x_j(c^k) = 0 \text{ if } j > k.$$

Note that since the n-dimensional cost vectors, c^k, $k = 1,2,\ldots,n$, form a basis of R^n, any $c \in R^n$ can be written as $c = \sum_{k=1}^{n} c_k \cdot c^k = (c_1, c_2,\ldots,c_n)$. *Additivity* implies

$$\begin{aligned} x_j(c) &= x_j\left(\sum_{k=1}^{n} c_k \cdot c^k\right) \\ &= \sum_{k=1}^{n} c_k \cdot x_j(c^k) \\ &= \sum_{k=j}^{n} c_k \cdot \frac{a_{jk}}{\sum_{l=1}^{k} a_{lk}} \quad \text{since } x_j(c^k) = 0 \text{ if } j > k. \\ &= x_j^{PP}(c). \end{aligned}$$

for all $j \in N$. The proposition is proved. □

This result clearly indicates that the *Polluter Pays* method treats all firms fairly, that is, the cost assigned to each firm is calculated in accordance with the proportion of its contribution to the concentration of pollution in its own segment and in all its downstream segments.

To analyze the relationship between the solution of the *Polluter Pays* method with values, such as the (weighted) Shapley value and the τ - value, we need to model the *pollution cost allocation problem* as a coalitional game with transferable payoff.[5] This type of game is

[5] For more details on coalitional games see, for example, Osborne and Rubinstein (1994).

described by a pair $G=(N,v)$, where N is a finite set of players and $v(S)$ is a real number that represents the *worth* of the coalition S ($S \subseteq N$). In the context of a cost game, *worth* represents the *costs* incurred by the coalition S. Now, letting $N=\{1,2,...,n\}$ be the set of agents (polluting firms) and letting $S \subseteq N$ be any coalition from the n polluters, all we need is to define the *characteristic function* $v(S)$. To do this, we denote by $\min S$ the most upstream polluting firm in the coalition S. Under the polluter pays principle, each member of $S \subseteq N$ is responsible for the costs to reduce pollution in its own segment and in all downstream segments in accordance with its contribution to the level of pollution. Therefore, the cost of the coalition S, for any given $c \in R_+^n$, should be

$$v^c(S) = \sum_{j=\min S}^{n} c_j \ . \tag{3}$$

Now, for any given $c \in R_+^n$, we have a coalitional game, (N, v^c), that satisfies $v^c(\emptyset)=0$. This game is identical to that proposed by Ni & Wang (2007) under the UTI doctrine. Since both the (weighted) Shapley value and the τ - value identify a single result in a coalitional game, they are examples of values. A *value* is a function ϕ defined on the space of games G^N, $\phi: G^N \to R^n$, that identifies a feasible allocation, $\sum_{i\in N}\phi_i(v)=v(N)$, for every coalitional game. The weighted Shapley value can be characterized by the axioms of *Dummy Player*, *Asymmetry*, *Efficiency* and *Additivity* and can be defined using unanimity games by the following expression (see, for example, Haeringer, 2006)

$$\phi_i(v,\omega)=\sum_{i\in T}\alpha_T \cdot \phi_i(u_T(S),\omega)=\sum_{i\in T}\alpha_T \frac{\omega_i}{\sum_{j\in S}\omega_j}$$

where $\phi_i(v,\omega)$ is the allocation to player i at the result $\phi(v,\omega)$, $u_T(S)$ is the unanimity game defined by $u_T(S)=1$ if $S \supseteq T$ and $u_T(S)=0$ otherwise, and ω is a weight vector $(\omega_i)_{i\in N}$ being $\omega_i \in R_+$ the weight associated to player $i \in N$; $\alpha_T=\sum_{T\subseteq S}(-1)^{s-t}v(T)$, being s and t the number of players in coalitions S and T, respectively.

In the next proposition, we show that the *Polluter Pays* method is consistent with the weighted Shapley value of the game (N,v^c), that is, the solution of the *Polluter Pays* method (2) coincides with the weighted Shapley value ϕ of the game (N,v^c) for all $c \in R_+^n$.

Proposition 2. *For all $c \in R_+^n$ and v^c defined by (3), we have $x_j^{PP}(c)=\phi_j(v^c,\omega)$ for all polluting firms $j \in N$.*

Proof. Consider again the n-dimensional cost vector $c^k=(0,...,0,1,0,...,0)$, $k=1,2,...,n$, where 1 is the kth component of c^k. The coalitional games associated with these cost vectors c^k, $k=1,2,...,n$, are given by

$v^{c^k}(S)=0$ if $\min S > k$ and $v^{c^k}(S)=1$ otherwise.

Note that for the games (N, v^{c^k}), all polluting firms $j>k$ are dummies and all polluting firms $j \le k$ are not treated symmetrically, that is, different weights are assigned to them according to their contribution to the concentration of pollution in their own segments as well as downstream segments. If these weights are defined in accordance with the transfer coefficients presented previously, the weighted Shapley values of the games (N, v^{c^k}) are

$$\phi_j(v^{c^k},\omega)=0 \text{ if } j>k \qquad \text{and} \qquad \phi_j(v^{c^k},\omega)=\frac{a_{jk}}{\sum_{l=1}^{k} a_{lk}} \text{ otherwise.} \tag{4}$$

Since any $c \in R_+^n$ can be obtained by $c=\sum_{k=1}^{n} c_k \cdot c^k$, we have, for all coalitions $\varnothing \neq S \subseteq N$ of the game (N, v^c),

$$\begin{aligned} v^c(S) &= \sum_{i=\min S}^{n} c_i \\ &= \sum_{i=\min S}^{n} c_i \cdot c^i \\ &= \sum_{i=\min S}^{n} c_i \cdot v^{c^i}(S), \end{aligned}$$

By *Additivity* and expressions in (4), we get

$$\begin{aligned} \phi_j(v^c,\omega) &= \sum_{j=\min S}^{n} c_j \cdot \phi_j(v^{c^j},\omega) \\ &= \sum_{k=j}^{n} c_k \cdot \frac{a_{jk}}{\sum_{l=1}^{k} a_{lk}} \\ &= x_j^{PP}(c) \end{aligned}$$

for all $j=1,2,\ldots,n$. The proposition is proved. □

Tijs (1987) uses three axioms to characterize the τ - value, being *Efficiency* one of them. The others two axioms are called *Minimal Right Property* and *Restricted Proportionality Property*. The *Minimal Right Property* is implied by Additivity together with Individual Rationality ($x_i \leq v(\{i\})$ in the case of a cost game) and Efficiency while the *Restricted Proportionality Property* implies that the gains or costs assigned to a given player is proportional to his(her) marginal contribution to the grand coalition, i.e., the coalition formed by all players.

For the next proposition, that establishes the relationship between the *Polluter Pays* method and the τ - value of the corresponding cost game, we need some additional notions. These notions are used to establish the expression of the τ - value for the cost game.[6] The first of them is the *marginal vector* $m(v^c)$ of the game (N, v^c) that is defined as the vector with i th coordinate

$$m_i(v^c)=v^c(N)-v^c(N-\{i\}).$$

In a cost allocation problem, the real number $m_i(v^c)$ measures the smallest contribution of player i to the worth of the grand coalition $v^c(N)$ if he(she) joins the coalition of all the others $N-\{i\}$ players. In a division of the total cost of cleaning up a polluted river, for example, player i must not expect to contribute less than his(her) marginal value $m_i(v^c)$. Thus, we can see $m_i(v^c)$ as a lower bound for player i in the allocation of the total cost $v^c(N)$ of the game (N, v^c).

It is easy to show that, in all possible coalitions $0 \neq S \subseteq N$ that player i can join, $\sum_{i\in S} m_i(v^c) \leq v^c(S)$. The gap $g(S)=v^c(S)-\sum_{i\in S} m_i(v^c)$ plays an important role in the

[6] For more details on the τ - value see, for example, Tijs (1987).

definition of the τ - value of the game (N, v^c). In fact, for coalitional games with $g(S) \geq 0$, we must construct a payoff vector x such that $m_i(v^c) \leq x_i \leq m_i(v^c) + \lambda_i(v^c)$, where $\lambda_i(v^c) = \max_{S; i \in S} g(S)$ and the expression $m_i(v^c) + \lambda_i(v^c)$ represents an upper bound for player i in the allocation of the total cost $v^c(N)$ of the game (N, v^c). For games (N, v^c) with $g(N) \geq 0$, the τ - value is defined as follows.

$$\tau(v^c) = \begin{cases} m(v^c) & \text{if } g(N) = 0 \\ m(v^c) + \alpha(v^c) g(N) & \text{if } g(N) > 0 \end{cases} \tag{5}$$

where $\alpha(v^c) \in [0,1]$ is chosen such that $\sum_{i \in N} \tau_i(v^c) = v^c(N)$. Taking into account this *Efficiency* axiom of the τ - value and the characteristic function of the coalitional game defined by (3), we define the weights $\alpha(v^c)$ in a similar way as we did in the Proposition 2.

Proposition 3. *For all* $c \in R_+^n$ *and* v^c *defined by (3), we have* $x_j^{PP}(c) = \tau_j(v^c)$ *for all polluting firms* $j \in N$.

Proof. For any coalitional game (N, v^c) with cost vector $c = (c_1, c_2, ..., c_n) \in R_+^n$, we have $g(N) = \sum_{k=2}^n c_k$. In this game, the marginal contribution of the polluting firms are $m_j(v^c) = c_1$ if $j = 1$, and $m_j(v^c) = 0$, if $j = 2,3,...,n$. Note that, if we want to charge firms i and j, $i \neq j$, $i, j \geq 2$, according to their contribution to the concentration of pollution in their own segments as well as all downstream segments, the parameter $\alpha(v^c)$ of the equation (5) cannot be constant, but should vary according to each firm and each segment of the river. So, for any coalition $\varnothing \neq S \subseteq N$, $v^c(S)$, we should have

$$\alpha_j(v^c) = 0 \quad \text{if } \min S > k \quad \text{and} \quad \alpha_j(v^c) = f(a_{jk}) \in (0,1] \text{ otherwise,} \tag{6}$$

where $j = 1,...,n$ refers to the jth polluting firm, $k = 1,...,n$ refers to the kth segment of the river, and a_{jk} ($j = 1,...,n$; $k = 1,...,n$) are the coefficients of the pollutants transport model defined previously. Taking this into account, we can calculate the τ - value for jth polluting firm the of the coalitional game (N, v^c) with $g(N) > 0$ as follows

$$\tau_j(v^c) = m_j(v^c) + \sum_{k=2}^n \alpha_j(v^c) \cdot c_k . \tag{7}$$

Equation (7) is equivalent to the following equation since it satisfies expressions (6), $m_j(v^c) = c_1$ if $j = 1$, and $m_j(v^c) = 0$, if $j = 2,3,...,n$, and the axiom of *Efficiency*, that is,

$$\begin{aligned} \tau_j(v^c) &= \sum_{k=j}^n c_k \cdot \frac{a_{jk}}{\sum_{l=1}^k a_{lk}} \\ &= x_j^{PP}(c) \end{aligned}$$

for all $j = 1,2,...,n$. The proposition is proved. □

Finally, we show in the next proposition that both the weighted Shapley and the τ - value, and therefore the *Pollution Pays* method solution, are in the core of the corresponding cost game (N, v^c). We show that indirectly, using the notion of convexity in games. In convex games the core is always nonempty and the Shapley value is a core allocation (see, for example, Moulin, 1988). Monderer et al. (1992) show also that the set of all weighted

Shapley values contains the core of a coalitional game. For cost sharing games, the convexity property is equivalent to the concavity of the game (Ni and Wang, 2007). A game $G=(N,v)$ is called *concave* if and only if

$$v(S\bigcup\{i\})-v(S)\geq v(T\cup\{i\})-v(T) \qquad (8)$$

for every $i\in N$ and for every $S\subset T\subset N\setminus\{i\}$.

We recall that the core of a cost allocation game, $C(v)$, is defined by

$$C(v)=\left\{x\in X:\sum_{i\in S}x_i\leq v(S)\ \text{ for every } S\subseteq N\right\}$$

where $X=\left\{x\in R^n:\sum_{i\in N}x_i=v(N)\right\}$.

Proposition 4. *For all* $c\in R_+^n$, *both the weighted Shapley value and the* τ *- value of the cost game* (N,v^c) *are in the core, i.e.,* $\sum_{i\in S}\phi_i(v^c,\omega)=\sum_{i\in S}\tau_j(v^c)=\sum_{i\in S}x_j^{PP}(c)\leq v^c(S)$ *for every* $S\subseteq N$.

Proof. We need only to show that the game (N,v^c) is concave, i.e., for every $i\in N$ and for every $S,T\subset N\setminus\{i\}$, if $S\subset T$, then (rearranging (8))

$$v^c(T\cup\{i\})-v^c(S\cup\{i\})\leq v^c(T)-v^c(S) \qquad (9)$$

There are three cases to consider:

Case 1. If $i<\min T=\min S$, $\min T=\min S<i$ or $\min T<\min S<i$, then (9) is satisfied trivially as an equality.

Case 2. If $i<\min T<\min S$, then (9) is satisfied as a strict inequality since $v^c(T\cup\{i\})=v^c(S\cup\{i\})$ and $v^c(T)>v^c(S)$.

Case 3. If $\min T<i<\min S$, then (9) is satisfied as a strict inequality since $v^c(T\cup\{i\})-v^c(S\cup\{i\})<v^c(T)-v^c(S)$.

These three cases cover all possibilities and show that the game (N,v^c) is concave. The proposition is proved. □

4. Concluding remarks

In this paper we investigate the allocation cost problem of cleaning up a polluted river among the agents located along it. We propose a method based on the polluter pays principle and a pollutant transport model to solve it. This method is characterized by four axioms. We also show that the solution of the proposed method coincides with the (weighted) Shapley value and the τ - value to the corresponding cost game that is induced according to the polluter pays principle. Another nice property of this solution is that it belongs to the core of the cost game analyzed and, therefore, is a stable allocation.

The information required to use the proposed method (*Polluter Pays* method) include the type of water pollutant (whether biodegradable or not), the transfer coefficients calculated for type of water pollutant and segment of the river, and the concentration of pollution in each segment. With this information, it is possible to calculate indirectly the levels of emissions of the firms in each segment of the river. The cost allocation suggested by this method is based on these coefficients and the estimated cost to clean pollutants from the river in each segment. Note that the method is based only on technical information that is specific for each polluted river. The more accurate this technical information is, the more reliable the application of this method will be.

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