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The dynamics of Stock price adjustment to fundamentals: an empirical essay via STAR models in the Tunisian stock market

Ramzi Boussaidi
Faculty of Law, Economics and Management of Jendouba, University of Jendouba, Tunisia

Abaoub Ezzeddine
College of Administrative and Financial Studies, Taif University, Kingdom of Saudi Arabia

Abstract

This paper examines the dynamics of stock prices adjustment to fundamental value proxied by dividend per share and earnings per share in the Tunisian stock market based on the cointegration techniques. First, the linear cointegration between stock prices and fundamental values is examined by using the Johansen's cointegration test. The empirical results indicate a linear cointegrating relationship between stock prices and dividend per share, and not between stock prices and earnings per share, in support of a linear mean reversion of stock prices towards its fundamental value proxied by dividend. To further investigate the cointegration between stock prices and earnings per share in a nonlinear context, we modelled the deviation of stock prices away from EPS by a logistic smooth transition autoregressive (LSTAR) model. Our results indicate that this model cannot capture the nonlinearity of this deviation, failing, then, to give evidence of a nonlinear cointegration between stock prices and EPS. These results suggest that when selecting stocks, Tunisian investors should focus on the underlying performance of stocks only in terms of their dividend per share.

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Contact: Ramzi Boussaidi - boussaidi.ramzi@yahoo.fr, Abaoub Ezzeddine - abaoub.ezzeddine@planet.tn.
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1. Introduction

Since the work of De Bondt and Thaler (1985) giving evidence of a long-run return reversal, several empirical studies have shown that stock prices revert back to their fundamental values ensuring, thus, a stock price predictability. This reversal, also called mean reversion, is a phenomenon which corrects the deviation between stock prices and their fundamental values. Such a corrective movement implies the existence of a cointegrating relationship between stock prices and fundamental values, and reflects an error correction mechanism which adjusts the variations in stock prices towards fundamentals. The nature of this relationship was initially pioneered by Campbell and Shiller (1987) who developed the present value model relating the stock price and its fundamental value measured by dividends. This model implies that stock price is fundamentally determined by a discounted value of its expected future dividends. To check the validity of the model, the authors have shown that there should exist a stationary linear combination between the stock price and the dividends, and therefore, a cointegration between the two variables even though their processes are not stationary.

However, early studies have led to mixed evidence on the presence of a cointegration between stock prices and fundamentals. This controversy is due to the fact that this relationship has been checked in a linear framework using conventional methods of stationarity and cointegration, while new nonlinear econometric techniques have been developed such as Smooth transition Autoregressive models (STAR). These models have empirically shown a good performance in reflecting the generating process of some financial and macroeconomic series (Teräsvirta, 1995; Teräsvirta and Anderson, 1992; Sarantis, 2001; and Singh, 2014) and, particularly, in modeling the dynamics of relations other than the stock market mean reversion to fundamentals such as the adjustment of the exchange rate to its equilibrium measured by the purchasing power parity (Michael Nobay and Peel, 1997; and Dufrénot *et al*., 2004). Recently, empirical studies have shown that the conventional linear cointegration techniques are insufficient to take into account the dynamic of stock price adjustment to fundamentals. The recent financial and economic literature offers several explanations for the nonlinearity of the stock price adjustment to fundamental values, part of which is initially borrowed from works on foreign exchange markets. These explanations include the existence of transaction costs (Dumas, 1992; Sercu, Uppal and Van Hulle, 1995, Michael, Nobay and Peel, 1997, Kapetanios, Shin and Snell, 2006), the presence of noise traders in financial markets (Enders and Granger, 1998 and Enders and Siklos, 2001), the existence of speculative bubbles in stock prices (McMillan, 2009), the firms change of their dividend policy (Ackert and Hunter, 1999 Boudoukh *et al* ., 2007, Robertson and Wright, 2006, Lettau and Nieuwerburgh, 2008 and Park, 2010) and the uncertainty about the quality of the manager (Kiyotaki, 1990). In this paper, we try to examine the dynamics of the relationship between stock prices and fundamentals in the Tunisian stock market using new econometric concepts of cointegration.

The remainder of the paper is organized as follows. The second section presents a brief literature review on the dynamics of stock price adjustment to fundamental value. The third one examines empirically the dynamics of price adjustment to fundamental value in the Tunisian stock market in linear and non linear frameworks. The fourth section concludes the paper.

2. Literature Review

The widely-used approach to study the linearity of the stock price mean reversion to fundamental value is essentially based on the cointegration tests of Engle and Granger (1987) and Johansen (1991) between stock prices and dividends or the stationarity tests of the log dividend-price ratio. For example, Nasseh and Strauss (2004) applied a cointegration test on panel data between stock price and dividend for a sample of 84 U.S. firms over the period 1979 -1999 and revealed a long-term relationship between the two variables. Using a similar approach but with time series data, Chen, Kim and Chen (2007) found that the Taiwanese market index does not show a mean reversion towards its fundamental values measured by the DPS or EPS over the period 1995-2004. However, they found a cointegration between the stock price index and EPS of only 2 out of 7 industry sectors (hospitality and building). Sing, Liow and Chan (2002) found that only 9 out of 19 firms listed on the Singapore Stock market for the period 1989-1999 show a mean reversion of stock prices to at least one of their fundamental value measured by earnings per share, dividend per share or net asset value per share.

However, other empirical studies have analyzed very large data in several countries and have failed to detect a stock price mean reversion using conventional tests of stationarity and cointegration in a linear framework. For example, McMillan (2007) found that 9 out of 13 emerging markets show non-stationary dividend to price ratio indicating no cointegration between stock prices and dividends. Kapetanios, Shin and Snell (2006) detected a linear cointegration between the two variables only in 2 countries among 11 over the period 1974-2002. Balke and Wohar (2002) found that the quarterly log price-dividend ratio over the period 1953-1999 in the US is non-stationary. Similar results were found by Lamont (1998) for the period 1947-1994. Using monthly data, Kanas (2005) applied the two-stage test of Engle and Granger (1987) but he did not find a cointegration between stock prices and dividends for U.S.A, U.K, Japan and Germany, thus indicating the absence of such a relationship in a linear framework. However, when generating nonlinear transformations in the stock price and dividend variables through the Alternating Conditional Expectation (ACE) algorithm, he found that the Engle and Granger test give evidence of a significant cointegration between the two variables. He concluded that the long-term relationship between prices and dividends is nonlinear and the failure of the linear dividend discount model to explain this relationship is attributed to the lack of suitable nonlinear transformation of the two variables.

According to Bohl and Siklos (2004), the conflicting empirical findings are most likely due to the fact that the conventional stationary and cointegration tests are not appropriate because they assume a unit root as the null and a linear process under the alternative. Several studies show that this relation should be examined in a nonlinear context. Advanced explanations to this non-linearity include the existence of transaction costs, the presence of noise traders in financial markets, the existence of speculative bubbles in stock prices, the change in the dividend policy of firms and the uncertainty related to the quality of the manager.

The impact of the presence of transaction costs on the equilibrium relationship between stock prices and fundamentals was initially analyzed in the context of foreign exchange markets by Dumas (1992) and then, Sercu, Uppal and Van Hulle (1995). Dumas (1992), for example, showed that the deviation of the exchange rate from its equilibrium given by the Purchasing Power Parity (PPP) follows a nonlinear mean reversion process with a speed of adjustment towards equilibrium depending on the extent of this deviation from PPP. In the transaction band and in the absence of trading, the process is divergent inducing a deviation of the exchange rate from its equilibrium level. Deviations of the exchange rate relative to the PPA remain uncorrected as long as they are below the transaction costs. The mean reversion occurs only when these deviations have become large enough. Michael, Nobay and Peel (1997) examined this relationship in 4 countries (United States, Great Britain, Germany and France) with monthly and annual frequencies covering different periods and showed that in the presence of transaction costs, the adjustment of exchange rates to their fundamental value defined by the PPP follows a nonlinear Exponential Smooth Transition Autoregressive (ESTAR) model. Kapetanios, Shin and Snell (2006) developed a non linear Exponential Smooth Transition Autoregressive error correction model (ESTAR-ECM) based on the two-step cointegration method of Engle and Granger (1987). They showed that this model reflects the adjustment mechanism of monthly stock price towards dividends for 7 out of 11 international markets over the period 1974-2002.

Motivated by Shleifer's analyses of the interactions between fundamental traders and noise traders, McMillan (2007) underlines that the nonlinearity of the stock price adjustment to fundamental value, can be explained by the asymmetries between regimes of rising and falling prices. In fact, in bull markets noise traders tend to be overconfident and therefore they overreact to good news which makes prices rise above their fundamental value; however in bear markets they tend to exhibit a conservative and anchored behavior. Consequently, the correction of the deviations or the mean reversion is not the same when prices are above their fundamental value as when they are below it, indicating an asymmetric adjustment of stock prices to fundamentals.

Bohl and Siklos (2004) underline that the presence of speculative bubbles in stock prices causes an asymmetry in the change in log dividend to price ratio. This is because speculative bubbles in a real financial market are generally positive bubbles induced by the persistent increase in prices relative to fundamentals followed by a crash, but not negative bubbles induced by a persistent decline in prices relative to fundamentals, which remain, according to the authors, *theoretical bubbles*. Such a bubble behavior could be detected by a change in the log dividend-price ratio above the long-run equilibrium level followed by an increase to restore stock prices to that equilibrium level. The authors propose a Momentum Threshold Autoregressive (MTAR) model designed to detect this asymmetric behavior. Enders and Granger (1998) and Enders and Siklos (2001) showed that the traditional cointegration approach in the presence of an asymmetric deviation from equilibrium leads to biased results. Therefore, Bohl and Siklos (2004) used a Momentum Threshold Autoregressive (MTAR) model that takes into account the asymmetry caused by the presence of bubbles to examine the evolution of the monthly Standard & Poor's index log Dividend-Price ratio over the period 1871-2001. They detected a faster price adjustment to equilibrium when they are larger than dividends than when they are lower.

McMillan (2007) explains his nonlinear study through the presence of noise traders and transaction costs to explore the concept of asymmetry highlighted by Bohl and Siklos (2004), but through an asymmetric Exponential STAR model. The author estimated, first, a symmetric ESTAR model on monthly data of log Dividend-Price ratio for 13 countries and found an insignificant mean reversion for most countries (8 out of 13). However when the asymmetric tendency of Dividend/Price ratio and, consequently, the asymmetry in the speed of adjustment, are taken into account through an asymmetric ESTAR model, a significant asymmetric mean reversion is detected for 11 countries out of 13.

Moreover, Boudoukh *et al.* (2007) and Robertson and Wright (2006) showed that the transition from a dividend distribution policy to a share repurchase policy results in a break of the cointegrating relationship between stock prices and dividends and makes the dividend-price ratio lose its predictive power. This break is also reflected in a change in the average dividend-price ratio. Lettau and Nieuwerburgh (2008) showed that such a change explains the sensitivity of the predictive power of the dividend to price ratio. Park (2010) found that the dividend to price ratio series in the United States over the period 1871-2007 shows a change in persistence and the break point of this persistence is associated with the fourth quarter of

1979, which approximately coincides with the period in which the practice of share repurchase became important in the United States. In another paper, McMillan (2009) adopted the ESTAR model developed by Kapetanios, Shin and Snell (2003) to test the dynamics of the Dividend to Price ratio in 8 stock markets (Canada, France, Germany, Hong Kong, Japan, Singapore, the United Kingdom, and the United States). He reported that the Dividend to Price series is characterized not only by a nonlinearity, but also by the equilibrium level of the mean reversion changes over time due to the change in the corporate dividend policy of firms. He suggested that both characteristics can be detected through a Time Varying Exponential Smooth Transition Autoregressive (ESTAR-TV) model. Based on this model, he further reported a significant mean reversion for the 8 countries, a significant change in the speed of the nonlinear stock price adjustment to its fundamental value and a change in the mean reversion equilibrium level over time.

Ackert and Hunter (1999) showed, through an extension of the Froot and Obstfeld (1991) model, that the nonlinearity of the relationship between stock prices and dividends arises from the way managers choose the dividend payout. They developed a theoretical model where the evolution of the long-run stock price results from the observed behavior of the managers when dealing with dividend distribution. Another theoretical model was proposed by Kiyotaki (1990) which takes into account the market perception of the quality of the manager to explain the nonlinearity of the relationship between stock prices and fundamentals.

3. Empirical issues

The purpose of this empirical investigation is to examine whether stock prices revert back to their fundamental value in the Tunisian stock market and to identify the nature of this adjustment, i.e. whether it's linear or nonlinear. For this purpose, our first objective is to examine the hypothesis of mean reversion using the linear concept of cointegration developed by Johansen (1991) between stock prices and dividend per share (DPS) and between stock prices and earnings per share (EPS). Evidence of cointegration between stock price and fundamental value would imply that the mean reversion hypothesis holds; however evidence of non-cointegration between the two variables could imply that the mean-reversion hypothesis is rejected, or that the cointegration is nonlinear. To test the existence of a potential nonlinear cointegration, we use smooth transition autoregressive (STAR) models developed by Luukkonen, Saikkonen and Teräsvirta (1988) and Teräsvirta (1994).

3.1. The Tunis Stock Exchange

The Tunis Stock Exchange is a limited company created in 1969 having a capital equally and exclusively held by the stock market intermediaries. It includes a main market for large successful firms, an alternative market for small and medium-sized firms, a bond market; and a market of special purpose vehicles. The TSE's main function is to manage the stock market using an electronic quotation system, called "Stock Exchange Management System". However, the control function of the market is ensured by the public authority through the Financial Market Council. Among its principal tasks are the control of financial reporting and the penalty of the stock market regulations' violations. At the end of 2013, the last year of our sample period, the number of firms listed on the TSE is 71 firms forming 9 industries: financial, consumer goods, basic materials, manufacturing, services, oil and gas, health care, telecommunication, and technology. The market capitalization reached 14093 million dinars, representing 19.76% of the Gross Domestic Product, against 13780 million dinars in 2012, which indicates an increase of 2.27%. Moreover, the share of foreign participation in the market capitalization represents 20.04% against 20.51% in 2012. The volume of shares' issue has recorded a remarkable increase of 42% from 222 million dinars in 2012 to 316 million dinars in 2013 including 71.2% of cash issue.

3.2. Data and descriptive statistics

The data used in this study are annual stock prices (end of the year prices), dividend per share, earnings, and number of shares of all firms listed on the Tunisian Stock Market for the period 1971 to 2013. We consider all firms for which these data are simultaneously available. Their number has grown to 48 in 2013. The price index is the value weighted price of the firms composing our sample; the dividend index is the value weighted dividend per share; and the earnings index is the value weighted earnings per share. The variables are expressed in natural logarithms.

Table I. Descriptive statistics

	Mean	Median	Max	Min	Std. dev.	Skewness	Kurtosis	Jarque-Bera	Prob. (JB)
p	3.122	3.334	4.214	2.338	0.598	0.068	1.499	4.069	0.131
dps	-0.224	-0.182	0.563	-1.113	0.404	-0.061	2.151	1.319	0.517
eps	0.737	0.767	1.17	0.25	0.206	-0.452	3.03	1.465	0.481

Notes : *p*, *dps*, *eps* are stock price index, dividend per share and earnings per share, respectively, expressed in natural logarithm

Table I displays the descriptive statistics of the stock price index (*p), dividend per share (dps)* and *earning per share (eps)* indexes over the sample period. The mean of each of the three variables is 3.122; -0.224 and 0.737, respectively. The standard deviation of *p* is 0.598 however it is 0.404 for *dps* and 0.206 for *eps* indicating that stock prices are slightly more volatile than dividends, and earnings seem to be less volatile than dividends. The Jarque and Béra (1984) test shows that the series tend to be normally distributed given that the probability of JB statistic is higher than the conventional levels of significance.

3.3. Unit root tests

A precondition for cointegration tests is that all variables should have a unit root. To examine the stationarity of the three series, we use Augmented Dickey-Fuller (Dickey and Fuller, 1981), Phillips and Perron (1988), Kwiatkowski and *al.* (1992) and, Ng and Perron (2001) unit root tests. For the last one, we use the MZà test which is based on a modified Phillips-Perron test using GLS detrending. Using four unit root tests provides robustness check to our stationarity results because of the importance of this step in cointegration tests.

Table II displays results of the four unit root tests for the three series. For the stock price *p* and dividend per share *dps* in level, the tests indicate that each of these two variables has a unit root and it is, therefore, non-stationary. For the earnings per share *eps*, ADF, PP and Ng-Perron tests support the null of the presence of unit root at the 1% and 5% levels, however, the KPSS test supports the null of the absence of a unit root at these levels or, similarly, the null of stationary. But, at 10% level (not mentioned in the table) the KPSS test indicates that the null of stationary can be rejected at this level. In first difference, all the four unit root tests concur with the conclusion of the absence of a unit root for each of the three variables. Given that the series are stationary in first difference indicating that the variables *p*, *dps* and *eps* are integrated of order 1, *I(1)*, we proceed to the examination of the cointegration between *p* and *dps*, and *p* and *eps*.

Table II. Unit root tests				
Panel A. Variables in level				
	ADF	PP	Ng-Perron (MZa)	KPSS
P	-2.330	-2.369	-9.845	0.958**
	(-4.19/-3.52)	(-4.19/-3.52)	(-23.8/-17.3)	(0.73/0.46)
dps	-3.160	-3.162	-13.32	0.951**
	(-4.19/-3.52)	(-4.19/-3.52)	(-23.8/-17.3)	(0.73/0.46)
eps	-2.736	-0.433	-12.201	0.122
	(-3.59/-2.93)	(-2.62/-1.94)	(-23.8/-17.3)	(0.21/0.14)
Panel B. Variables in first difference				
Δp	-5.668**	-5.608**	-20.86*	0.091
	(-2.62/-1.94)	(-2.62/-1.94)	(-23.8/-17.3)	(0.73/ 0.46)
Δdps	-9.262**	-9.830**	-26.05**	0.114
	(-2.62/-1.94)	(-2.62/-1.94)	(-23.8/-17.3)	(0.73/ 0.46)
Δeps	-6.683**	-6.337**	-57.07**	0.059
	(-2.62/-1.94)	(-2.62/-1.94)	(-23.8/-17.3)	(0.73/ 0.46)

Notes : *p*, *dpa* and *eps* denote stock price, dividend per share and earnings per share in natural logarithm. *Δ* denotes the first difference of the variable. Numbers in parentheses are MacKinnon (1996) critical values associated with each statistic at 1% and 5% levels. Asterisks **,* denote rejection of the null hypothesis at 1 % and 5 % levels, respectively.

3.4. Linear Cointegration tests and error correction model

Table III summarizes the Johansen (1991) test results of the bivariate cointegration between stock prices and *dps* (Panel A) and, stock prices and *eps* (Panel B) for one optimal lag based on the Schwartz criterion. We assume the presence of a linear trend in the series and a constant in the cointegrating relation. The Johansen statistic associated with the Trace test is greater than the critical value at the 5% level (16.02 > 15.41) indicating that the null hypothesis of no cointegrating relation between *p* and *dps* can be rejected. However, the null hypothesis of the existence of at most one cointegrating relation between the two variables cannot be rejected at 5% level. This result is confirmed by the maximum eigenvalue test statistics. In sum, both trace test and maximum eigenvalue test concur with the conclusion of the existence of one cointegrating relation between stock price and *dps*. Panel B shows that based on the trace test there exists one cointegrating relation between stock price and *eps*, a result not supported by the maximum eigenvalue test. This divergence between the two tests may make the cointegration result less robust as in the case of *dps.*

Table III. cointegration tests between stock price and fundamental value			
Panel A. *Cointegration between p and dps*			
Trace test		Max-eigenvalue test	
None	At most 1	None	At most 1
16.02* (15.41/20.04)	1.536 (3.76/6.65)	14.48* (14.07/18.63)	1.536 (3.76/6.65)
Panel B. *Cointegration between p and eps*			
16.42* (15.49/19.93)	2.995 (3.76/6.63)	13.43 (14.07/18.52)	2.995 (3.76/6.63)

Note : numbers in parentheses are Osterwald-Lenum (1992) critical values for rejection of the null at the 5% and 1% levels. Asterisk * indicates rejection of the null at the 5%.

Given that there exists one cointegrating vector between stock prices and each of the two proxies of fundamental values, we estimate the Vector Error Correction Model (VECM) between the two variables. The estimation results of this model for optimal lag 1 are reported in table IV. We report only the equation corresponding to the regression of the stock price variation (Δp_t) on the lagged stock price variation (Δp_{t-1}), lagged changes in fundamentals (Δf_{t-1}) and error correction terms (ect_{t-1}).

$$\Delta p_t = \alpha + \beta \Delta p_{t-1} + \theta \Delta f_{t-1} + \gamma\, ect_{t-1} + \varepsilon_t$$

Table IV. Estimation of the VECM

	α	β	θ	γ	R^2
f = dps	0.018	0.284*	-0.265	-0.384***	19.5%
	(0.49)	(1.70)	(-1.35)	(-2.94)	
f = eps	0.023	0.073	0.064	-0.010	0.95%
	(0.56)	(0.04)	(0.20)	(-0.16)	

Notes: *f* denotes fundamental value; *Δ*, first difference. Asterisks ***, **,* denote significance at 1%, 5% and 10% levels, respectively.

If the fundamental value is proxied by dividend per share (*dps*), the coefficient associated with the error correction term has an expected negative sign significantly different from zero at 1% level. This indicates that there is a long-run convergence relationship of stock prices with its fundamental value measured by *dps*. In other words, although stock prices have diverged away from their fundamental value measured by *dps* from time to time, there exists an error correction mechanism which adjusts stock prices to return to their fundamental values. This result, also, provides support in favor of the long-run validity of the present value model. We further note that, lagged changes in stock price contribute to the prediction of changes in stock price while changes in *dps* do not. The explanatory power of the regerssors given by R^2 can reach 19.5%.

However, when fundamental value is proxied by earnings per share (*eps*), the coefficient associated with the error correction term is not significant, although it is negative. Furthermore, none of the other regeressors can predict the stock price variation which makes the explanatory power of the model very poor (0.95%). Unlike the case where fundamental value is proxied by dividend per share, this result does not support the existence of a mean reversion of stock prices to the earnings per share in a linearity context.

In sum, these results indicate that there is a mean reversion of stock prices to fundamental value when the latter is measured by dividend per share in support of the existence of a linear adjustment of stock prices to dividend per share. However, there is no linear mean reversion of stock prices to earnings per share. The absence of such a relation can be explained either by the existence of a nonlinear adjustment dynamic to earnings and, therefore, by the inability of conventional cointegration and linear VECM tests to detect this adjustment, or simply by the absence of a cointegration between the two variables. In the next section, we examine the hypothesis of the existence of a nonlinear cointegration between stock prices and earnings per share.

3.5. Nonlinear adjustment of stock prices to earnings

In order to examine non-linear adjustment of stock prices to earnings, we use univariate Smooth Transition Autoregressive model STAR developed by Luukkonen, Saikkonen and Teräsvirta (1988).

For p order, the *STAR (p)*, is as follows:

$$y_t = \pi_0 + \sum_{i=1}^{p} \pi_i y_{t-i} + \left(\theta_0 + \sum_{i=1}^{p} \theta_i y_{t-i} \right) \times F\big(\gamma(y_{t-d} - c) \big) + \varepsilon_t \tag{1}$$

Where y_t is a stationary process and $F(\gamma(y_{t-d} - c))$ is the transition function from a regime to another with $0 \le F \le 1$. This function allows for a smooth transition between the extreme regimes and depends on three parameters: y_{t-d}, the transition variable where d is the delay parameter $(1 \le d \le p)$; γ , the speed of transition between two extreme regimes, and c, the half-way point between the two regimes. ε_t are independently $N(0,\sigma_\varepsilon^2)$.

Luukkonen, Saikkonen and Teräsvirta (1988), Teräsvirta and Anderson (1992) and Teräsvirta (1994) define two types of STAR models depending on the nature of the transition function: Logistic STAR model and Exponential STAR model.

In the case of Logistic STAR model, the logistic function is given by:

$$F(y_{t-d}) = \frac{1}{1 + e^{-\gamma(y_{t-d} - c)}} \tag{2}$$

In the case of Exponential STAR model, the transition function is assumed to be an exponential function given by:

$$F(y_{t-d}) = 1 - e^{-\gamma(y_{t-d} - c)^2} \tag{3}$$

To investigate the nonlinear cointegration between stock prices p and the fundamental value f we should focus on the series of the residuals extracted from the long-term relationship between the two variables given by:

$$p_t = \alpha + \beta f_t + \varepsilon_t \tag{4}$$

The extracted residual series from equation (4) will be denoted y_t.

To study the nonlinear cointegration between stock prices and fundamental value, we assume that the stock price deviations from fundamental value can be described by the *STAR* model given by equation (1). In order to detect a possible cointegration relationship, Michael Nobay and Peel (1997) suggest modifying the STAR model as follows:

$$\Delta y_t = k + \lambda y_{t-1} + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + \left(k' + \lambda' y_{t-1} + \sum_{i=1}^{p-1} \phi_i' \Delta y_{t-i} \right) \times F\big(\gamma(y_{t-d} - c) \big) + \mu_t \tag{5}$$

This model is simply a STAR error correction model (ECM-STAR), where λ and λ' denote parameters that bring stock prices to the long-run equilibrium.

To validate the estimation of the model, some conditions must be fulfilled:

i) The mean reversion hypothesis implies that the larger the deviation of stock prices from fundamental value is, the larger the restoring force of the stock prices to its fundamental value will be. It means that for positive values of λ, ($\lambda \ge 0$), we should have $\lambda' < 0$ and $\lambda + \lambda' < 0$. This condition ensures the global stability of the nonlinear process. In fact, for small deviations, y_t can have a unit root or an explosive behavior, but for important deviations it follows a mean reversion process.

ii) The specification given by equation (5) is simply a nonlinear extension of the conventional Augmented Dickey Fuller (ADF) model specified as follows:

$$\Delta y_t = k'' + \lambda'' y_{t-1} + \sum_{i=1}^{p-1} \phi_i'' \Delta y_{t-i} + \upsilon_t \qquad (6)$$

According to Michael, Nobay and Peel (1997), if the true process is given by the nonlinear model (5), then the parameter λ'' in (6) should vary between λ and $\lambda + \lambda'$.

iii) The speed of transition between the inner (random walk) and the outer (reverting) regimes must be significantly different from zero. Otherwise, the stock price adjustment to its fundamental value is linear.

iv) The threshold c should have a realistic value. It should vary between the minimum and the maximum of the series.

The modeling procedure for STAR models specification is done through four steps:

Selecting the appropriate lag of the linear AR model

This first stage is crucial in the specification process of the nonlinear model because the next steps depend on the structure of the autoregressive model. We estimate a linear autoregressive model for different lags, *AR(k)*, and then determine the optimal lag based on Akaike (1974) and Schwarz (1978) information criteria. However, these criteria may lead to a model where the estimated residuals are autocorrelated. That's why Teräsvirta (1994) recommends using the Ljung and Box (1978) test of residual autocorrelation with one of these criteria.

These criteria are displayed in Table V. The AIC and SIC are minimized for a lag of $k = 4$. The Q statistics of lag 4 and 12 indicate that the null hypothesis of the absence of residual autocorrelation is already accepted starting from a lag 2. Then, the lag 4 ensures the elimination of residual autocorrelation. As a consequence, it's convenient to select 4 lags for the linear autoregressive model.

$$\Delta y_t = C + \lambda y_{t-1} + \sum_{j=1}^{k} \phi_j \Delta y_{t-j} + \upsilon_t$$

Table V. Selecting the optimal lag k for the AR model

lag	AIC	SC	Q(4)	p-value	Q(12)	p-value
1	1.432	1.562	30.216	0.000	37.237	0.000
2	1.112	1.288	7.1828*	0.127	9.0863*	0.696
3	1.196	1.419	6.8796	0.142	8.9292	0.709
4	0.983*	1.252*	0.5987	0.963	2.8911	0.996

Asterisk * indicates the optimal lag based on each criterion. AIC and SC denote Akaike and Schwarz criteria.. Q is the Ljung-Box statistic.

Linearity tests and specification of the delay parameter

To solve some econometric problems associated with the parameters identification in the original STAR models when testing for linearity against STAR models nonlinearity, Luukkonen, Saikkonen and Teräsvirta (1988) suggested three tests to get a linear approximation of the transition function using Taylor approximation. These tests resulted in an auxiliary model used to test the linearity against the alternative of a STAR model specified as follows:

$$y_t = \gamma_0 + \sum_{j=1}^{p} \gamma_{1j} y_{t-j} + \sum_{j=1}^{p} \gamma_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \gamma_{3j} y_{t-j} y_{t-d}^{2} + \sum_{j=1}^{p} \gamma_{4j} y_{t-j} y_{t-d}^{3} + \mu_t \quad (7)$$

Where μ_t are independently $N(0,\sigma_{v''}^{2})$.

The null hypothesis of linearity is: $H_{O1} : \gamma_{2j} = \gamma_{3j} = \gamma_{4j} = 0, \quad j = 1,\ldots,k$

Once linearity is rejected, we determine the delay parameter *d* using Teräsvirta (1994) method: We firstly select the order *p* of the autoregressive model, we vary the parameter *d,* and then we choose the value that minimizes the probability of the linearity test. If the linearity is rejected for many values of *d*, then we choose the one for which the linearity is strongly rejected, where *Pr* is the probability of the Fisher statistic associated with the null hypothesis of linearity based on the model (7).

Column 2 of table VI indicates that the null hypothesis of linearity H_{01} is rejected for the only plausible delay $d = 2$ at 5% level, given that the probability of Fisher statistic is lower for this lag [Pr (F) = 3, 69%].

Choosing between LSTAR and ESTAR

To identify the nature of the STAR model that reproduces the best process of the series when the linearity is rejected, Teräsvirta and Anderson (1992) and Teräsvirta (1994, 1995) proposed a sequence of Fisher tests performed on the auxiliary model (7):

$$\begin{cases} H_{O4} : \gamma_{4j} = 0, \quad j = 1,\ldots,k \\ H_{O3} : \gamma_{3j} = 0 / \gamma_{4j} = 0, \quad j = 1,\ldots,k \\ H_{O2} : \gamma_{2j} = 0 / \gamma_{3j} = \gamma_{4j} = 0, \quad j = 1,\ldots,k \end{cases}$$

The logic of these tests is based on the interpretation of the coefficients expressed in terms of the initial parameters in the original models. The rejection of H04 implies rejecting ESTAR model and selecting LSTAR model. If H04 is accepted while H03 is rejected, then we choose the ESTAR model. Accepting H04 and H03 and rejecting H02 supports the choice of LSTAR model.

The probabilities associated with the three tests are reported in columns 3, 4 and 5 of table VI. The p-value of *F4* statistic is larger than 5% indicating that we cannot reject the null hypothesis H_{04} at this level. In addition, p-value (*F3*) is greater than 5%, which also allows us to accept the hypothesis H_{03}. However, we reject the hypothesis H_{02}. These tests suggest an LSTAR model.

Table VI. Linearity test and specification of the nonlinear model

D	*Pr(F)* H_{01}: $\gamma_{2j}= \gamma_{3j}= \gamma_{4j}= 0$	*Pr(F4)* H_{04}: $\gamma_{4j}= 0$	*Pr(F3)* H_{03}: $\gamma_{3j}= 0/\gamma_{4j}= 0$	*Pr(F2)* H_{02}: $\gamma_{2j}= 0/\gamma_{3j}= \gamma_{4j}= 0$	Model
1	2.0950e-01	7.0684e-01	1.8636e-01	7.5086e-02	Linear
2*	3.6946e-02	8.7683e-01	1.3385e-01	4.2938e-03	LSTR
3	2.6681e-01	6.5724e-01	7.6825e-01	1.7813e-02	Linear
4	2.3722e-01	4.8023e-01	4.9108e-01	5.6735e-02	Linear

Note: asterisk * indicate the delay for which the p-value(F) is minimal.

LSTAR model estimation

The estimated LSTAR model by the Nonlinear Least Squares is as follows:

$$\Delta y_t = (0.67 - 1.02\Delta y_{t-1} + 1.04\Delta y_{t-2} + 0.24\Delta y_{t-3} - 1.16\Delta y_{t-4} - 0.25 y_{t-1})$$
$$(3.82)\ (2.71)\qquad (2.12)\qquad (0.91)\qquad (3.19)\qquad (0.95)$$
$$+ (-0.74 + 1.12\Delta y_{t-1} - 0.94\Delta y_{t-2} - 0.13\Delta y_{t-3} + 1.02\Delta y_{t-4} - 0.14 y_{t-1}) \times \hat{F}$$
$$(3.43)\ (2.20)\qquad (1.51)\qquad (0.30)\qquad (2.34)\qquad (0.39)$$
$$\hat{F} = [1 + exp(-0.06(y_{t-2} - 0.01))]^{-1}$$
$$(1.19)\qquad (0.38)$$

R^2 = 0.61 ; S^2 = 0.07 ; Q(2) = 0.54 ; Prob [Q(2)] = 0.59 ; Q(4) = 0.24 ; Prob[Q(4)] = 0.91 ; ARCH(2) = 1.26 ; Prob[ARCH(2)] = 0.53 ; JB = 1.82 ; Prob(JB) = 0.40.

The figures in parentheses denote the *t*-statistics. S^2 is the variance of the residuals. Q(k) is the Ljung-Box statistic used to test the joint significance of the autocorrelation up to 2 and 4 lags for the residuals. ARCH (2) is the ARCH test statistic of Engle (1982) of lag 2. JB is the Jarque and Béra test statistic.

Based on the Jarque and Béra test statistic, the diagnostic of the residuals of the model shows that the null hypothesis of the normality of the residuals cannot be rejected at conventional significance levels. Similarly, the ARCH test statistic shows that the null hypothesis of homoscedasticity of the residuals cannot be rejected. Furthermore, the Ljung-Box statistics of order 2 and 4 are insignificant indicating the absence of autocorrelation in the series of the residuals. These diagnostics argue that the residuals of the model have the good statistical properties. Next, we focus on the conditions supporting the existence of an LSTAR-based nonlinear cointegration.

Coefficients corresponding to the constant, Δy_{t-1} and Δy_{t-4} are significant in both regimes and the one associated with Δy_{t-2} is significant only in the first regime. However, the coefficient on Δy_{t-3} is not significant in both regimes. Both parameters λ and λ' are negative but not significantly different from zero indicating a negative sum that does not support the condition of the global stability of the process. This implies that the process characterizing the stock price deviations from earnings does not show a restoring force to the long-term equilibrium. We found that the minimum and the maximum of the series Δy_t are -0,691 and 0,878, respectively. The threshold c is therefore between these two values, but not significantly different from zero. In addition, the speed of transition between the inner regime and the outer regime is not only very small but also not significantly different from zero. We can therefore conclude that the LSTAR model is not able to capture the dynamics that govern the stock prices deviation from earnings and does not support the nonlinear cointegrating relationship between the two variables.

4. Conclusion

Financial literature has provided a conflicting empirical evidence on the relation between stock prices and fundamental value in a linear context. We extended the analysis of this relation in a nonlinear context using the smooth transition autoregressive models in order to examine the nature of the price adjustment to fundamental value measured by the dividend per share (DPS) or earnings per share (EPS) in the Tunisian stock market. The results indicate a linear cointegrating relationship between the stock price and the DPS supported by an error correction model showing an error correction mechanism in the market. Such a mechanism adjusts the stock prices that have diverged away from the fundamental value proxied by DPS in support of a linear mean reversion behavior of stock prices. However, a weak evidence of linear cointegration was found between stock prices and earnings per share. This relation is not robust enough to generate a valid error correction model which implies a rejection of a linear mean reversion to fundamental value proxied by earnings per share.

In order to further investigate this relation in a nonlinear context, we have modelled the deviation of stock prices away from EPS by a logistic smooth transition autoregressive (LSTAR) model. The model estimation results indicate that this model cannot capture the nonlinearity of this deviation, failing, then, to give evidence of a nonlinear cointegration between stock prices and EPS. As a consequence, stock prices and earnings per share still appear non-cointegrated.

The absence of a cointegrating relationship between stock prices and earnings per share in both linear and nonlinear contexts implies that there is no a mean reversion process that adjusts stock prices to return to their fundamental values proxied by earnings per share. Evidence of stock prices mean reversion towards their fundamental values measured by DPS indicates that, contrary to the EPS, DPS can serve as a better proxy of fundamental values of stock prices in the Tunisian stock market. When selecting stocks, Tunisian investors should then focus on the underlying performance of stocks in terms of the dividend per share.

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