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Volatility estimation for cryptocurrencies: Further evidence with jumps and structural breaks

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Abstract

In this paper we study the daily volatility of four cryptocurrencies (BitCoin, Dash, LiteCoin, and Ripple) from June 2014 to November 2018. We first show that the cryptocurrency returns are strongly characterized by the presence of jumps as well as structural breaks (except Dash). Then, we estimate four GARCH-type models that capture short memory (GARCH), asymmetry (APARCH), strong persistence (IGARCH), and long memory (FIGARCH) from (i) original returns, (ii) jump-filtered returns, and (iii) jump-filtered returns with structural breaks. Results indicate the importance to take into account the jumps and structural breaks in modelling volatility of the cryptocurrencies. It appears that the cryptocurrency returns are well modelled by infinite persistence (BitCoin, Dash, and LiteCoin) or long memory (Ripple) with a Student-t distribution.

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1. Introduction

Cryptocurrencies are a new breed of digital currency systems built on computer cryptology and decentralized (peer-to-peer) network architecture. Policymakers, investors, and researchers monitor their behavior. Cryptocurrency prices are characterized by high volatility and some drastic shocks (see Figure 1), such as the hacking of Mt.Gox exchange platform with a negative return of -49% for Bitcoin. Financial-market participants and policy-makers can benefit from a better understanding of how shocks can affect volatility over time, especially whether the shocks are persistent or short lived.

A number of studies apply GARCH-type models to describe the volatility of cryptocurrencies (Katsiampa, 2017; Chu et al., 2017; Baur and Dimpfl, 2018). However, as shown by Catania and Grassi (2017), Chaim and Laurini (2018), Scaillet et al. (2018), Charles and Darné (2019) and Trucíos (2019) cryptocurrencies are subject to a number of drastic shocks (called large shocks, outliers or jumps). These shocks may pose difficulties for the identification and estimation of GARCH models governing the conditional volatility of returns (Carnero et al., 2007, 2012; Charles and Darné, 2005; Laurent et al., 2016). Further, some studies find that cryptocurrency returns exhibit regime changes (Ardia et al., 2018; Mensi et al., 2018) or structural breaks in the volatility process (Charfeddine and Maouchi, 2019). Ignoring regime changes or structural breaks in the volatility dynamics can bias the estimated persistence of volatility (see, e.g., Lamoureux and Lastrapes, 1990; Hillebrand, 2005). These previous studies do not take into account both jumps and structural breaks in the volatility process.
The aim of this paper is to fill this gap. For that, we first employ the semi-parametric procedure to detect jumps in GARCH models proposed by Laurent et al. (2016) on four major cryptocurrencies (Bitcoin, Dash, LiteCoin and Ripple), spanning June 1st, 2014 to November 11th, 2018. Second, we use an appropriate methodology to identify breakpoints and sudden shifts in volatility with the modified iterative cumulative sums of squares (ICSS) algorithm developed by Sansó et al. (2004). Nevertheless, Rodrigues and Rubia (2011) show that the asymptotic distribution of the ICSS test statistics varies under jumps, indicating that neglected jumps tend to bias the ICSS test. Therefore, we then apply the modified ICSS algorithm on filtered returns.

To show the importance to take into account both jumps and structural breaks in modelling volatility of cryptocurrencies we estimate four GARCH-type models (GARCH, APARCH, IGARCH and FIGARCH) from (i) original returns, (ii) jump-filtered returns, and (iii) jump-filtered returns with structural breaks.

The remainder of this article is organized as follows. Section 2 presents the methodology. Section 3 describes the data whereas the empirical results are presented in Section 4. Finally, Section 5 concludes.

2. Methodology

2.2. Outlier detection

There are several methods for detecting jumps in nonlinear setting based on intervention analysis as originally proposed by Box and Tiao (1975) and the iterative procedure of Chen and Liu (1993) (e.g., Sakata and White, 1998; Franses and Ghijsels, 1999; Doornik and Ooms, 2005; Grané and Veiga, 2010; Hotta and Tsay, 2012), the local influence method (Zhang and King, 2005; Zevallos and Hotta, 2012) or a weighted forward search approach (Crosato and Grossi, 2017), among others.[1] Here we use the semi-parametric procedure to detect jumps proposed by Laurent et al. (2016) [LLP hereafter].

Consider that the return series with an independent jump component $a_t I_t$, defined as

$$r_t^* = r_t + a_t I_t \tag{1}$$

where r_t^* denotes the observed returns, I_t is a dummy variable for a jump on day t, and a_t is the jump size. r_t denotes the unaffected returns, defined by $r_t = \log P_t - \log P_{t-1}$, where P_t is the observed price at time t, and described by a Normal GARCH(1,1) model:

$$\begin{aligned}
r_t &= \mu_t + \varepsilon_t, \qquad\qquad \mu_t = c, &(2)\\
\varepsilon_t &= z_t \sqrt{\sigma_t^2}, \\
\varepsilon_t &\sim N(0, \sqrt{\sigma_t^2}), \qquad\qquad z_t \sim i.i.d.N(0,1), \\
\sigma_t^2 &= \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2 &(3)
\end{aligned}$$

In equation (1) a jump $a_t I_t$ will not affect σ_{t+1}^2 (the conditional variance of r_{t+1}), so that we can have non-Gaussian fat-tailed conditional distributions of r_t^*.
LLP use the bounded innovation propagation (BIP) to obtain robust estimates of μ_t and σ_t^2 in equations (1) and (2), respectively. These are denoted by $\widetilde{\mu}_t$ and $\widetilde{\sigma}_t$ and are robust to potential jumps $a_t I_t$. They detect the presence of jumps by testing the null hypothesis $\mathrm{H}_0 : a_t I_t = 0$ against the alternative $\mathrm{H}_1 : a_t I_t \neq 0$. The null is rejected if

$$\max_T |\widetilde{J}_t| > g_{T,\lambda}, \qquad t = 1, \ldots, T \tag{4}$$

where $g_{T,\lambda}$ is the suitable critical value. If H_0 is rejected a dummy variable is defined as $\widetilde{I}_t = I\left(|\widetilde{J}_t| > k\right)$, where $I(.)$ is the indicator function, with $\widetilde{I}_t = 1$ when a jump is detected at time t and 0 otherwise. The filtered returns $\widetilde{r}_t$ are obtained as follows

$$\widetilde{r}_t = r_t^* - (r_t^* - \widetilde{\mu}_t)\widetilde{I}_t \tag{5}$$

[1] See Hotta and Zevallos (2013) for a review on the detection of jumps in GARCH models.

2.3. Sudden change detection

The most popular statistical methods specifically designed to detect breaks in volatility are CUSUM-type tests. We apply the modified ICSS algorithm test proposed by Sansó et al. (2004) which is robust to conditional heteroscedasticity by explicitly considering the fourth moment properties of the disturbances and the conditional heteroscedasticity. Assume that the variance within each interval is denoted by σ_j^2, $j = 0, 1, \ldots, N_T$, where N_T is the total number of variance changes, and $1 < \kappa_1 < \kappa_2 < \cdots < \kappa_{N_T} < T$ are the set of breakpoints. Then the variances over the N_T intervals are defined as

$$\sigma_t^2 = \begin{cases} \sigma_0^2, & 1 < t < \kappa_1 \\ \sigma_1^2, & \kappa_1 < t < \kappa_2 \\ \ldots & \\ \sigma_{N_T}^2, & \kappa_{N_T} < t < T \end{cases}$$

The cumulative sum of squares is used to estimate the number of variance changes and to detect the point in time of each variance shift. The cumulative sum of the squared observations from the beginning of the series to the kth point in time is expressed as $C_k = \sum_{t=1}^{k} r_t^2$ for $k = 1, \ldots, T$. To test the null hypothesis of constant unconditional variance, the adjusted statistic is given by

$$AIT = \sup_k |T^{-0.5} G_k| \tag{6}$$

where $G_k = \hat{\lambda}^{-0.5}\left[C_k - \left(\frac{k}{T}\right)C_T\right]$, with C_T is the sum of the squared residuals from the whole sample period, $\hat{\lambda} = \hat{\gamma}_0 + 2\sum_{l=1}^{m}\left[1 - l(m+1)^{-1}\right]\hat{\gamma}_l$, $\hat{\gamma}_l = T^{-1}\sum_{t=l+1}^{T}(r_t^2 - \hat{\sigma}^2)(r_{t-l}^2 - \hat{\sigma}^2)$, $\hat{\sigma}^2 = T^{-1}C_T$, and the lag truncation parameter m is selected using the procedure in Newey and West (1994). The value of k that maximizes AIT is the estimate of the break date.

3. Data and summary statistics

The dataset that we analyze consists of daily closing prices of four cryptocurrencies, namely Bitcoin (BTC), Dash, LiteCoin (LTC), and Ripple (XRP). The data were collected from coinmarketcap.com. The four cryptocurrencies were chosen based on their market capitalization and their large sample. Even if some cryptocurrencies are available before June 2014 our dataset covers the period from June 1st, 2014 to November 11th, 2018, to harmonize the number of observations across the cryptocurrencies (1 625 observations). This large sample allows to well estimate the GARCH-type models, especially the FIGARCH model.

Table 3 presents the summary statistics for cryptocurrency returns (Panel A). XRP displays the higher mean returns with about 0.296% per day whereas LTC exhibits the lower mean returns with about 0.095% per day. The XRP returns are also the more volatile, as measured by their standard deviation (7.07%), with the BTC returns being the less volatile (3.80%). All returns are highly non-Normal, and show evidence of significant positive skewness, except BTC with negative skewness, excess kurtosis, and strong conditional heteroscedasticity.

We find that the cryptocurrency returns are strongly characterized by the presence of jumps as found by Chaim and Laurine (2018), with a proportion of detected jumps more than 3%, except for Dash (2%). Charles and Darné (2019) show that the jumps in cryptocurrency returns are mainly due to attacks, hacks, thefts, closings or bankruptcies of cryptocurrency exchange platforms as well as technical issues.

Charles and Darné (2005, 2014) showed that financial assets are also affected by the presence of jumps. Therefore, we compare the occurrence of jumps in cryptocurrency returns to other assets, such as the spot prices of Brent crude oil market (Brent) and gold market (Gold), the S&P500 stock index (S&P500), and the euro-dollar exchange rate (EUR/USD). The results show that the cryptocurrency returns are more affected by jumps since we find between two to three times more jumps than for other assets (Gold and S&P500, 1.60%; Brent, 1.07%; EUR/USD, 0.80%) (Table 3, Panel C). Note that all the cryptocurrrencies exhibit higher mean returns and volatility than the other assets.
The jump-filtered returns also exhibit excess skewness, excess kurtosis and conditional heteroscedasticity, although the excess skewness and kurtosis decrease dramatically (Table 3, Panel B).

Finally, we test for the presence of structural breaks in volatility of cryptocurrency prices on the filtered returns by using the modified ICSS algorithms test. As suggested by Huyng et al. (2008) the minimum duration between two consecutive breaks is set at 20 days to reduce the possibility that any temporary shocks is being mistaken as a break. The results show two breaks for BTC (03/06/2017 and 08/10/2018), LTC (04/05/2017 and 09/27/2018) and XRP (04/21/2017 and 06/03/2017) whereas no break in found for Dash.[2] The BTC, LTC and XRP display a high volatility period beginning in March-April 2017 with the strong speculation on the cryptocurrencies. This high volatility regime is long for BTC and LTC, ending in August and April 2018, respectively, whereas it is very short (less than 2 months) for XRP. Table 2 presents some main characteristics of the four cryptocurrencies, giving possible explanations of the findings. BTC and LTC are very similar, and the major difference is the speed of transactions since LTC requires 2.5 minutes against 10 minutes for BTC (Table 2). This difference is stronger with XRP since its speed of transaction is 5 seconds due to its mechanism to validate transactions. The block generation mechanism to secure and validate transactions (blocks) in the blockchain is based on the proof-of-work (PoW or mining) for BTC and LTC, whereas XRP uses the proof-of-correctness (PoC) approach, namely an iterative consensus. Dash employs an hybrid system of the PoW and Proof-of-Stake (PoS) mechanisms, giving more security on the transactions.[3] Further, Ripple Labs owns two-thirds of the existing 100 billion XRP, giving a centralized control.[4] Finally, the strong differences between Dash and the three other cryptocurrencies are its low market capitalization and its low circulation supply.

4. Estimation results

We now estimate four different GARCH-type models, namely the (symmetric) GARCH model, the Asymmetric Power ARCH (APARCH) model capturing the asymmetric effect, the Integrated GARCH (IGARCH) model capturing the strong persistence, and the Fractionaly IGARCH (FIGARCH) model capturing long memory, with the Normal and Student-t distributions. A

[2] Note that we had found five breaks for BTC (03/06/2017, 12/08/2017, 03/14/2018, 08/10/2018 and 09/27/2018) and three breaks for LTC (04/05/2017, 04/27/2018 and 09/27/2018). Following Aggarwal et al. (1999), dummy variables representing the different volatility regimes were introduced in the condition variance equation of the GARCH-type models to account for the significant volatility level shifts. We found that all the dummy variables are not significant and included only the significant dummies.

[3] See Ciaian et al. (2018) for a discussion on the block generation mechanisms.

[4] All the XRP are not mineable and are considered as “pre-mined”.

brief presentation of the models is given in Table 1.[5] The parameters of the volatility models are estimated by the quasi-maximum likelihood (QML) method – producing robust standard errors – and the quasi-likelihood function is maximized using the BFGS algorithm from the G@RCH 8.0 package for Ox.
We compare the estimations in three ways: (1) original returns, (2) filtered returns, and (3) filtered returns with structural breaks identified from the modified ICSS algorithm. For the third approach, we introduce identified breaks into the GARCH-type models by incorporating dummy variables that take a value of one from each point of structural change of variance onwards and take a value of zero elsewhere. The comparison between the volatility models is evaluated from various in-sample criteria: LogLikehood (LL), Akaike (AIC) and Hannan-Quinn (HQ) criteria.

Tables 4-7 show the estimation results of the GARCH-type models on the raw returns (Panel A), filtered returns (Panel B), and filtered returns with dummies (Panel C) for the BTC, Dash, LTC and XRP returns, respectively.
When estimating GARCH-type models on original returns we find that the IGARCH process with a Student-*t* distribution captures the best temporal pattern of volatility for the BTC, LTC and XRP return series (Panel A, Tables 4, 6 and 7, respectively), whereas the APARCH model with a Student-*t* distribution is the best specification for the Dash return series (Table 5).
The estimation results of the GARCH-type models on the filtered returns (Panel B) show that the four cryptocurrency return series are better modelled by an IGARCH model with a Student-*t*. Finally, when introducing structural breaks in the GARCH-type models (Panel C) we find that the BTC and LTC return series are well specified by the IGARCH model with a Student-*t* distribution whereas the FIGARCH model with a Student-*t* distribution is the best specification for the XRP return series.
Overall, the results show the importance to take into account the jumps as well as the structural breaks in modelling volatility of the cryptocurrencies as shown by the in-sample criteria and also by the number of GARCH-type models rejected due to their regularity and non-negativity conditions not satisfied. Further, the BTC, Dash and LTC return series seem to be well modelled by an IGARCH model, implying that the shocks to the conditional variance persist indefinitely, whereas the XRP return series appears to be well specified by a FIGARCH model, implying a long-memory behavior and a slow rate of decay after a volatility shock. Moreover the Student-*t* distribution is the better distribution for the four cryptocurrency return series by capturing the fat tails. Note that all the asymmetric volatility models are rejected when estimating GARCH-type models on filtered returns, suggesting that the asymmetric effect is not appropriate for the cryptocurrencies.

5. Conclusion

We have studied the volatility of four cryptocurrencies (BitCoin, Dash, LiteCoin, and Ripple) from June 2014 to November 2018. Results show that the cryptocurrency returns are strongly characterized by the presence of jumps as well as structural breaks (except Dash). We have estimated four GARCH-type models (GARCH, APARCH, IGARCH and FIGARCH) from (i) original returns, (ii) jump-filtered returns, and (iii) jump-filtered returns with structural breaks.

[5]We have also considered GJR-GARCH, EGARCH and FIAPARCH models. To save space the results are given in the Online Appendix.

Results indicate the importance to take into account the jumps and structural breaks in modelling volatility of the cryptocurrencies. It appears that the cryptocurrency returns are well modelled by infinite persistence (BitCoin, Dash, and LiteCoin) or long memory (Ripple) with a Student-t distribution. These results are potentially relevant for financial-market participants and policy-makers to better understand of how shocks can affect volatility over time. Policy-makers and regulators are concerned about two important issues in volatility. First, what level of volatility is excessive, relatively to some agreed benchmark of acceptable or non-excessive volatility, knowing that the level of volatility which is considered to be excessive may will differ across different markets, especially for cryptocurrencies. Second, what measures can be adopted to reduce financial volatility without impacting adversely upon the efficient workings of the financial markets. Volatility is also important for cryptocurrency users and investors who are both concerned about managing the risk (e.g., Value-at-Risk) associated with sharp changes in cryptocurrency prices. Furthermore, the ability of modelling and forecasting cryptocurrency volatility has important application for hedging. Lastly, the Securities Exchange Commission (SEC) and the Commodity Futures Trading Commission (CFTC) set to collaborate to regulate Bitcoin ETFs and other investment products, making the volatility dynamic of cryptocurrencies to play a crucial role in any potential derivatives pricing and trading.

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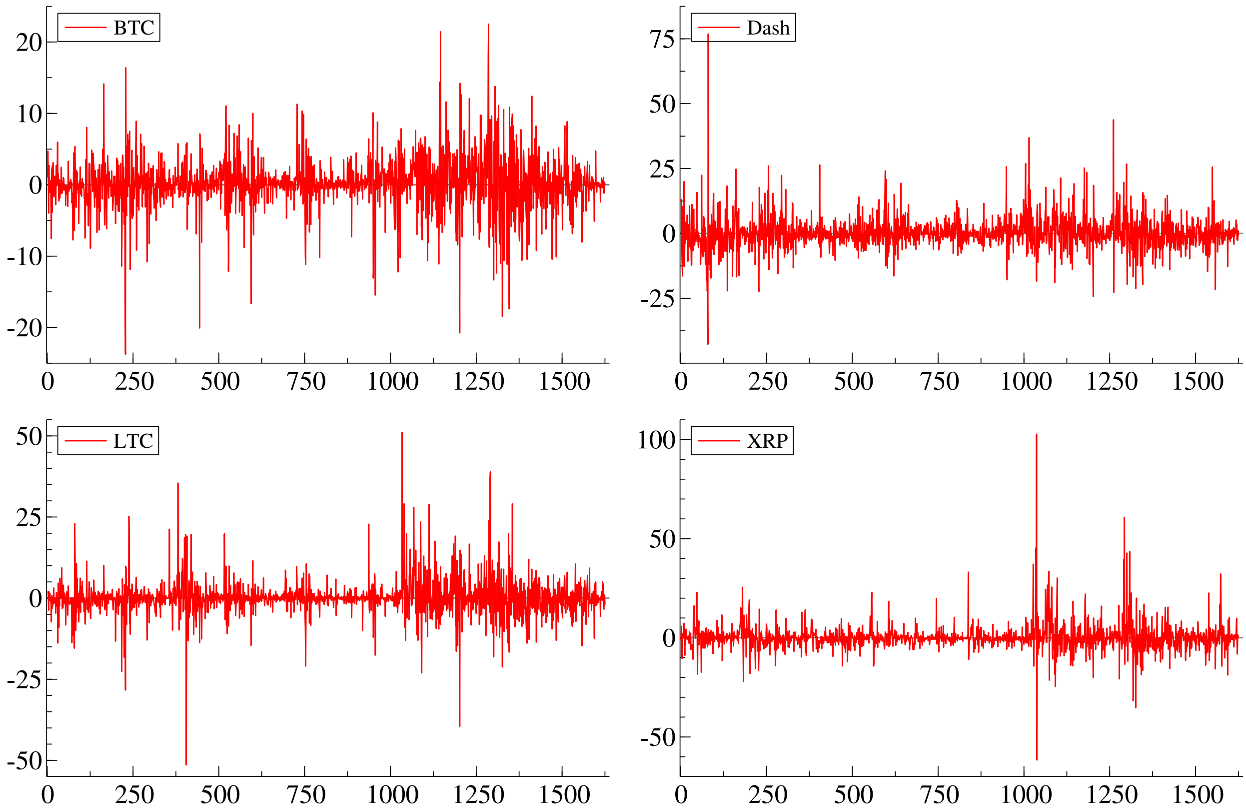

Figure 1: Bitcoin prices, returns and squared returns - July 2010 to October 2016.

Table 1: GARCH(1,1)-type models.

Models	Equations	Constraints positivity	stationarity
GARCH	$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$	$\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$	$\alpha + \beta < 1$
IGARCH	$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + (1-\alpha)\sigma_{t-1}^2$	$\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$	
APARCH	$\sigma_t^\delta = \omega + \alpha\left(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1}\right)^\delta + \beta\sigma_{t-1}^\delta$	$\delta > 0$, $-1 < \gamma < 1$	$\alpha E(\lvert z\rvert - \gamma z)^\delta + \beta < 1$ $\omega > 0$
FIGARCH	$\sigma_t^2 = \omega + \left\{1 - [1-\beta L]^{-1}(1-\alpha L)(1-L)^d\right\}\varepsilon_t^2$	$\omega > 0$, $\beta - d \leq \alpha \leq \frac{2-d}{3}$ $d\left(\alpha - \frac{1-d}{2}\right) \leq \beta(\alpha - \beta + d)$	$0 \leq d \leq 1$

Notes: The existence of the fourth moment for the GARCH(1,1) model implies that $E[\varepsilon_t^4] < \infty$, which is satisfied if $k\alpha^2 + 2\alpha\beta + \beta^2 < 1$. Under a Normal distribution $k = 3$. See Ling and McAleer (2002) for the Student-t distribution. For the APARCH model Ding et al. (1993) and Lambert and Laurent (2001) derived a closed form solution to $\kappa = E(\lvert z\rvert - \gamma z)^\delta$ for a Normal and Student-t distributions, respectively. The conditions on the FIGARCH(1,d,1) model are sufficient to ensure that the conditional variance is positive almost surely for all t. However, occasionally it fails to satisfy theses conditions that all parameters should be positive to ensure the nonnegativity of conditional variance. Therefore, Conrad and Haag (2006) introduced another set of conditions that guarantees the nonnegativity of the conditional variance in all situations.

Table 2: Characteristics of cryptocurrencies.

	Date of release	Duration of a block creation	Block generation mechanism	Maximum supply	Market capitalization	Circulation supply
BTC	2009	10 min	Proof-of-Work	21 million	\$70.85 billion	17.6 million
DASH	2014	2.5 min	Proof-of-Work & Proof-of-Stake	22 million	\$0.80 billion	8.7 million
LTC	2011	2.5 min	Proof-of-Work	84 million	\$3.66 billion	61.0 million
XRP	2012	5 sec	Proof-of-Correctness	100 billion	\$12.81 billion	41.7 billion[a]

Notes: The information on the market capitalization and the circulation supply comes from coinmarketcap in March 25, 2019. [a] denotes that all the XRP are not mineable.

Table 3: Descriptive statistics.

	Mean (%)	Min (%)	Max (%)	Std. (%)	Skew	Ex.Kurt	JB	$Q^*(10)$	ARCH(10)	jumps
Panel A: Original										
BTC	0.143	-23.8	22.5	3.796	-0.346^*	5.904^*	2392.2^*	10.5	18.6^*	3.2%
DASH	0.163	-42.7	76.8	6.589	1.370^*	15.62^*	17023.0^*	5.60	14.8^*	2.0%
LTC	0.095	-51.4	51.0	5.818	0.726^*	14.64^*	14659.0^*	18.5^*	7.49^*	3.7%
XRP	0.296	-61.6	102.7	7.073	2.834^*	39.25^*	106500^*	13.0	19.0^*	3.5%
Panel B: Filtered										
BTC	0.192	-17.4	14.4	3.188	-0.226^*	3.350^*	773.9^*	8.0	28.2^*	
DASH	-0.044	-22.1	26.0	5.393	0.134^*	2.415^*	400.0^*	10.9	14.6^*	
LTC	-0.034	-21.2	20.1	4.256	0.330^*	4.279^*	1269.0^*	11.9	23.6^*	
XRP	-0.026	-25.4	33.5	4.949	0.844^*	7.703^*	4210.1^*	10.1	47.5^*	
Panel C: Other assets										
Brent	-0.052	-8.69	8.82	2.147	0.099	1.915^*	173.5^*	17.91	16.89^*	1.07%
Gold	0.002	-3.41	4.65	0.868	0.276^*	2.965^*	425.6^*	24.1^*	2.94^*	1.60%
EUR/USD	-0.015	-2.84	2.97	0.548	0.024	2.264^*	240.0^*	10.38	3.07^*	0.80%
S&P500	0.031	-4.18	3.83	0.810	-0.603^*	3.505^*	642.8^*	13.4	24.62^*	1.60%

Notes: * denotes significance at the 5% level. Original and Filtered indicate that the descriptive statistics are calculated on the original and jump-filtered returns, respectively. $Q^*(10)$ denotes heteroscedasticity-robust Ljung-Box statistic at 10 lags. jumps denotes the proportion of detected jumps.

Table 4: Estimates of GARCH-type models for BTC returns.

		Parameters						*In-sample criteria*
	Distrib.	ω	α	β	γ	δ	d	LL
Panel A: Original								
GARCH	Gauss[a]	0.260^{*}	0.137	0.861				-4236.8
		(1.90)	(4.80)	(31.6)				
	Student[a]	0.138^{**}	0.301	0.838				-4019.7
		(1.53)	(3.71)	(33.1)				
APARCH	Gauss	nc						
	Student	0.051^{*}	0.226	0.865	-0.090^{**}	1.005		-4012.2
		(1.68)	(4.41)	(38.3)	(-1.23)	(3.81)		
IGARCH	Gauss	0.253	0.139	0.861				-4236.8
		(2.30)	(5.13)	$(--)$				
	Student	0.159	0.154	0.846				-4028.0
		(2.07)	(6.22)	$(--)$				
FIGARCH	Gauss	0.250	0.264	0.802			0.753	-4228.0
		(1.97)	(2.76)	(17.7)			(6.75)	
	Student	0.390^{**}	-0.887	-0.868			0.340	-4042.0
		(0.49)	(-6.53)	(-5.72)			(16.5)	
Panel B: Filtered								
GARCH	Gauss[a]	0.082	0.159	0.848				-3832.8
		(2.40)	(7.04)	(47.4)				
	Student[a]	0.047^{**}	0.209	0.839				-3758.4
		(1.60)	(6.19)	(42.7)				
APARCH	Gauss[a]	0.085	0.157	0.845	-0.007^{**}	2.126		-3832.7
		(2.23)	(6.39)	(39.1)	(-0.13)	(5.06)		
	Student[a]	0.047^{**}	0.207	0.836	-0.036^{**}	2.147		-3758.1
		(1.45)	(5.89)	(30.4)	(-0.76)	(3.95)		
IGARCH	Gauss	0.091	0.151	0.849				-3833.2
		(2.78)	(8.45)	$(--)$				
	Student	0.069	0.154	0.846				-3762.8
		(2.27)	(7.94)	$(--)$				
FIGARCH	Gauss	0.100	0.084^{**}	0.788			0.862	-3831.7
		(2.45)	(1.33)	(18.6)			(10.6)	
	Student	nc						
Panel C: Breaks								
GARCH	Gauss	0.132	0.147	0.836				-3823.9
		(2.77)	(6.93)	(44.3)				
	Student[a]	0.107	0.192	0.820				-3742.0
		(2.33)	(6.24)	(36.5)				
APARCH	Gauss	0.150	0.140	0.827	0.001^{**}	2.327		-3823.4
		(2.40)	(5.38)	(33.8)	(0.02)	(4.25)		
	Student	0.115	0.187	0.813	-0.029^{**}	2.255		-3748.8
		(2.01)	(5.56)	(27.7)	(-0.56)	(3.94)		
IGARCH	Gauss	0.106	0.164	0.836				-3825.2
		(2.89)	(8.72)	$(--)$				
	Student	0.114	0.180	0.820				**-3749.2**
		(2.64)	(7.97)	$(--)$				
FIGARCH	Gauss	0.117	0.084^{**}	0.757			0.827	-3823.3
		(2.36)	(1.09)	(14.3)			(7.97)	
	Student	nc						

Notes: nc means "not computed" due to convergence problem. [a] The conditions for positivity and stationarity constraints or the existence of the fourth moment are not observed. * and ** indicate that the null hypothesis is rejected at the 5% and 10% level, respectively. The robust t-ratios appear in parentheses. The best likelihood (LL) appear in bold.

Table 5: Estimates of GARCH-type models for Dash returns.

		Parameters						*In-sample criteria*
	Distrib.	ω	α	β	γ	δ	d	LL
Panel A: Original								
GARCH	Gauss[a]	2.570^{*}	0.219	0.748				-5155.3
		(1.32)	(2.67)	(7.12)				
	Student[a]	2.471	0.286	0.730				-4991.9
		(2.81)	(3.88)	(12.9)				
APARCH	Gauss	0.928^{**}	0.223	0.763	-0.053^{**}	1.380		-5150.9
		(1.46)	(3.16)	(7.94)	(−0.56)	(4.32)		
	Student	0.853^{*}	0.283	0.756	0.114^{*}	1.428		-4987.9
		(1.71)	(4.72)	(15.2)	(1.95)	(5.41)		
IGARCH	Gauss	1.985^{**}	0.239	0.761				-5156.8
		(1.55)	(2.74)	(−−)				
	Student	2.480	0.268	0.732				-4992.0
		(2.84)	(4.75)	(−−)				
FIGARCH	Gauss[a]	1.515^{**}	0.518^{**}	0.640^{**}			0.462	-5138.0
		(0.74)	(1.24)	(1.56)			(3.20)	
	Student	8.156	−0.981	−0.984			0.357	-4988.8
		(2.75)	(−103.8)	(−119.5)			(8.76)	
Panel B: Filtered								
GARCH	Gauss[a]	1.064^{**}	0.165	0.808				-4876.9
		(1.58)	(3.04)	(11.9)				
	Student[a]	1.216	0.212	0.774				-4825.7
		(2.46)	(4.13)	(15.0)				
APARCH	Gauss	1.400^{**}	0.168	0.797	0.044^{**}	2.164		-4876.3
		(1.17)	(3.00)	(11.0)	(0.89)	(6.41)		
	Student	0.952^{*}	0.225	0.773	0.090^{**}	1.812		-4824.1
		(1.74)	(4.76)	(16.4)	(1.61)	(6.02)		
IGARCH	Gauss	0.676^{*}	0.172	0.828				-4879.3
		(1.74)	(2.93)	(−−)				
	Student	1.137	0.225	0.775				**-4825.9**
		(2.47)	(4.31)	(−−)				
FIGARCH	Gauss	1.400^{*}	0.018^{**}	0.311^{**}			0.469	-4869.2
		(1.87)	(0.11)	(1.45)			(4.40)	
	Student	1.760	0.012^{**}	0.413			0.589	-4822.0
		(2.40)	(0.10)	(2.65)			(5.86)	

Notes: nc means “not computed” due to convergence problem. [a] The conditions for positivity and stationarity constraints or the existence of the fourth moment are not observed. * and ** indicate that the null hypothesis is rejected at the 5% and 10% level, respectively. The robust t-ratios appear in parentheses.The best likelihood (LL) appear in bold.

Table 6: Estimates of GARCH-type models for LTC returns.

		Parameters						*In-sample criteria*
	Distrib.	ω	α	β	γ	δ	d	LL
Panel A: Original								
GARCH	Gauss	1.576^{*}	0.089	0.865				-4953.3
		(1.89)	(3.97)	(26.7)				
	Student^{a}	0.545^{**}	0.670	0.860				-4449.4
		(1.16)	(1.97)	(26.0)				
APARCH	Gauss	0.334^{**}	0.100	0.875	-0.381^{**}	1.008^{**}		-4942.1
		(0.70)	(4.17)	(34.0)	(-0.78)	(1.07)		
	Student^{a}	0.106^{**}	0.382	0.866	-0.104^{**}	1.106		-4440.5
		(1.43)	(3.23)	(34.7)	(-1.30)	(6.18)		
IGARCH	Gauss	1.285	0.133	0.867				-4970.6
		(1.97)	(4.51)	$(--)$				
	Student	0.198^{*}	0.123	0.877				-4465.5
		(1.72)	(4.87)	$(--)$				
FIGARCH	Gauss	12.94^{*}	-0.770	-0.686			0.249^{*}	-4965.2
		(1.81)	(-7.17)	(-4.09)			(1.90)	
	Student^{a}	-0.041^{**}	0.358	0.712			0.594	-4456.4
		(-0.20)	(4.20)	(9.16)			(6.40)	
Panel B: Filtered								
GARCH	Gauss^{a}	0.234	0.186	0.819				-4255.9
		(2.99)	(7.38)	(40.7)				
	Student^{a}	0.211	0.340	0.794				-4155.2
		(2.23)	(4.12)	(23.8)				
APARCH	Gauss^{a}	0.262	0.183	0.815	-0.017^{**}	2.132		-4255.6
		(2.05)	(7.37)	(31.7)	(-0.32)	(6.70)		
	Student^{a}	0.166^{*}	0.325	0.805	-0.022^{**}	1.772		-4154.9
		(1.78)	(4.02)	(21.5)	(-0.32)	(7.07)		
IGARCH	Gauss	0.244^{*}	0.180	0.820				-4256.0
		(3.50)	(8.96)	$(--)$				
	Student	0.223	0.191	0.809				-4162.9
		(2.94)	(6.83)	$(--)$				
FIGARCH	Gauss	0.253	0.126^{*}	0.716			0.253	-4253.2
		(2.58)	(1.70)	(6.47)			(4.94)	
	Student	0.195^{**}	-0.985	-0.990			0.350	-4169.5
		(0.30)	(-116.5)	(-162.9)			(15.4)	
Panel C: Breaks								
GARCH	Gauss	0.295	0.174	0.802				-4244.3
		(3.02)	(7.09)	(31.8)				
	Student^{a}	0.363	0.305	0.754				-4421.9
		(2.43)	(4.13)	(16.8)				
APARCH	Gauss	0.396	0.163	0.789	-0.010^{**}	2.361		-4243.3
		(2.06)	(6.20)	(24.8)	(-0.17)	(6.97)		
	Student	0.316	0.301	0.762	-0.010^{**}	1.846		-4143.8
		(2.09)	(4.21)	(16.3)	(-0.13)	(7.41)		
IGARCH	Gauss	0.251	0.197	0.803				-4246.3
		(3.35)	(8.06)	$(--)$				
	Student	0.368	0.246	0.754				**-4144.8**
		(2.70)	(5.56)	$(--)$				
FIGARCH	Gauss	0.260	0.095^{**}	0.715			0.822	-4244.8
		(2.59)	(1.02)	(5.17)			(3.95)	
	Student	0.571^{**}	-0.735	-0.777			0.356	-4148.9
		(0.76)	(-2.47)	(-2.82)			(9.41)	

Notes: nc means "not computed" due to convergence problem. ^{a} The conditions for positivity and stationarity constraints or the existence of the fourth moment are not observed. * and ** indicate that the null hypothesis is rejected at the 5% and 10% level, respectively. The robust t-ratios appear in parentheses. The best likelihood (LL) appear in bold.

Table 7: Estimates of GARCH-type models for XRP returns.

		Parameters						*In-sample criteria*
	Distrib.	ω	α	β	γ	δ	d	LL
Panel A: Original								
GARCH	Gauss[a]	4.482 (2.20)	0.441 (2.04)	0.548 (3.62)				-5002.3
	Student[a]	4.738** (1.52)	1.364* (1.89)	0.566 (5.89)				-4682.7
APARCH	Gauss	1.846** (0.87)	0.404 (3.00)	0.562 (4.11)	−0.265* (−1.86)	1.374* (1.66)		-4986.3
	Student[a]	0.616* (1.89)	0.606 (3.51)	0.643 (9.39)	−0.049** (−0.76)	0.918 (4.64)		-4674.1
IGARCH	Gauss	4.490 (2.24)	0.458 (3.60)	0.542 (−−)				-5002.4
	Student	2.649* (1.82)	0.397 (3.26)	0.603 (−−)				-4694.1
FIGARCH	Gauss	9.501 (2.65)	−0.334 (−2.43)	−0.302* (−1.95)			0.615 (5.83)	-4970.6
	Student	0.754** (0.83)	0.225** (0.89)	0.118** (0.50)			0.344 (7.53)	-4662.1
Panel B: Filtered								
GARCH	Gauss[a]	0.937 (2.26)	0.210 (4.48)	0.769 (15.2)				-4255.9
	Student[a]	1.339 (2.28)	0.488 (3.37)	0.680 (10.1)				-4421.9
APARCH	Gauss	1.103* (1.85)	0.209 (3.95)	0.759 (13.4)	−0.065** (−1.08)	2.070 (5.87)		-4569.3
	Student[a]	0.716* (1.90)	0.435 (3.91)	0.709 (11.8)	0.016** (0.28)	1.559 (6.76)		-4420.8
IGARCH	Gauss	0.829 (2.53)	0.231 (4.58)	0.769 (−−)				-4571.8
	Student	1.280 (2.67)	0.309 (4.73)	0.691 (−−)				-4425.4
FIGARCH	Gauss	2.209* (0.71)	−0.302** (−0.25)	−0.041** (−0.03)			0.488 (2.82)	-4561.7
	Student	0.860** (1.51)	0.222** (1.49)	0.388 (2.66)			0.480 (6.39)	-4413.8
Panel C: Breaks								
GARCH	Gauss	1.063 (2.35)	0.223 (4.45)	0.716 (10.6)				-4544.9
	Student[a]	1.504 (3.10)	0.457 (4.39)	0.604 (10.0)				-4406.8
APARCH	Gauss	1.401 (2.07)	0.227 (3.78)	0.689 (8.55)	−0.088** (−1.33)	2.140 (5.35)		-4542.5
	Student	0.952 (2.25)	0.419 (4.67)	0.633 (10.4)	0.004** (0.06)	1.608 (5.68)		-4406.1
IGARCH	Gauss	0.830 (2.43)	0.286 (4.18)	0.714 (−−)				-4549.8
	Student	1.487 (3.29)	0.395 (6.58)	0.605 (−−)				-4407.2
FIGARCH	Gauss	1.287** (0.69)	0.020** (0.04)	0.446** (0.51)			0.707* (1.67)	-4547.7
	Student	2.985* (1.89)	−0.861 (−31.3)	−0.886 (−33.9)			0.419 (7.57)	*-4399.0*

Notes: nc means "not computed" due to convergence problem. [a] The conditions for positivity and stationarity constraints or the existence of the fourth moment are not observed. * and ** indicate that the null hypothesis is rejected at the 5% and 10% level, respectively. The robust t-ratios appear in parentheses. The best likelihood (LL) appear in bold.