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Lump-sum over Distortionary Taxation Dominance with Heterogeneous Individuals

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Abstract

The dominance of lump-sum over distortionary taxation for the single consumer case is a well-known proposition in microeconomics. This result implies that if the consumer is asked about what tax she would pay to bear a given tax burden, she would choose lump-sum taxation. This paper provides a version of this dominance of lump-sum taxation for the case of several heterogeneous individuals by means of a game where the government allows each individual to choose between the two tax regimes.

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1 Introduction

The comparison between lump-sum and distortionary taxation is one of the oldest issues in Public Economics since Barone (1912) showed that, keeping constant the utility of the taxpayer, the Exchequer could obtain a larger revenue from a lump-sum tax as opposed to a distortionary tax. Different versions of this lump-sum taxation dominance, such as Borgatta (1921), Joseph (1939), Hicks (1939), and Peacock and Berry (1951) among others, were subsequently published. Nowadays this dominance is taught in several microeconomics textbooks. In particular, Stigler (1987) and Varian (1992) show the version provided by Borgatta (1921) and Joseph (1939) for the single individual and two goods case. The proposition asserts that a given tax revenue yield would leave the taxpayer better off under a lump-sum tax than under a distortionary tax. The proof of this proposition relies crucially on the fact that both taxes have to collect the same tax revenue. As a consequence of that the bundle chosen under lump-sum taxation is directly revealed to be preferred to the bundle chosen under distortionary taxation. This allows us to assert that, if the individual is asked about what tax she would pay to bear a given tax burden, she would choose lump-sum taxation.

This paper extends a version of this preference for lump-sum taxation for the case of several heterogeneous individuals. The model conceives of two goods, a fixed quantity k of numeraire which has to be collected by the government and n heterogeneous individuals. This version of the preference for lump-sum taxation is a game where the government allows each individual to choose between two tax regimes: a lump-sum taxation or a distortionary taxation. In the case where the individual chooses the lump-sum taxation, and in accordance with the single individual case, she has to bear a constant tax rate on her wealth given by the ratio between k and the total endowment of the economy; if she decides distortionary taxation, she has to bear a tax on labour/income (equivalent in the model to an excise tax on the price of the consumption good). The tax revenue is the sum of both lump-sum and income taxation and the government keeps budgetary equilibrium. While the lump-sum tax rate is constant, the income/commodity tax depends on the number of individuals who are bearing it. As a result of that, strategic interdependence arises from the number of individuals who are paying the distortionary tax. As we will see, in the unique pure strategy Nash equilibrium of this game everyone ends up choosing the lump-sum taxation regime.

The paper is structured as follows: Section 2 presents the model. Section 3 is devoted to the description of the lump-sum dominance game and achieving its equilibrium. Finally Section 4, summarizes the results.

2 The Model

Let $J = \{1, 2, ..., n\}$ be the set of heterogeneous individuals. Each individual is endowed with a quantity w_j of *numeraire* (time endowment, for example), let

$W=\sum_{j\in J} w_j$ be the total endowment of *numeraire* (wealth) in the economy. The strictly-convex and monotonic preferences over the consumption of the good X and *numerarie* Y (leisure, for instance) of each individual, are represented by a strictly quasi-concave utility function $u_j(x_j, y_j)$. An amount $0<k<W$ of *numeraire* has to be collected by a Government; and the good X is produced in a competitive industry with constant returns to scale so that $C(x)=px$, where $x=\sum_{j\in J} x_j$. In this trend, the feasibility condition is given by:

$$W \geq \sum_{j\in J} y_j + C(x) + k. \tag{1}$$

For collecting the quantity k the Government gives two options to individuals. On the one hand, an ad-valorem tax $0\leq t\leq 1$ on the income, which is just the labour (distortionary taxation). In this case the j-th individual budget constraint is

$$B_j(t)=\left\{(x_j,y_j)\in R_+^2 : (1-t)\,(w_j-y_j)\geq px_j\right\}. \tag{2}$$

Let $(x_j(t), y_j(t))$ be the bundle that maximizes $u_j(x_j,y_j)$ subject to $B_j(t)$, and $v_j(t)=u_j\left(x_j(t),y_j(t)\right)$ her indirect utility function after the optimal decision under distortionary taxation.

On the other hand, the individual can choose to bear a proportional lump-sum tax rate $0\leq T\leq 1$ on her wealth (initial endowment). In this case her budget constraint is

$$B_j(T)=\left\{(x_j,y_j)\in R_+^2 : (1-T)w_j\geq px_j+y_j\right\}. \tag{3}$$

Let $(x_j(T), y_j(T))$ be the bundle that maximizes $u_j(x_j,y_j)$ subject to $B_j(T)$, and $v_j(T)=u_j\left(x_j(T),y_j(T)\right)$ her indirect utility function after the optimal decision under lump-sum taxation.

These budgetary sets represent the options allowed to each individual in such a way that the individual who chooses to pay one tax is waived from paying the other one.

The Government's tax policy is as follows: it sets a constant lump-sum tax rate, given by $T^*=k/W$, for those individuals who choose *to-bear-lump-sum-taxation,* whereas for those individuals who choose *to-bear-distortionary-taxation* the tax rate is determined by fulfilling the budgetary equilibrium. More precisely, calling $D\subseteq J$ the subset of individuals who choose *to-bear-distortionary-taxation* and $J\backslash D$ (complementary of D) the subset of individuals who choose *to-bear-lump-sum-taxation*, t is determined as the minimum value of t[1] which solves the Government´s budget equilibrium equation

$$\frac{tp}{1-t}\sum_{j\in D} x_j(t) = k - T^* \sum_{j\in J\backslash D} w_j. \tag{4}$$

[1] The l. h. s. of equation (4) is a Laffer curve which represents the tax revenue collected from distortionary taxation as a function of t, and the r. h. s. is a constant which represents the fiscal debt after lump-sum tax revenue, since the solution of Equation (4) can be multiple, we assume that the Government chooses the lowest positive one. That is, the solution locates in the positively slopped section of the Laffer curve, see Fullerton (1982) and Bender (1984).

The following lemma states that the budgetary equilibrium for the government, Equation (4), implies equilibrium in labor market, Equation (1), and vice versa.

Lemma 1 *The budget of the government is balanced if and only if the feasibility condition holds.*

The proof, which is in the Appendix, is an immediate consequence of Walras' Law. Moreover, since the solution of Equation (4) depends on whose individuals are bearing distortionary taxation, there is a unique ad-valorem tax rate $t(D) \in [0,1]\ \forall D \subseteq J$ which is solution of Equation (4). To illustrate this point let us consider the following example:

Example 1: *Let be two different Cobb-Douglas individuals* $u_j(x_j, y_j) = x_j^{\alpha_j} y_j^{1-\alpha_j}\ j \in J = \{1,2\}$, *where* $\alpha_1 = 1/3, w_1 = 1, \alpha_2 = 2/3, w_2 = 9$ *and* $k = p = 1$. *The constant lump-sum tax rate is* $T^* = 1/10$, *and the distortionary tax rates are* $t(\{1,2\}) = 3/19, t(\{1\}) = 3/10, t(\{2\}) = 3/20$ *and* $t(\{\emptyset\}) = 0$, *where* $\{1,2\}, \{1,2\}, \{1,2\}$ *and* $\{\emptyset\}$ *are the different subsets of individuals who choose to-bear-distortionary-taxation that can be formed from set* J.

Finally, taking into account that $W = \sum_{j\in D} w_j + \sum_{j \in I \setminus D} w_j$ and Equation (4), it will be useful through the paper to state that $t(D)$ is that ad-valorem tax rate which fulfils

$$\frac{t(D)}{1-t(D)} p \sum_{j\in D} x_j\left(t(D)\right) = k \frac{\sum_{j\in D} w_j}{W}. \tag{5}$$

3 The lump-sum taxation dominance game and its equilibrium

Given the tax policy previously defined our game is a one-shot game $\Gamma = \{J, \{S_j\}_{j\in J}, \{\pi_j\}_{j\in J}\}$, where J is the set of individuals, $S_j = S = \{l, d\}$ is the set of strategies of individual j, where l means *to-bear-lump-sum-taxation* and d means *to-bear-distortionary-taxation.* $\boldsymbol{S} = \prod_{j\in J} S$ is the set of strategic profiles. Note that given a subset D of players there is a unique profile $s \in \boldsymbol{S}$ such that $D(s) = D$. Thus, for each $D \subset J$ let $s(D)$ denote the strategy profile such that the players that chose d are those in D.[2] And

$$\pi_j(s) = \begin{cases} v_j(T^*), \text{ if } s_j = l \\ v_j(t(s(D))), \text{ if } s_j = d, \end{cases}$$

where $s = (s_1, s_2, ..., s_n) \in \boldsymbol{S}$, $s_i = l, d$, $i \in J$, is the payoff function for individual j. Therefore, in the lump-sum taxation dominance game each individual chooses

[2] For instance, in Example 1 the strategic profiles associated to the different subsets of individuals who choose *to bear distortionary taxation* are $s(\{1,2\}) = (d,d), s(\{1\}) = (d,l), s(\{2\}) = (l,d)$ and $s(\{\emptyset\}) = (l,l)$.

between *bearing lump-sum taxation* or *bearing distortionary taxation*. If the individual chooses *to-bear-lump-sum-taxation*, she has to pay a constant tax rate on her initial endowment given by $T^* = k/W$ and the payoff of this action is independent of the others' actions. On the other hand, if she chooses *to-bear-distortionary-taxation*, she has to pay a tax $t(D)$ on her labour income, where $t(D)$ is assessed accordingly with equation (5) and, in consequence, its value depends on who individuals are bearing it. Hence, the payoff of this action depends on the others' actions, that is, strategic interdependence arises only from the distortionary tax.

It should be noted that choosing between *bearing distortionary taxation* and *bearing lump-sum taxation* is equivalent to choosing one of the budget constraints between (2) and (3). The following propositions lead us to the pure strategy Nash equilibrium of this game.

Proposition 2 *Let $(T^*, t(D))$ be a tax policy and $s(D)$ a strategy profile such that $D \neq \emptyset$. Then, at least the individual $h \in D$ with the largest ratio between consumption of good X and wealth has incentives to deviate to l.*

Proof: Let $h \in D$ and $(x_h(t(D)), y_h(t(D)))$ be her consumption bundle which, due to the monotonicity, exhausts the budget constraint given by (2), that is

$$w_h = \frac{p}{1-t(D)} x_h(t(D)) + y_h(t(D)). \tag{6}$$

Let us find out the conditions under which this consumption bundle is affordable under lump-sum taxation. Thus, plugging $(x_h(t(D)), y_h(t(D)))$ into constraint (3) with $T = T^*$ and operating we can write

$$w_h \geq \left[p + T^* \frac{w_h}{x_h(t(D))} \right] x_h(t(D)) + y_h(t(D)). \tag{7}$$

Comparing (6) and (7) $(x_h(t(D)), y_h(t(D)))$ is affordable under the lump-sum tax regime if and only if

$$\frac{p}{1-t(D)} \geq p + T^* \frac{w_h}{x_h(t(D))},$$

taking into account (5) and clearing, this condition can be written as

$$\frac{x_h(t(D))}{w_h} \geq \frac{\sum_{i \in D} x_i(t(D))}{\sum_{i \in D} w_i}, \tag{8}$$

a condition which holds for at least the individual in D with the largest ratio between her consumption of good X and her wealth. In fact, letting $h \in D$ be this individual, i. e. h is such that $\frac{x_h(t(D))}{w_h} \geq \frac{x_i(t(D))}{w_i}$ $\forall i \in D$, reordering, $w_i x_h(t(D)) \geq w_h x_i(t(D))$ and adding with respect to $i \in D$ we have $x_h(t(D)) \sum_{i \in D} w_i \geq w_h \sum_{i \in D} x_i(t(D))$, which is just the condition (8).

Finally, since, $(x_h(T^*), y_h(T^*))$ would be the bundle chosen under the lump-sum tax regime and, for individual h, $(x_h(t(D)), y_h(t(D)))$ is, in turn, affordable under this budget constraint, $v_h(T^*) > v_h(t(D))$ and, thus, individual h deviates to l.■

Note that, on the one hand, Proposition 1 states conditions for which the bundle chosen under distortionary taxation is also affordable under lump-sum taxation. This means that the bundle chosen under lump-sum taxation is directly revealed preferred to the bundle chosen under distortionary taxation. For the particular case in which every individual is endowed with the same quantity of numerarie ($w_i = w, \forall i \in J$) condition (8) becomes $x_h(t(D)) \geq \frac{\sum_{i \in D} x_i(t(D))}{|D|}$, where $|D|$ denotes the cardinal of D. Thus, in this case, Proposition 1 would state that the individuals in set D whose consumption of good X is above the average consumption in this set would be willing to deviate to lump-sum taxation. Note that Proposition 1 holds $\forall D \subseteq J$, thus, we can state the following Proposition.

Proposition 3 *The strategy profile $s(D)$ such that $D = \emptyset$, or $s_i = l \quad \forall i \in J$, is the pure strategy Nash equilibrium for the game Γ.*

Proof: The first part of the proof consists in to show that an strategy profile $s(D)$ such that $D \neq \emptyset$ is not a Nash equilibrium for the game Γ. Let us suppose this is not true, that is, in equilibrium there is a number $|D| \geq 1$ of individuals who choose *to-bear-distortionary-taxation*. $D \neq \emptyset$ means $|D| = 1, 2, ..., n$. Thus, according to Proposition 1 this is not an equilibrium because there is at least one individual in D (anyone with the largest ratio between consumption of good X and wealth) whose dominant strategy is *to-bear-lump-sum-taxation.* Moreover, Proposition 1 applies for any size of set D, hence, the same argument works for $|D| = n-1, n-2, ..., 1$.

The second part of the proof consists in to show that when every individual is bearing lump-sum taxation, that is when $D = \emptyset$, none of them has incentives to deviate to d. The proof is similar to that of Borgatta-Joseph: if being bearing lump-sum taxation the individual h deviates to d, she will choose the bundle $(x_h(t\{h\}), y_h(t\{h\}))$ maximizing $u_h(x_h, y_h)$ subject to $B_h(t\{h\})$ so that, according to (5), $\frac{t\{h\}}{1-t\{h\}} px(t\{h\}) = T^*w$. It is fairly easy to see that $(x_h(t\{h\}), y_h(t\{h\}))$ is also affordable under $B_h(T^*)$, thus $v_h(T^*) > v_h(t\{h\})$. Thus, when $D = \emptyset$, nobody has incentives to bear distortionary taxation.■

Proposition 2 is based on the fact that in our game there is always at least one individual in D whose dominant strategy is *bearing lump-sum taxation.* According to Proposition 1, this individual is the one whose ratio consumption of good X-wealth is larger among the individuals belonging to D. But this outcome occurs for each possible size of the set of individuals who bear distortionary taxation. Hence, in equilibrium, the set of individuals who bear distortionary taxation is empty. Note that, in equilibrium, when every consumer pays T^*, the quantity k is financed through lump-sum taxes, so the equilibrium is just a Marginal Cost Pricing Equilibrium and, as is well-known, this is enough for Pareto optimality. Finally, to remark that, in this environment of many heterogeneous individuals, when everybody is bearing distortionary taxation, *bearing lump-sum taxation* is not a dominant strategy for every single individual. This is what makes different the game Γ with the single-individual Borgatta-Joseph case. To illustrate this point let us continue with the Example 1.

Example 2: *The payoffs matrix for the individuals of the Example 1 is*

	d	l
d	$0.4996, 4.2467$	$0.4698, 4.2859$
l	$0.4762, 4.2732$	$0.4762, 4.2859$

As we see, if l were not available for individual 2, d would be the best response of individual 1. Instead, since $\frac{x_2(t\{1,2\})}{w_2} > \frac{x_1(t\{1,2\})}{w_1}$ when both individuals are bearing distortionary taxation, l is a dominant strategy for individual 2 (Proposition 1). Given this dominant strategy for individual 2, the best response for individual 1 is l, and the pure strategy Nash equilibrium is (l, l).

4 Conclusion

As we have seen, a version of the preference for lump-sum taxation has been proved for the case of several heterogeneous individuals. Although the proof is similar to that used for the single individual case, the result is not a mere translation of this case to an environment with many heterogeneous individuals since when everybody bears distortionary taxation there are individuals who prefer this tax scheme. When lump-sum taxation is available as an alternative, heterogeneity introduces strategic interdependence by means of the number of individuals who choose to pay the distortionary tax rate in such a way that those individuals whose ratio between consumption of the good assessed for the distortionary tax and their wealth is larger would have incentives to change to the lump-sum tax. This effect prevails independently of the size of the set of individuals who are bearing the distortionary tax whenever it is non empty. Finally, if all the individuals except one decide to pay the lump-sum tax, the best option left is also to pay that tax. Therefore the unique pure strategy Nash equilibrium is that in which all individuals bear lump-sum taxation, which yields a Pareto optimal allocation by means of a marginal cost pricing equilibrium.

5 Appendix

Proof of Lemma 1.

Let the Government tax policy $(t(D), T^*)$ and $D \neq \emptyset$ any possible subset of J so that $\forall h \in D$ $(x_h(t(D)), y_h(t(D)))$ is the consumption bundle which maximizes

$u_j(x_j, y_j)$ subject to $B_j(t(D))$, and $\forall j \in J \backslash D$, $(x_j(T^*), y_j(T^*))$ is the bundle that maximizes $u_j(x_j, y_j)$ subject to $B_j(T^*)$. Due to the monotonicity, all these consumption bundles exhaust their respective budget constraints. Summing up these individual budget constraints for each subsets D and $J \backslash D$,

$$\begin{aligned} \sum_{j \in D} w_j &= p \sum_{j \in D} x_j(t(D)) + \frac{pt}{1-t} \sum_{j \in D} x_j(t(D)) + \sum_{j \in D} y_j(t(D)), \\ \sum_{j \in J \backslash D} w_j &= p \sum_{j \in J \backslash D} x_j(T^*) + \sum_{j \in J \backslash D} y_j(T^*) + T^* \sum_{j \in J \backslash D} w_j. \end{aligned}$$

Summing up the former equalities, we hold

$$\begin{aligned} W &= p \left(\sum_{j \in D} x_j(t(D)) + \sum_{j \in J \backslash D} x_j(T^*) \right) + \sum_{j \in D} y_j(t(D)) + \sum_{j \in J \backslash D} y_j(T^*) + \\ & \quad ... + \frac{pt}{1-t} \sum_{j \in D} x_j(t(D)) + T^* \sum_{j \in J \backslash D} w_j. \end{aligned}$$

It is clear that when Equation (5) holds Equation (1) holds, and vice versa.■

6 References

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