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Recursive preferences, long-run risks, and stock valuation

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Abstract

In this note, we develop a stock valuation model with recursive preferences and long-run risks. The model is based on the Epstein and Zin (1989, 1991) and Weil (1989) recursive utility framework. Our main result indicates that the intrinsic value of a stock is negatively related to (i) the long-run covariance between dividends and aggregate consumption, and (ii) the long-run covariance between dividends and market returns. This theoretical finding suggests that the sensitivity of dividends to market returns and aggregate consumption affects the long-run risk of a firm and its equity value.

1. Introduction

According to Chen et al. (2013, p. 39), a large and growing body of theoretical work in macroeconomics and finance models the preferences of economic agents using a recursive utility function similar to the one explored by Epstein and Zin (1989, 1991) and Weil (1989). One reason for the growing interest in such preferences is that they provide a useful generalization of the standard power utility method. Under the recursive representation, the coefficient of relative risk aversion does not need to equal the inverse of the elasticity intertemporal substitution, as it must in standard power utility models with constant relative risk aversion.

In this note, we develop a stock valuation model with recursive preferences and long-run risks.

As noted by Ferson et al. (2013), the long-run risk model following Bansal and Yaron (2004) and Bansal et al. (2005) has been a phenomenal success. Indeed, Bansal and Yaron (2004) revealed that long-run risks represent a potential resolution of asset pricing puzzles. In addition, Bansal et al. (2005) showed that cash flow betas, obtained from the long-run covariance between dividends and aggregate consumption, account for more than 60% of the cross-sectional variation in risk premia. Moreover, many subsequent studies confirmed the importance of the long-run risk approach. Examples include, Hansan et al. (2008), Da (2009), Bansal and Kiku (2011), Bergeron (2013a, and 2013b), Bansal et al. (2016), and Li and Zhang (2017).

However, none of the above-mentioned studies integrates the long-run concept of risk into the stock valuation process, using recursive preferences.

In this regard, our motivation comes from the following observations: (1) the growing interest in recursive preferences; (2) the success of the long-run risk approach; (3) the numerous studies related to stock valuation;[1] and (4) the absence of a theoretical stock valuation model with recursive utility and long-run risks.

Our main result indicates that the intrinsic value of a stock is negatively related to (*i*) the long-run covariance between dividends and aggregate consumption, and (*ii*) the long-run covariance between dividends and market returns. This theoretical finding suggests that the sensitivity of dividends to market returns and aggregate consumption affects the long-run risk of a firm and its equity value.

2. The model

In this section, we construct our extension model. We first refer to the fundamental equation of asset pricing with stochastic discount factors. Next, we introduce a recursive

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[1] As mentioned by Sharafoddin and Emisia (2016, p. 128), one of the most significant issues in investment management is stock valuation (*Procedia Economics and Finance*).

utility function. Then, we sum over several periods to integrate two long-run risk measures into the valuation process. Our development follows Bergeron (2013a, and 2013b). However, in this paper, we do not assume a restrictive power utility function, a constant relative risk aversion, a time separable utility function or a specific linear dividend multifactor process.

2.1 Long-lived asset prices

At time t ($t = 0, 1, 2, \ldots, T - 1$), for a long-lived asset such as a stock, the *law of one price* stipulates that:[2]

$$P_{it} = E_t \sum_{\tau=1}^{\infty} \tilde{S}_{t+\tau} \widetilde{D}_{i,t+\tau}, \tag{1}$$

where P_{it} is the price of asset i at time t, $\widetilde{D}_{i,t+\tau}$ is the dividend of asset i at time $t + \tau$, and $\tilde{S}_{t+\tau}$ is the stochastic discount factor (SDF) between time t and $t + \tau$ ($\tau = 1, 2, 3, \ldots, \infty$).[3] Since the dividend value of asset i at time t, D_{it}, is known, we can write:

$$P_{it} = D_{it} E_t \sum_{\tau=1}^{\infty} \tilde{S}_{t+\tau} \widetilde{D}_{i,t+\tau} / D_{it} = D_{it} E_t[\tilde{F}_{it}], \tag{2}$$

where the stochastic variable $\tilde{F}_{it}$ represents the sum of the discounted dividend growth ($\tilde{F}_{it} \equiv \sum_{\tau=1}^{\infty} \tilde{S}_{t+\tau} \widetilde{D}_{i,t+\tau} / D_{it}$). To simplify the price valuation, we assume, as in Bergeron (2013a, and 2013b), that the sequence of variables $\tilde{F}_{it}$ is independent and identically distributed (*i.i.d.*). More precisely, we assume that $\tilde{F}_{it} = \tilde{F}_i$ (for $t = 0, 1, 2, \ldots, T - 1$). This assumption is similar to the classic assumption of a stationary process for the price-dividend ratio.[4] Thereby, we can establish that:

$$P_{it} = D_{it} E_t[\tilde{F}_i], \tag{3}$$

or, taking the expectation on each side of equation (3), that;

$$P_{it} = D_{it} E[\tilde{F}_i] = D_{it} \phi_i, \tag{4}$$

with $E[\tilde{F}_i] \equiv \phi_i$. Given the available information at time t, we also have:

$$\tilde{P}_{i,t+1} = \widetilde{D}_{i,t+1} \phi_i, \tag{5}$$

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where $\tilde{P}_{i,t+1}$ represents the random price of asset i at time $t + 1$.

This last relationship simply reveals that prices and dividends are stochastically related, as in Rubinstein (1976, p. 410).

2.2 Recursive preferences and asset pricing

Following Restoy and Weil (2011), we consider a standard asset pricing model with a representative agent and recursive preferences as in Epstein and Zin (1989) and Weil (1989).[5] Indirect utility at time t, V_t, is given recursively as:

$$V_t = \left\{ (1 - \delta) C_t^{1-\rho} + \delta (E_t[\tilde{V}_{t+1}])^{1/\theta} \right\}^\theta, \tag{6}$$

where C_t is consumption at time t, $0 < \delta < 1$ is the time discount factor, $\rho > 0$ is the elasticity parameter of intertemporal substitution, $\tilde{V}_{t+1}$ is the indirect utility at time $t + 1$, and θ is such that $\theta \equiv (1 - \gamma)/(1 - \rho)$, where $\gamma > 0$ denotes the relative risk aversion coefficient. Using the above preferences, Epstein and Zin (1989) demonstrate that the return of asset i, $\tilde{R}_{i,t+1}$, must satisfy the following agent's Euler equation:

$$1 = E_t \left[\delta^\theta \tilde{G}_{t+1}^{-\rho\theta} \tilde{R}_{w,t+1}^{\theta-1} \tilde{R}_{i,t+1} \right], \tag{7}$$

where $\tilde{G}_{t+1}$ represents consumption growth, and $\tilde{R}_{w,t+1}$ represents the return on wealth, between time t and $t + 1$. More precisely, we have:

$$\tilde{R}_{i,t+1} \equiv \frac{\tilde{P}_{i,t+1} + \tilde{D}_{i,t+1}}{P_{it}} \ , \ \ \tilde{G}_{t+1} \equiv \frac{\tilde{C}_{t+1}}{C_t}, \ \ \tilde{R}_{w,t+1} \equiv \frac{\tilde{W}_{t+1}}{W_t - C_t},$$

where $\tilde{C}_{t+1}$ denotes consumption at time $t + 1$, W_t represents the wealth level at time t, and $\tilde{W}_{t+1}$ is the wealth value at time $t + 1$. In accordance with Epstein and Zin (1991), the return on wealth corresponds to the return on the *market portfolio* ($\tilde{R}_{m,t+1}$), which includes many types of assets. In this manner, equation (7) indicates that:

$$1 = E_t \left[\delta^\theta \tilde{G}_{t+1}^{-\rho\theta} \tilde{R}_{m,t+1}^{\theta-1} \tilde{R}_{i,t+1} \right], \tag{8}$$

where the SDF between time t and $t + 1$ ($\tilde{S}_{t+1}$) now corresponds to the following value: $\tilde{S}_{t+1} = \delta^\theta \tilde{G}_{t+1}^{-\rho\theta} \tilde{R}_{m,t+1}^{\theta-1}$.

Taking the expectation on each side of equation (8) allows us to release index t of the conditional operator, to exhibit:

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[5] See also Chung (2012), or Campbell et al. (2018).

$$1 = E\left[\delta^\theta \tilde{G}_{t+1}^{-\rho\theta} \tilde{R}_{m,t+1}^{\theta-1} \tilde{R}_{i,t+1}\right]. \tag{9}$$

If consumption and returns are jointly lognormal, and if we accept the existence of a safe one-period bond with a return equal to $R_{f,t+1}$, we can obtain, after log-linearizing, the usual expression for the approximate return on any asset, that is:[6]

$$E\left[\tilde{r}_{i,t+1}\right] - r_{f,t+1}$$
$$= \rho\theta COV\left[\tilde{g}_{t+1}, \tilde{r}_{i,t+1}\right] + (1-\theta)COV\left[\tilde{r}_{m,t+1}, \tilde{r}_{i,t+1}\right] - V\left[\tilde{r}_{i,t+1}\right]/2, \tag{10}$$

where lowercase letters denote the logarithm of their uppercase counterpart, namely:

$$\tilde{r}_{i,t+1} \equiv \ln(\tilde{R}_{i,t+1}), \ r_{f,t+1} \equiv \ln(r_{f,t+1}), \ \tilde{g}_{t+1} \equiv \ln(\tilde{G}_{t+1}), \ \tilde{r}_{m,t+1} \equiv \ln(\tilde{R}_{m,t+1}).$$

As mentioned by Restoy and Weil (2011), this relationship is often interpreted as a combination of the Capital Asset Pricing Model (CAPM) and the Consumption-CAPM, since each asset's risk premium must equal a weighted average of the asset's covariance with consumption growth (as in the Consumption-CAPM) and market return (as in the CAPM).

Introducing equations (4) and (5) into the definition of asset returns, indicates, after simple manipulations, that:

$$\tilde{R}_{i,t+1} = \frac{\phi\tilde{D}_{i,t+1} + \tilde{D}_{i,t+1}}{\phi D_{i,t}} = (1 + 1/\phi_i)\tilde{G}_{i,t+1},$$

where $\tilde{G}_{i,t+1}$ represents the dividend growth value of asset i, between time t and $t+1$, ($\tilde{G}_{i,t+1} \equiv \tilde{D}_{i,t+1}/D_{it}$). Taking the logarithm of returns shows that:

$$\tilde{r}_{i,t+1} = \ln(1 + 1/\phi_i) + \tilde{g}_{i,t+1},$$

with $\tilde{g}_{i,t+1} \equiv \ln(\tilde{G}_{i,t+1})$. Thus, introducing equations (4) and (5) into equation (10), and using basic covariance or variance properties, allows us to write, after simplification:

$$E\left[\ln(1 + 1/\phi_i) + \tilde{g}_{i,t+1}\right] - r_{f,t+1}$$
$$= \rho\theta COV\left[\tilde{g}_{t+1}, \tilde{g}_{i,t+1}\right] + (1-\theta)COV\left[\tilde{r}_{m,t+1}, \tilde{g}_{i,t+1}\right] - V\left[\tilde{g}_{i,t+1}\right]/2. \tag{11}$$

Equation (11) identifies an equilibrium asset pricing condition in which the only future random variable of a particular asset corresponds to dividend growth.

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[6] See, for example, Restoy and Weil (2011, p. 4).

Multiplying by $V[\tilde{g}_{t+1}]$ and $V[\tilde{r}_{m,t+1}]$ on each side of equation (11) indicates, after manipulations, that:

$$E\big[\ln(1 + 1/\phi_i) + \tilde{g}_{i,t+1}\big] - r_{f,t+1} = \lambda_{ct}b_{cit} + \lambda_{mt}b_{mit} - V[\tilde{g}_{i,t+1}]/2, \tag{12}$$

where

$$\lambda_{ct} \equiv \rho\theta V[\tilde{g}_{t+1}] > 0,$$

$$\lambda_{mt} \equiv (1 - \theta)V[\tilde{r}_{m,t+1}] > 0,$$

$$b_{cit} \equiv COV[\tilde{g}_{t+1}, \tilde{g}_{i,t+1}]/V[\tilde{g}_{t+1}],$$

$$b_{mit} \equiv COV[\tilde{r}_{m,t+1}, \tilde{g}_{i,t+1}]/V[\tilde{r}_{m,t+1}].$$

Here, resulting coefficient b_{cit} represents the dividend sensitivity to aggregate consumption, for asset i at time t, while coefficient b_{mit} represents the dividend sensitivity to market returns, for asset i at time t. These coefficients are defined for a specific period only. In the next subsection we define the corresponding coefficients (b_{ci} and b_{mi}) over many periods.

2.3 Market and consumption risks over the long-run

In the long run, the upper equilibrium condition can be easily extended by summing from time zero ($t = 0$) to time $T - 1$ ($t = T - 1$), as shown below:

$$\sum_{t=0}^{T-1} E\big[\ln(1 + 1/\phi_i) + \tilde{g}_{i,t+1}\big]$$

$$= \sum_{t=0}^{T-1} r_{f,t+1} + \sum_{t=0}^{T-1} \lambda_{ct}b_{cit} + \sum_{t=0}^{T-1} \lambda_{mt}b_{mit} - \sum_{t=0}^{T-1} V[\tilde{g}_{i,t+1}]/2. \tag{13}$$

Rearranging equation (13), we can write:

$$T \; x \; \ln(1 + 1/\phi_i)$$

$$= \sum_{t=0}^{T-1} r_{f,t+1} + \sum_{t=0}^{T-1} \lambda_{ct}b_{cit} + \sum_{t=0}^{T-1} \lambda_{mt}b_{mit} - \frac{1}{2}\sum_{t=0}^{T-1} V[\tilde{g}_{i,t+1}] - \sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}]. \tag{14}$$

Multiplying by scalar values $\sum_{t=0}^{T-1} \lambda_{ct}$ and $\sum_{t=0}^{T-1} \lambda_{mt}$ on each side of equation (14), implies that:

$$T \times \ln(1 + 1/\phi_i) = \sum_{t=0}^{T-1} r_{f,t+1} + \sum_{t=0}^{T-1} \lambda_{ct} \sum_{t=0}^{T-1} \omega_{ct} b_{cit} + \sum_{t=0}^{T-1} \lambda_{mt} \sum_{t=0}^{T-1} \omega_{mt} b_{cit}$$

$$-\frac{1}{2} \sum_{t=0}^{T-1} V[\tilde{g}_{i,t+1}] - \sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}], \tag{15}$$

where $w_{ct} \equiv \lambda_{ct} / \sum_{t=0}^{T-1} \lambda_{ct}$, with $0 < w_{ct} < 1$, and where $w_{mt} \equiv \lambda_{mt} / \sum_{t=0}^{T-1} \lambda_{mt}$, with $0 < w_{mt} < 1$. Dividing by T on each side of equation (15), we get:

$$\ln(1 + 1/\phi_i) = \frac{1}{T} \sum_{t=0}^{T-1} r_{f,t+1} + \frac{1}{T} \sum_{t=0}^{T-1} \lambda_{ct} \sum_{t=0}^{T-1} \omega_{ct} b_{cit} + \frac{1}{T} \sum_{t=0}^{T-1} \lambda_{mt} \sum_{t=0}^{T-1} \omega_{mt} b_{cit}$$

$$-\frac{1}{2}\frac{1}{T} \sum_{t=0}^{T-1} V[\tilde{g}_{i,t+1}] - \frac{1}{T} \sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}], \tag{16}$$

or, equivalently:

$$\ln(1 + 1/\phi_i) = r_f + \lambda_c b_{ci} + \lambda_m b_{mi} - 0.5\sigma_i^2 - g_i \tag{17}$$

where:

$$g_i \equiv \sum_{t=0}^{T-1} E[\tilde{g}_{i,t+1}] / T, \ \ r_f \equiv \sum_{t=0}^{T-1} r_{f,t+1} / T, \ \ \sigma_i^2 \equiv \sum_{t=0}^{T-1} V[\tilde{g}_{i,t+1}] / T,$$

$$\lambda_c \equiv \sum_{t=0}^{T-1} \lambda_{ct} / T, \ \ \lambda_m \equiv \sum_{t=0}^{T-1} \lambda_{mt} / T,$$

$$b_{ci} \equiv \sum_{t=0}^{T-1} \omega_{ct} b_{cit}, \ \ b_{mi} \equiv \sum_{t=0}^{T-1} \omega_{mt} b_{mit}.$$

Estimators g_i, r_f, σ_i^2, λ_c, and λ_m correspond to arithmetic averages, while b_{ci}, and b_{mi} correspond to weighted averages (over T periods). In particular, b_{ci} and b_{mi} are viewed as the *long-run* sensitivity of the asset's dividends to aggregate consumption and market returns. Taking the exponential on each side of equation (17), we have:

$$1/\phi_i = exp\{r_f + \lambda_c b_{ci} + \lambda_m b_{mi} - 0.5\sigma_i^2 - g_i\} - 1. \tag{18}$$

At time $t = 0$, equation (4) indicates that: $\phi_i = P_{i0}/D_{i0}$. Integrating equation (4) into equation (18) allows us to write:

$$D_{i0}/P_{i0} = exp\{r_f + \lambda_c b_{ci} + \lambda_m b_{mi} - 0.5\sigma_i^2 - g_i\} - 1. \tag{19}$$

Therefore, we can easily isolate the price of a long-lived asset, as is shown below:

$$P_{i0} = \frac{D_{i0}}{exp\{r_f + \lambda_c b_{ci} + \lambda_m b_{mi} - 0.5\sigma_i^2 - g_i\} - 1}. \tag{20}$$

Equation (20) represents our main result. Using recursive utility, this equation indicates that the equilibrium price of a stock is negatively related to the long-run sensitivity of the asset's dividends to market returns, as well as to aggregate consumption. Given that the two sensitive parameters (b_{ci} and b_{mi}) are negatively related to the theoretical asset value, they thus represent a rightful measure of risks. To estimate these two measures, investors must make a prediction for each period, and then compute the average. If the covariance between dividends and market returns is equal to zero ($b_{mi} = 0$), then the corresponding stock value is similar to the one obtained in Bergeron (2013a). Under conditions of certainty, variances and covariances equal zero, and equation (20) is similar to the classic *Gordon Growth Model*.

Note that the derivation of equation (20) does not require the restrictive constant growth assumption for dividends. This approach is consistent with the familiar notion that firms have different stages of growth and risk, in the long run (see, for example, Mac an Bhaird and Lucey, 2011). This approach is also consistent with the long-run measure of risk adopted by Bansal et al. (2005), for which the covariance between dividend growth and aggregate consumption varies over time.

In sum, our model indicates that the intrinsic current value of a stock can be easily estimated with a practical formula that integrates two dimensions of risk: market risk and consumption risk, over the long run.

3. Conclusion

The recursive utility framework and long-run risk concept represent two major innovations in financial economics. In this paper, we integrate the long-run risk concept into the stock valuation process, using recursive preferences. Our model reveals that stock values are influenced by the long-run covariance between dividends and market returns, as well as by the long-run covariance between dividends and consumption. The main contributions of this paper can be summarized as follows. First, we combine recursive preferences and long-run risk for stock valuation. Second, we suggest that long-run risk is defined by two factors: market returns and aggregate consumption. Third, we demonstrate that two long-run risk parameters affect the market equity value of a firm.

Our extension model also presents other interesting characteristics. For example, our development indicates the value of the dividend-price ratio under equilibrium conditions. Moreover, the final formula is easy to apply for common stock investors. Furthermore, the model does not assume a constant dividend growth rate, which allows us to refine the valuation process (period by period). Likewise, the model offers new predictions to test the validity of the long-run concept of risk empirically.

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