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Safe strategic voting and three approaches for choosing a strategic preference statement under the Borda Rule

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Abstract

Numerous situations exist when a group of voters could manipulate the election´s outcome. We change one assumption for safe strategic vote of Slinko and White (2014), by allowing voters with the same top k ranks to form a manipulative group, and we analyse how to detect manipulative groups. Furthermore, we introduce possible ways to determine a strategic preference statement for such groups. We introduce a "safe" way and a "greedy" way to cast a strategic preference statement. Concluding, we use data from the 2015 Styrian parliamentary elections to apply safe strategic voting.

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1 Introduction

For as long as voting rules have existed there has been a debate on how to cast a vote. The reason is the possibility of acting strategically by misreporting preferences. A lot of literature exists on manipulation and strategic voting. A manipulation occurs whenever someone secures an outcome he/she prefers to the honest outcome by misrepresenting his preferences. For example, Gibbard (1973) and Satterthwaite (1975) early recognized that all non-trivial social decision rules are manipulable. Barberà (2010) provided a survey on strategy-proofness in social choice. Several studies on specific voting rules with strategic behaviour exist (see, e.g., Van der Straeten, Laslier and Blais, 2016; Ganser and Veuger, 2010; Fisher and Myatt, 2014). In addition to individual manipulation, researchers introduced coalitional manipulability of voting procedures and group strategy-proofness (see, e.g., Barberà, Berga and Moreno, 2010; Slinko, 2004). These topics describe situations where a group of voters can manipulate the election by misrepresenting their preferences while all other voters do not change their preference statement. Slinko and White (2014) extended the Gibbard-Satterthwaite theorem and defined safe strategic voting. Safe strategic voting means that a voter, of a manipulative group with identical preferences, has an incentive to act strategically and cannot strategically overshoot or undershoot with that strategic vote. Strategic overshooting (undershooting) occurs whenever a manipulative group is able to change the outcome to their favor, but it is also possible that too many (few) voters of a manipulative group act strategically which results in a worse outcome than the current outcome. Finally, Slinko and White (2014) prove that an onto (surjective), non-dictatorial social choice rule is safely manipulable. In this paper we will change one single assumption of safe strategic voting to enable more realistic manipulative groups, which can vote strategically and change the social outcome, and state possibilities to create such a strategic preference statement.

In particular, we change the assumption that a manipulative group has to be a group of voters with identical preferences. This means, we allow all voters with the same top k ranks to form a manipulative group. Furthermore, we state possible ways to create a strategic preference statement whenever we know the truthful preference profile, the manipulative group and the candidate who should win. We introduce the "safe" way to determine a strategic preference statement, the "greedy" way and the "mixed" way. In the "greedy" way the point distance between the candidate who should win and the current winner is maximized in the strategic preference statement for the manipulative group. This enables a manipulation more easily. However, this way has the risk of overshooting and undershooting. The "safe" way enables the group to vote strategically without the risk of getting worse off. But, the manipulative group only agrees on a strategy prescribing changes for each individual and not on a single preference statement for the whole group. Furthermore, there are more situations where a manipulation is not possible in comparison to the "greedy" way. The "mixed" way combines the "safe" and the "greedy" way. Again, it only states a strategy how the manipulative group should change their preference statements and not one single preference statement for the whole group. After discussing the possible ways of strategic voting, we will apply all concepts on a dataset of the 2015 Styrian parliamentary elections. For the whole analysis we use the Borda Rule, which is the widest known positional scoring rule and easy to understand. Furthermore, this voting rule is less susceptible to being manipulated by one single voter than other positional voting procedures (see, e.g., Saari, 1990; Brams and Fishburn, 2002).

We observe that it is hard to find large enough groups with identical preferences to manipulate the election, especially when the number of candidates is higher. So we need a criterion which enables voters to form more easily a manipulative group. Furthermore, we find that there exist situations in real elections such that the "safe" way to determine a strategic preference statement can be applied.

The paper is structured as follows: Section 2 introduces the framework. Section 3 outlines the possibilities of strategic voting and group building. Different possible strategic preference statements and how to choose them will be analysed in Section 4. The algorithm which combines these steps will be explained in Section 5. In Section 6 the theoretical findings will be applied on a dataset. Finally, Section 7 concludes the paper.

2 Framework

Let $X = \{a, b, c, ...\}$ be a finite set of m candidates and I be a finite set of n individuals. Each individual $i \in I$ has a preference relation, i.e., a transitive and complete binary relation on X. For further analysis we will use the following notation. Individual i has a preference of type ab if individual i strictly prefers a over b, a preference of type ba if individual i strictly prefers b over a, and a preference of type (ba) if individual i is indifferent between a and b.

The different possibilities of strategic voting depend on the voting situation. The voting situation is the truthful preference statement of all voters. The collection of all preference statements is also called a preference profile, an example can be seen in table 1. This example consists of 15 voters and 5 candidates, $\{a, b, c, d, e\}$. Three different truthful preference statements exist. For example, five voters are of type $abcde$ which means that the voters prefer a to b to c to d to e. In this example only strict rankings exist. For indifference we write the candidates in brackets, e.g., $a(bc)$ which means a is preferred to b and c, but the voter is indifferent between b and c.

5	5	5
a	b	a
b	c	c
c	a	b
d	d	d
e	e	e

Table 1: Preference profile of 15 voters.

For further analysis on strategic voting we need to determine a Social Choice Rule. We choose the Borda Rule because it is the widest known positional scoring rule. Additionally, it is easy to understand and apply. Furthermore, this voting rule is less susceptible to being manipulated than other positional voting procedures (see, e.g., Saari, 1990; Brams and Fishburn, 2002). The Borda Rule assigns points to the candidates according to their positions in the preference ranking. It starts with zero points for the least preferred candidate up to $m - 1$ points for the top choice. The total scores, which is the sum of all scores, for each candidate over all voters, reveal the winner of the election [1]. However, our three approaches can be applied for all scoring rules.

[1] Several tie-breaking rules, in case of multiple winners, exist in the literature see, e.g., Dummett and Farquharson (1961) .

3 Strategic voting and possible manipulative groups

Strategic voting can occur in various ways. Let us start with the original approach of Gibbard and Satterthwaite which takes changes in the preference statement of one individual into account. This means one voter can change his/her preference statement to change the winner of the election to his/her favor. However, we already know that such situations, where a voter is pivotal, meaning that the voter can change the outcome, hardly exist (for more detailed information on the pivotal voter model see, e.g., Palfrey and Rosenthal (1983) and Ledyard (1984)). For this reason we include the possibility of coalitional manipulation. Coalitional manipulation means that a group of voters comes to a mutual agreement to change the preference statement in order to change the winner of the election (in favor of this group). However, if we allow voters to deviate from the agreement, voters could in principle also get worse off by misstating their preferences. Therefore, Slinko and White (2014) introduced the concept of a safe strategic vote. Safe strategic vote means that there is a group of voters who has an incentive to vote strategically with no risk of over- or undershooting. Over- and undershooting occurs when too many (few) voters of a given group change their preference statements which leads to a worse outcome of the election for the manipulating voters than the outcome with truthful preference statement and the voters have an incentive to act strategically.

Table 2 shows a voting situation where overshooting might occur. Under truthful preference statements candidate c wins the election with 30 points over candidate b with 28 points and candidate a with 26 points. However, voters of type $bcdae$ prefer candidate b to c and might try to manipulate the outcome in their favor. Therefore, they change their preference statements to $bdaec$. If all five voters of type $bcdae$ change their preference statements, this leads to candidate a winning the election, which is even worse for the five voters of type $bcdae$ than the current outcome. However, if only one voter changes her preference statement candidate b wins the election. But, already two strategic voters can change the outcome to a tie between candidate a and b. This means there is a switching point between successful manipulation and overshooting in this situation at two strategic votes.

In the preference profile of table 3 the opposite holds, not too many voters act strategically but too few. Truthful preference statements lead to candidate b as winner of the election with following scores: 110 points for candidate b, 109 points for candidate c, 102 points for candidate a, and so on. If all 10 voters of type $dabec$ decide to vote strategically and state $adebc$ instead of their truthful preference, candidate a wins the election. However, if only two of them vote strategically candidate c wins the election. The switching point between successful manipulation and undershooting in this situation is at nine strategic votes.

5	5	1		*scores*	*x*
a	*b*	*e*	*c*	30	*b*
c	*c*	*b*	*b*	28	*d*
d	*d*	*d*	*a*	26	*a*
b	*a*	*a*	*d*	22	*e*
e	*e*	*c*	*e*	4	*c*

Table 2: Overshooting

In the next paragraphs we will introduce two ways of determining the group of voters (manipulators) and its size. The first way of determining the group of voters is to count how many voters have the same preferences, because only voters with identical preferences are allowed

15	14	2	10		scores	x
b	c	c	d	b	110	a
c	a	e	a	c	109	d
a	b	d	b	a	102	e
d	e	b	e	d	59	b
e	d	a	c	e	30	c

Table 3: Undershooting

to form such a manipulative group (see Slinko and White, 2014). An example can be seen in table 4. Truthful preference statements lead to the following scores: 50 points for a, 45 points for b, 40 points for c, 15 points for d and no points for e. In this scenario candidate a wins the election. Now, assume that the five voters of type $bcade$ change their preference statements to $bcdea$. This results in candidate b winning the election. Even if only some voters change their preference statement, the outcome would not be worse than in the truthful case for all voters of type $bcade$.

5	5	5		scores	5	5	5		scores
a	b	a	a	50	a	b	a	b	45
b	c	c	b	45	b	c	c	a	40
c	a	b	c	40	c	d	b	c	40
d	d	d	d	15	d	e	d	d	20
e	e	e	e	0	e	a	e	e	5

Table 4: Safe strategic vote

However, the case of the restriction to identical preferences delimits the group size in such a way that no favorable changes might occur. Another approach, which enables larger groups, is to determine the group by only looking at the top k ranks. Voters with the same top k ranks form the manipulative group and all other ranks worse than rank k are not taken into account. As already mentioned, this could enable changes in cases where a group with identical preferences is not able to change the outcome. An example can be seen in table 5. The truthful outcome is the following: a receives 119 points, b 108 points, c 92 points, d 31 points and e 10 points. Candidate a wins the election under truthful preference statements. Now, all voters of type $bcade$ could form a group and change their preference statement to $bcdea$. However, this would not change the winner. Additionally, there is the possibility to form a group with identical preferences $bcead$. However, also this would not change the winner. The new approach of looking at the top k ranks, where k equals two, allows us to form a group consisting of voters of type $bcade$ and voters of type $bcead$, since the first two ranks are identical. Considering the case that all these 10 voters change their preference statement to $bcdea$, b would win the election with a score of 108 points and a only receives 104 points. Furthermore, we could include voters with incomplete preference statements, who have all ranked candidates in common. [2]

As the previous paragraph shows, it might be interesting to form a group of manipulative voters with only the k top ranks being the same. The reason is the possibility of forming larger groups. However, we do not know how to decide which preference statement should be made instead

[2]For incomplete preference statements all not ranked candidates are treated as ranked last. Whenever there are indifferences in preference statements every candidate in this indifference group receives the average points. For example, three candidates are ranked last, which means no candidate is ranked second to last and third to last, each candidate receives one point.

16	5	5	10		*scores*	16	**5**	5	10		*scores*
a	*b*	*b*	*a*	*a*	119	*a*	**b**	*b*	*a*	*a*	109
b	*c*	*c*	*c*	*b*	108	*b*	**c**	*c*	*c*	*b*	108
c	*a*	*e*	*b*	*c*	92	*c*	**d**	*e*	*b*	*c*	92
d	*d*	*a*	*d*	*d*	31	*d*	**e**	*a*	*d*	*d*	36
e	*e*	*d*	*e*	*e*	10	*e*	**a**	*d*	*e*	*e*	15
16	**5**	**5**	**10**		***scores***	**16**	**10**	**10**			***scores***
a	*b*	**b**	*a*	*a*	114	*a*	**b**	*a*		*b*	108
b	*c*	**c**	*c*	*b*	108	*b*	**c**	*c*		*a*	104
c	*a*	**d**	*b*	*c*	92	*c*	**d**	*b*		*c*	92
d	*d*	**e**	*d*	*d*	41	*d*	**e**	*d*		*d*	46
e	*e*	**a**	*e*	*e*	5	*e*	**a**	*e*		*e*	10

Table 5: No safe strategic vote with identical preferences

of the truthful preference statement. In the next section we will show possible ways to choose the strategic preference statement.

4 Three approaches for choosing a strategic preference statement

Many results on strategic voting assume individual preferences to be common knowledge. Everyone knows the preferences of all voters and therefore the outcome of the election. A trial and error strategy determines the strategic preference statement. Sometimes the strategy follows an algorithm like the greedy algorithm of Zuckerman et al. (2009), which assigns voter by voter a strategic preference statement. This means that each voter gets an individual strategic preference statement and this requires a high level of organization and the knowledge of the exact preference profile. But, in real situations it would be hard to coordinate such individual strategic preference statements and hence one statement or at least one strategy specifying what each voter should change for the whole manipulative group would be desirable. Of course there exist situations which do not permit strategic voting at all. Furthermore, the possibility exists that the manipulation leads to a worse candidate as winner of the election for the manipulative group. We define three possible ways to determine the strategic preference statement for a given manipulative group and a fixed candidate who should win the election.

The "greedy" way to determine the strategic preference statement starts with ranking the candidate who should win the election first. All other candidates get ranked according to their Borda scores in ascending order, which means the candidate with the highest Borda score gets ranked last. This way enables the preferred winner to pass the current winner more easily because of the maximal difference in points between the preferred candidate and the current winner in the strategic preference statement. Furthermore, because of the ranking in ascending order according to the Borda scores the additional points from the manipulative group for all candidates with high scores are lower than in the truthful statement. Additionally, this strategy uses the same strategic preference statement for the whole manipulative group. However, this way might be prone to over- or undershooting.

Proposition 1: Whenever a strategic vote is possible and over- or undershooting can not occur then by applying the greedy way the manipulative group can make a preferred candidate win the election.

Second, the "safe" way to determine the strategic preference statement distinguishes between the situation when a manipulative group with identical preferences is used and the situation when a manipulative group with same top k ranks is used. Whenever a manipulative group with same top k ranks is used we need to fix all ranks below the rank k. All other steps are the same for both situations. The next step is to fix the ranks of the current winner and all candidates ranked below, whenever using manipulative groups with same top k ranks the current winner is either under the top k ranks and now fixed or already fixed. Furthermore, we rank the candidate who should win the election first. Again, all other candidates get ranked according to their Borda scores in ascending order in terms of total Borda scores. This way of determining the strategic preference statement enables a preference statement without risk of electing a worse candidate than the current winner. Because every candidate which is ranked in truthful preference statements below the current winner and the current winner itself receive the same points than before. This ensures that no candidate which is preferred by the current winner gets higher points. Furthermore, through the rearrangement of the ranks above the current winner some of those preferred candidates might get more points than before and possibly even more points than the current winner. But there could be situations where the manipulative group is not able to change the winner of the election when using the "safe" way because the difference in points between the preferred candidate and the current winner in the strategic preference statement might be lower than in the "greedy" way. Such an example is illustrated at the end of this section.

Proposition 2: Whenever the "safe" way is used to determine the strategic preference statement, strategic voting is safe in the sense of no over- or undershooting.

The "mixed" strategy maximizes the difference in points between the preferred candidate and the current winner in the strategic preference statement while trying to hold the risk of getting worse off low. For this strategic preference statement we fix the ranking of the candidates below the current winner. All those candidates now get ranked one rank better and the current winner gets ranked last. Now we can rearrange all preferred candidates (ranked above the current winner) and rank the candidate who should win the election first. All other candidates again get ranked according to their Borda scores in ascending order among the remaining ranks.

Table 6 shows an example of a voting situation which leads to candidate a as winner of the election with 45 points and candidate b as second best candidate with 40 points. However, voters of type $edbac$ prefer candidate b over candidate a, and therefore could decide to form an alliance and vote strategically to help candidate b. They have three options to vote strategically as mentioned above. The "greedy" way is to change the preference statement to $bedca$. The change in the preference statement results in the following changed scores: a receives 40 points, b 50 points, c 30 points, d 15 points and e 15 points. Now candidate b wins the election and this represents a better result for the manipulative group of voters. The second option is to choose the "safe" way to change the preference statement and declare $bedac$ as preference statement instead of $edbac$. This also leads to b winning the election. However, the difference in scores between the winner and second best candidate is lower. The third alternative is the "mixed" method with the strategic preference statement $bedca$. Again this leads to candidate b winning the election.

In the previous paragraph we have seen an example of a voting situation which enables all ways, "greedy", "safe" and "mixed", to change the outcome of the election. The aim was to help candidate b, who is second ranked to win the election. However, we can also have situations like table 7 where every possible way can help candidate b win the election but possibly voters

10	5			10	**5**		*scores*	10	**5**		*scores*	10	**5**		*scores*
a	*e*	*a*	45	*a*	**b**	*b*	50	*a*	**b**	*b*	50	*a*	**b**	*b*	50
b	*d*	*b*	40	*b*	**e**	*a*	40	*b*	**e**	*a*	45	*b*	**e**	*a*	40
c	*b*	*d*	25	*c*	**c**	*c*	30	*c*	**d**	*c*	20	*c*	**d**	*c*	25
d	*a*	*c*	20	*d*	**d**	*d*	15	*d*	**a**	*d*	20	*d*	**c**	*d*	20
e	*c*	*e*	20	*e*	**a**	*e*	15	*e*	**c**	*e*	15	*e*	**a**	*e*	15

Table 6: Possible preference statements and social outcomes: truthful, greedy, safe, mixed

try to help candidate *c*, whom all voters of type *ecbad* prefer over *b*. Table 7 shows the three possible rankings, w.r.t. the considered ways. The "greedy" way and the "mixed" way result in candidate *c* as winner of the election. The "safe" way results in no change in the winner of the election because the changes in points are too low. However, the aim is to help candidate *c* to win the election. This shows that the "greedy" way enables smaller groups to change the outcome.

10	6		*scores*	10	**6**		*scores*	10	**6**		*scores*	10	**6**		*scores*
a	*e*	*a*	46	*a*	**c**	*c*	44	*a*	**c**	*a*	46	*a*	**c**	*c*	44
b	*c*	*b*	42	*b*	**d**	*a*	40	*b*	**e**	*c*	44	*b*	**e**	*b*	42
c	*b*	*c*	38	*c*	**e**	*b*	36	*c*	**b**	*b*	42	*c*	**b**	*a*	40
d	*a*	*e*	24	*d*	**b**	*d*	28	*d*	**a**	*e*	18	*d*	**d**	*e*	18
e	*d*	*d*	10	*e*	**a**	*e*	12	*e*	**d**	*d*	10	*e*	**a**	*d*	16

Table 7: Another possible preference statements and social outcomes: truthful, greedy, safe, mixed

The "greedy" way seems to be the best choice for determining the strategic preference statement. But, there might exist situations where the "greedy" way is the riskiest way as the example in table 8 shows. Truthful preference statement results in candidate *a* winning the election. All 10 voters of type *edbac* decide to vote strategically and help candidate *d* win the election. If we apply the "greedy" way to determining the strategic preference statements, all voters of type *edbac* state *dceba* as their preferences and this leads to the following scores: 44 points for candidate *a*, 43 points for *b*, 52 points for *c*, 51 points for *d* and 20 points for *e*. The winner of the election is candidate *c*, which is even worse than candidate *a*. If we use the "safe" way, they have to choose the strategic preference statement *debac*. Candidate *a* wins the election and therefore no change occurs. The "mixed" way with preference statement *debac* leads to candidate *b* winning the election, which is better than candidate *a* but not the original aim of the manipulation.

11	10		*scores*	11	**10**		*scores*	11	**10**		*scores*	11	**10**		*scores*
a	*e*	*a*	54	*a*	**d**	*c*	52	*a*	**d**	*a*	54	*a*	**d**	*b*	53
b	*d*	*b*	53	*b*	**c**	*d*	51	*b*	**e**	*b*	53	*b*	**e**	*d*	51
c	*b*	*d*	41	*c*	**e**	*a*	44	*c*	**b**	*d*	51	*c*	**b**	*a*	44
d	*a*	*e*	40	*d*	**b**	*b*	43	*d*	**a**	*e*	30	*d*	**c**	*c*	32
e	*c*	*c*	22	*e*	**a**	*e*	20	*e*	**c**	*c*	22	*e*	**a**	*e*	30

Table 8: Preference statements and social outcomes: truthful, greedy, safe, mixed

To conclude, this section provides possible situations in which we apply the different ways of determining strategic preference statements and analyze the problems of the different options.

Which way is the best to choose depends on the risk the manipulative group accepts compared to better chances to change the election outcome.

5 An algorithmic approach for changing the election outcome

The previous sections considered possibilities to find a manipulative group and to determine the strategic preference statement. There are various possible situations where this knowledge can be applied. In our analysis we focus on the goal to change the election outcome. This analysis of strategic voting requires a combination of determining the manipulative group and the strategic preference statement, and therefore, we formulate an algorithmic approach. The algorithmic approach starts with calculating the Borda scores for the truthful situation. Then we fix a candidate who should win the election, which later on may be changed to another one, and calculate the necessary number of manipulative voters by dividing the point distance between the actual winner (x^1) and the favored candidate (x^*) by the maximum points gain in one single ranking (max). After checking, whether there are enough voters which could act strategically in their favor, we can determine the manipulative group. The two considered ways to determine the manipulative group are shown in section 3. After the manipulative group is fixed, the manipulative statement can be chosen. Herefore, we can use one of the ways shown in section 4. However, when using the "safe" way there is no guarantee for a change in the outcome. On the other hand the "greedy" and the "mixed" way may face the problem of overshooting or undershooting. Whenever the preferred candidate wins the election the algorithm is finished, but if not another group needs to be detected. If all possible manipulative groups are tested the preferred candidate can be changed and the algorithm starts again.

We now define some notations which we will use in the algorithmic approach. Let $B(x)$ be the Borda score of candidate x, $x^1 = argmax\{B(x)|x \in X\}$ the actual winner and L_i the binary relation over X of individual i. Consider $x^* \in X$ arbitrary (fixed). Set the maximum points gain $max = m - 1$.

1. Fix the preferred candidate x^* and calculate the necessary number of manipulative voters, $necessary = (B(x^1) - B(x^*))/max$.

   (Determine the manipulative group)

2. Check whether enough voters exist who fulfill the following properties.
   (i) All voters of the manipulative group prefer x^* over x^1.
   (ii) The distance between these two candidates in the voters ranking is larger than max.
   If there are enough voters proceed with Step 4, else Step 3.

3. If max equals one, STOP (no manipulation possible). Else set max to $max - 1$. Update $necessary = (B(x^1) - B(x^*))/max$ and proceed with Step 2.

4. For safe strategic voting with identical preferences (with same top k ranks) determine the manipulative group by searching a group of voters with identical preferences (same top k ranks) and size of at least $necessary$.

   (Determine the strategic preference statement)

5. (i) "safe" way: fix the ranks of x^1 and all candidates ranked below for all voters in the manipulative group.
   (ii) "mixed" way: fix the ranking of all candidates ranked below x^1 for all voters in the manipulative group and rank x^1 last.
   (iii) "greedy" way: no rank is fixed.
6. Set x^* on the first rank (in all three options).
7. Rank all other candidates according to their total Borda scores in ascending order.
8. In case of the "mixed" and the "greedy" way check whether over- or undershooting occur.
9. Check whether the preferred candidate wins the election. If not, check if all possible groups have already been tried. If there are still groups which have not been tried already, proceed with Step 4. Else start again with Step 1.

6 Empirical Analysis

In this section we will apply the algorithm of the previous section on a dataset collected during election day of the 2015 Styrian parliamentary elections. We conducted an exit poll right in front of five voting stations and invited voters to participate in our survey. The participants were asked several questions about the election which included but were not limited to the preference statement. A total of 730 voters participated in the exit poll and stated their preferences. There was no need to state a strict or complete ranking. However, this will not falsify the results because it allows more general statements.

The starting point is again the truthful voting situation. Let us use the Borda rule to elect the winner of the election. The total scores of the candidates are: 3754 for candidate a, 3668 for candidate b, 2602 for candidate c, 2808 for candidate d, 2262 for candidate e, 1709 for candidate f, 2323 for candidate g and 1314 for candidate h. The winner of the election is candidate a and the second best candidate is candidate b. The distance between those two candidates equals 86 points. Hence, at least 15 voters have to rearrange their preference ordering to elect candidate b. Further steps need to be taken to check whether strategic voting in favor of candidate b is possible for a group of voters.

First, we take a look at the ways to determine the manipulative group. The theoretical concepts of section 3 are safe strategic voting with identical preferences and with same top k ranks. To start, we check how many voters have an identical preference statement. Remember, we need at least 15 voters to change the outcome in favor of candidate b. Another important assumption is that such a manipulative group has to prefer candidate b over a. Otherwise they would not have an incentive to vote strategically. The minimum required group size is sufficient in the case where candidate b is ranked directly before candidate a. Unfortunately, there exist only groups of a maximum size of 6 voters with such truthful preference statements. This means that these groups are not able to change the outcome alone. However, we now can enlarge the group of voters by increasing the distance between candidates a and b in the preference statement and check whether large enough groups exist to change the outcome. After checking all possible cases we conclude that a safe manipulation with identical preferences is not possible in this dataset. The other possibility is to take groups with same top k ranks into consideration. The smallest deviation from identical preferences are groups with same top m ranks, in our case 8. The higher the value of k is the more values are the same within the manipulative group.

However, these groups need not to be the same, because the group with same top m ranks includes all cases with same strict ranking but also include indifferences. Therefore, we find a group of 29 voters with truthful preference statement $badgehfc$ or the similar ranking including indifferences. This manipulative group is able to change the outcome in favor of candidate b. However, in real elections it is much easier to find a group of voters who agree on the first two ranks, which means k equals 2. For this reason we are looking for voters who have at least the first two ranks in common. For example, the dataset includes a group of 54 voters who start their preference statement with cb. Another possible group has a preference statement starting with bc and has size 55. We used $k = 2$ because it enables the possibility to use a "safe" strategic preference statement and the low value of k also enables larger groups because it is less restrictive. Now we can change the preference statement in order to manipulate the outcome of the elections.

Second, we consider the different ways of determining strategic preference statements. The "greedy" way of determining the strategic preference statement is to rank candidate b first and candidate a last. All other candidates get assigned ranks according to their Borda scores in ascending order. The strategic preference statement is $bhfegcda$ for all 54 voters who start their truthful preference statement with cb. The scores for candidate a and b now change to 3765 points for candidate b and 3527 points for candidate a. Candidate b wins the election. The group is able to change the outcome in their favor. Another possibility to determine the strategic preference statement is the "safe" way. We again use the group of 54 voters with truthful preference statements starting with cb to manipulate the election. All 54 voters agree on changing their preference statement to bc instead of cb and leave the remaining ranking as it is because every voter could have ranked candidate a on a different rank. The changed scores for a and b are: 3765 points for candidate b and 3747 points for candidate a. All remaining scores stay the same. Again, the new winner of the election is candidate b.

Additionally, we can check further group options with same k top preferences to help candidate b to win the election, where k could be all possible numbers between 1 and m. Remember, lower values for k are less restrictive and larger groups of voters will be found. However, if we know more about group preferences we can possibly use "safe" strategic preference statements more easily. When $k = 1$ we can use the group of voters with b as top preference which is the case for 203 voters. For $k = 3$ the group with truthful preferences starting with bad is an option for a manipulative group with 56 voters. Even for $k = 4$ we find a group of voters which can change the election outcome, namely the group starting with $badg$ which consists of 40 voters. We can also find groups which can form a manipulative group for k equals 5, 6 or 7. On top, the group with truthful preference statement $badgehfc$ consists of 29 voters and is able to change the outcome in favor of candidate b. This is possible because we allow voters to be within this group even if they have not ranked all candidates but the same candidates are on all placed ranks. For all possible groups, we can find a strategic preference statement according to the algorithm stated in section 4.

Furthermore, there might exist a possibility to help other candidates to win the election. Therefore, we calculate the distance in points between the candidates and the current winner of the Borda rule. The distances in points are 86 for candidate b, 1152 points for candidate c, 946 points for candidate d, 1492 points for candidate e, 2045 points for candidate f, 1432 points for candidate g and 2440 points for candidate h. This equals a minimum group size of 15 voters for candidate b to possibly win the election. For candidate c a minimum group size of 192 voters is necessary. Candidate d needs at least 158 voters to build a manipulative group. At least 249 voters are needed for candidate e, 341 for candidate f, 239 for candidate g and 406 for candidate h. The easiest way to eliminate all unrealistic situations is to take into account the number of pairwise comparisons the candidate wins against candidate a, who is the current winner of the election. In this dataset only 75 voters prefer candidate h to candidate a. This means finding a minimum group size of 406 voters is not possible because only 75 voters prefer candidate h to a. The same holds for all candidates except candidates b, c and d. Candidate d wins 187 pairwise comparisons and needs a minimum group size of 158 voters. However, there is no large enough group which enables the best case scenario. Only a maximum of 56 voters ranked candidate a one rank lower than candidate d. The second best scenario, when candidate a is ranked at most two ranks lower than candidate d requires a size of 190 voters which is larger than the possible size of 187. Although candidate c only requires 231 voters in the second best scenario and 242 voters prefer candidate c to a, no group which is able to change the outcome exists. The election outcome can only be changed in favor of candidate b.

7 Conclusion

In this paper we analyzed possibilities to coalitionally manipulate voting outcomes. First we outlined the possibilities to form such coalitions or manipulative groups. We changed one assumption of safe strategic vote (Slinko and White, 2014) by allowing voters with same top k ranks to form a manipulative group instead of voting truthfully. This enables us to find larger groups. Larger groups are more often able to change the outcome of elections. Second, we stated three possible ways to determine a strategic preference statement for a manipulative group. We distinguished between a "safe", a "greedy" and a "mixed" way to determine such a preference statement. We combined these two steps to show a possible algorithmic approach. The empirical results in the paper show that it is not always possible to find large enough groups with identical preferences to manipulate the outcome safely. However, we found various possible groups with top k same ranks and analyzed the ways of determining the strategic preference statement. In our dataset every way was possible for a specific group, but only for one candidate. However, we know that it is harder to find a large enough group to change the election outcome in a "safe" way.

Possible topics for future research could be a method which quickly detects whether there is over- or undershooting for a specific manipulative group and when to choose which way to determine the strategic preference statement.

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