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The Brazilian granular business cycle

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Abstract

We investigate whether the granular hypothesis holds for the Brazilian business cycle and find that idiosyncratic shocks to net revenues of the top 100 companies explain about one-third of GDP fluctuations for annual data. Quarterly data cannot dismiss the granular hypothesis either. However, the granular hypothesis seems to break down after the 2008 financial crisis.

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1. Introduction

Empirical research over the last decade has revived the hypothesis that idiosyncratic firm-level shocks matter for aggregate business cycles (see Miranda-Pinto and Shen (2019) and references therein for a full account). This “granular” hypothesis (Gabaix, 2011) contrasts with the more established view that shocks to individual firms diversify away by the law of large numbers and thus have negligible aggregate effects. In line with the granular hypothesis, empirical firm-size distributions documented for several countries are fat-tailed, which means a few firms (the big “grains”) are disproportionately large and, as a result, firm-level shocks do not cancel out.

Two measures are usually considered to investigate the granular hypothesis: 1) the “granular residual” that gauges shocks to the largest companies, and 2) the “granular volatility” that tracks GDP volatility. The “granular size” of an economy is an extra metric of the optimal number of firms to be considered in the calculation of the granular residual.

The granular hypothesis cannot be rejected right away for the Brazilian economy, as the tails of its firm-size distribution follows Zipf’s law (Da Silva et al., 2018). This follow-up study aims to quantify the impact of idiosyncratic shocks to the largest Brazilian firms on GDP fluctuations by measuring the granular residual and volatility as well as the granular size of the Brazilian economy.

2. Data and basic measures

We collect annual data of net revenues for the 500 largest Brazilian companies since 1999 from *Exame* magazine, and also quarterly data for publicly listed companies since 1997 from *Economatica*.

The granular residual is

$$\Gamma_t = \sum_{i=1}^{K} \frac{R_{i,t-1}}{\mathrm{GDP}_{t-1}} \left(g_{it} - \overline{g}_t \right), \tag{1}$$

where $R_{i,t-1}$ is net revenue of firm *i* at time $t-1$, GDP_{t-1} is gross domestic product, and $g_{it} - \overline{g}_t$ captures an idiosyncratic shock to firm *i*. In particular,

$$g_{it} = \ln R_{it} - \ln R_{i,t-1} \tag{2}$$

and

$$\overline{g}_t = Q^{-1} \sum_{i}^{Q} g_{it} \ , \tag{3}$$

for $Q \geq K$.

The impact of firm i is given by its size as gauged by its sales, that is, gross output rather than net output. The weights add up to more than 1, reflecting the fact that a growth rate of 1 percent in a firm generates an increase in the value produced equal to 1 percent multiplied by its sales.

Individual productivity shocks are better measured by a firm’s revenue divided by its number of employees in a year (Gabaix, 2011). Alas, this sort of microdata is not available for all the companies in our sample, and thus we consider firm revenue shocks only, as in Di Giovanni et al. (2014) and others.

One alternative is to compute the granular residual by using the deviation of the sales growth rate from the industry-specific averages, $\bar{g}_{I_i t}$:

$$\bar{\Gamma}_t = \sum_i^K \frac{R_{i,t-1}}{\text{GDP}_{t-1}} (g_{it} - \bar{g}_{I_i t}), \tag{4}$$

and then compute GDP growth as

$$\text{GDP growth}_t = \beta_0 + \beta_i \Gamma_{t-p} + u_t . \tag{5}$$

The explanatory power of equations (1) and (4) are evaluated by adjusted R^2 s, which equal zero if only aggregate shocks matter.

The granular volatility is

$$\sigma_{Gt} = \sqrt{\sum_i^K \left(\frac{R_{it}}{\text{GDP}_t} \right)^2 \sigma_i^2} , \tag{6}$$

where σ_i is firm-level volatility (Carvalho and Gabaix, 2013).

In a framework with endogenous labor supply (see Gabaix (2011) for more details), the relationship between granular volatility and GDP volatility is:

$$\sigma_{Yt} = \mu \cdot \sigma_{Gt} . \tag{7}$$

The productivity multiplier is

$$\mu = \frac{1+\phi}{\alpha} , \tag{8}$$

where α is labor share, and ϕ is Frisch elasticity of labor supply.

3. Results

Figure 1 displays the sum of net revenues of the 50 and 100 largest Brazilian firms, as a proportion of GDP, from 1999 to 2018. On average, revenues of the top 50 companies make up 18 percent of GDP, while revenues of the top 100 companies represent 24 percent. In comparison, the average sales of the top 100 companies in the U.S. and E.U. economies represent about 29 percent of GDP (Gabaix, 2011; Ebeke and Eklou, 2017).

After gauging firm concentration with the Herfindahl index (Gabaix, 2011, p. 738)

$$h = \sqrt{\sum_{i=1}^N \left(\frac{R_{it}}{\text{GDP}_t} \right)^2} , \tag{9}$$

we find h = 4.3 percent, on average, for the 1999-2018 period. Compared with the U.S. economy, where h = 5.3 percent in 2008 (Gabaix, 2011) and Spain's, where h = 4.8 percent for 1995-2016 (Blanco-Arroyo et al., 2018), the Brazilian economy has a relatively greater number of small firms.

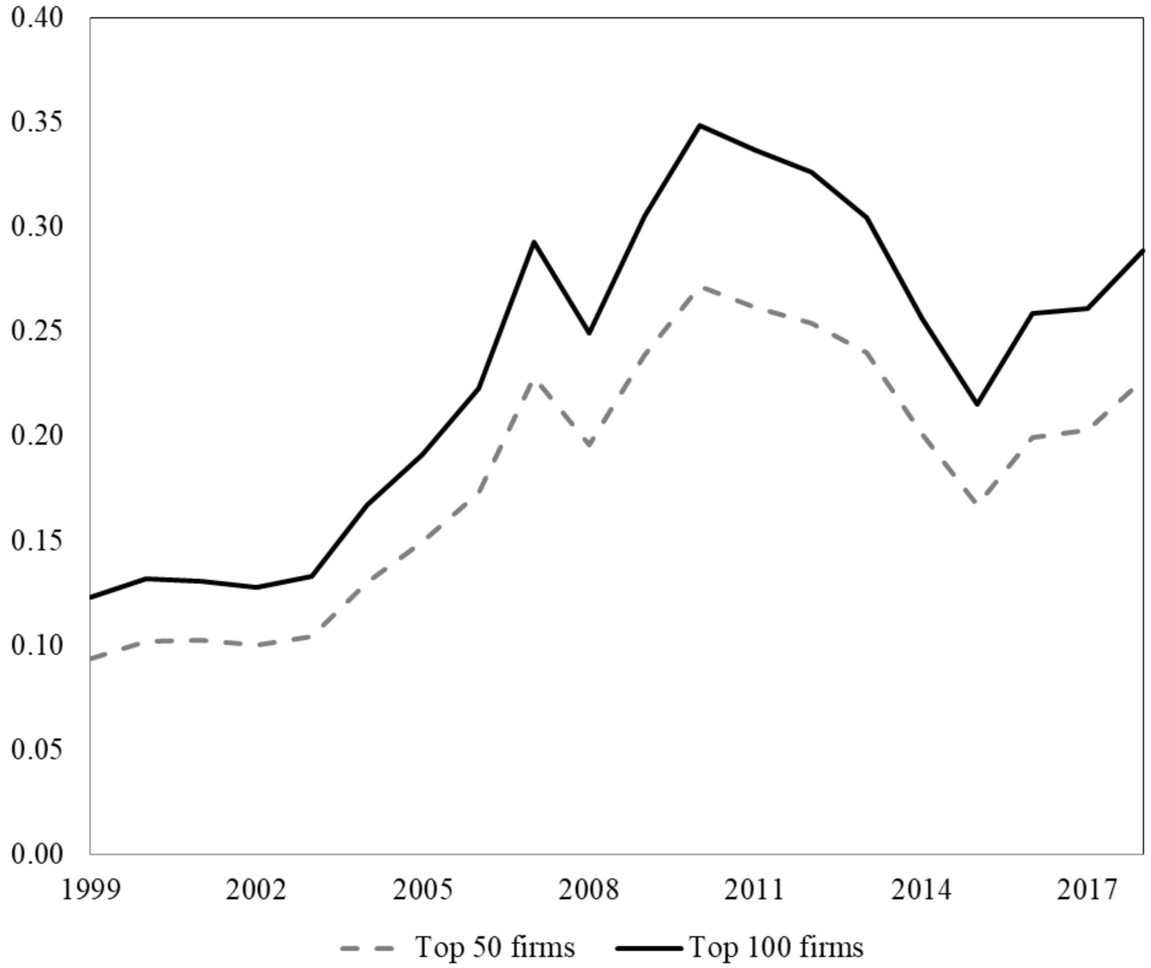

Figure 1. Sum of the net revenues of the top 50 and 100 Brazilian firms, as a percentage of GDP, 1999-2018

We first check whether the Brazilian economy is granular by assessing the explanatory power of the granular residual for the top 100 firms. Table 1 presents linear regressions of annual GDP growth on equation (1) for K = 100. Adjusted R^2s are reasonably high, at 32.9 percent (for one lag and Q = 100) and at 36.3 percent for Q = 500. If only aggregate shocks did matter, the adjusted R^2s would be zero. So our findings show a good explanatory power of the granular residual and are in line with the international evidence (Gabaix (2011) for the U.S.; Ebeke and Eklou (2017) for the E.U.; and Fornaro and Luomaranta (2018) for Finland). Thus, idiosyncratic shocks to net revenues of the 100 largest Brazilian firms explain about one-third of GDP fluctuations.

A positive correlation between granular residual and GDP growth can still reflect the fact that aggregate shocks drive both aggregate GDP and firm revenues. To rule out this reverse causality, we calculate the sample correlation of the net revenue growth rates of the 100 largest firms for each year t:

$$\rho_t = \frac{\frac{1}{K(K-1)}\sum_{i \neq j} g_{it} g_{jt}}{\frac{1}{K}\sum_i g_{it}^2}. \tag{10}$$

We find $\rho_t = 0.065$, which is significantly small, and therefore most variation in firm revenues is likely to be idiosyncratic.

Table 1. Explanatory power of the granular residual

	Q = 100		Q = 500
	(1)	(2)	(3)
Γ_t	3.485***	5.664***	2.864**
	(0.743)	(0.688)	(0.512)
Γ_{t-1}		3.017***	
		(1.109)	
Intercept	0.017***		0.045***
	(0.005)		(0.004)
N	19	18	19
R^2	0.229	0.369	0.398
Adj. R^2	0.183	0.329	0.363

For the years t = 2000 to 2018, GDP growth is regressed on the granular residual Γ_t for Q = 100 and Q = 500 firms using equation (1). Firms are the largest by net revenues from the previous year. Standard errors are in parentheses.
**$p < 0.05$.
***$p < 0.01$.

An industry-demeaning model could provide a more appropriate estimate of the granular residual, and so we re-estimate the linear regression using equation (4) as an explanatory variable. Table 2 presents similar results to the ones reported in Table 1, though the adjusted R^2 is lower than the ones in Table 1, as in Fornaro and Luomaranta (2018), but not like in Gabaix (2011).

Table 2. Explanatory power of the granular residual, industry demeaned

	GDP growth$_t$
$\bar{\Gamma}_t$	2.146***
	(0.715)
Intercept	0.038***
	(0.006)
N	19
R^2	0.236
Adj. R^2	0.191

For the years t = 2000 to 2018, GDP growth is regressed on the granular residual $\bar{\Gamma}_t$ for the top 100 firms using equation (4). Firms are the largest by net revenues from the previous year. Standard errors are in parentheses.
***$p < 0.01$.

If Brazilian business cycles come largely from microeconomic shocks, then granular volatility should track aggregate volatility. Because the top 100 firms are very large, most of the variation in σ_{Yt} is governed by them, as in Carvalho and Gabaix (2013). Thus, we compute the granular volatility in equation (6) and compare it with estimates of aggregate volatility.

Estimating σ_i on a firm-by-firm basis generates unstable values, and so we decide to take the constant value of 5.36 percent. This value is obtained by first taking the cross-sectional variance of growth rates for K = 100:

$$\sigma_t^2 = K^{-1}\sum_i^K g_{it}^2 - \left(K^{-1}\sum_k^K g_{it} \right)^2, \tag{11}$$

and then computing the average standard deviation

$$\bar{\sigma} = \left(K^{-1}\sum_i^K \sigma_t^2 \right)^{\frac{1}{2}}. \tag{12}$$

We pick from the Penn World Table the labor share $\alpha = 0.5422$ that is the average of the 2000-2014 period. We also consider $\phi = 0.25$ for Frisch elasticity of labor supply, a value usually employed in studies of the Brazilian economy. For these values, the Brazilian productivity multiplier ends up as $\mu = 2.3$. The granular volatility is calculated for this value and also alternatively using Carvalho and Gabaix's $\mu = 4.5$.

The baseline estimate of aggregate volatility is calculated using the trend deviations obtained from a Hodrick-Prescott filter of the quarterly real GDP, 1995:q1 to 2018:q4 (smoothing parameter = 1,600). The standard deviation for a quarter q is computed after taking a continuous 16-quarter window centered on the quarter of interest. To construct the volatility of a given year t, the average is calculated over the four quarters of that year. Because of this rolling window, aggregate volatility can only be obtained for 1999-2016.

Figure 2 plots the evolution of the Brazilian granular volatility for the two demeaned values of σ_{Yt}. The granular volatility seems to track the swings in GDP volatility.

The contribution of the granular residual to GDP fluctuations can be under- or overestimated if we do not optimally calibrate the number of granular firms K^* in equation (5) (Blanco-Arroyo et al., 2018). Thus, we investigate how the explanatory power of the granular residual behaves when we gradually increase K in equation (1), considering the range $1 \le K \le Q = 300$. Figure 3 shows the evolution of R^2. The "granular curve" shows that R^2 initially increases, wobbles and then stabilizes.

An "equal-weight benchmark curve" is also devised by replacing the empirical weights in equation (1) with constant weights for all firms, 300 in total, while keeping unchanged the corresponding idiosyncratic shocks. This benchmark quantifies the (negligible) contribution of the granular residual to GDP fluctuations from equal-size firms (Figure 3).

Figure 3 shows the transition from the granular curve to the equal-weight benchmark curve, when we progressively remove the L largest firms in Γ_t. In particular, curve $R^2(K,190)$ almost collapses to the equal-weight benchmark curve, indicating that the remaining heterogeneity across the firms has a negligible impact on aggregate fluctuations.

Then, we evaluate the sensitivity of the $R^2(K,L)$ curves to the gradual exclusion of large firms (increasing values of L). Figure 4 plots the average cumulative explanatory power as a function of L:

$$C(L) = \tfrac{1}{Q}\sum_{K=1}^{Q} R^2(K,L). \tag{13}$$

The $C(L)$ curve intersects the curve of average cumulative explanatory power of the equal-weight benchmark at point 4, indicating that the granular size of the Brazilian economy is $K^* \approx 130$ firms.

The timespan of our annual time series is arguably short. This problem plagues other studies too, such as Ebeke and Eklou (2017) ($N = 14$) and Blanco-Arroyo et al. (2018) ($N = 22$). To prevent small sample problems, we consider the quarterly data to expand our annual series from $N = 19$ to $N = 87$ because micro level shocks can have large effects in the short run, but their impacts on aggregate fluctuations are attenuated when considering data at lower frequencies (Fornaro and Luomaranta, 2018). For example, a strike occurring in one company in a period can have a substantial effect on aggregate output for that period, but its impact can vanish due to time aggregation when we consider the whole year. One advantage of using quarterly data is to assess the granular hypothesis for different subsamples and to spot any events that can possibly affect it. This makes possible the analysis presented in Section 4.

Table 3 shows the explanatory power of the granular residual for the quarterly data only to reinforce the conclusion that idiosyncratic movements from the top 100 firms can explain a significant chunk of GDP fluctuations, that is, 21.3 percent. Compared with the annual data (Table 1), the explanatory power of the granular residual is weakened, however.

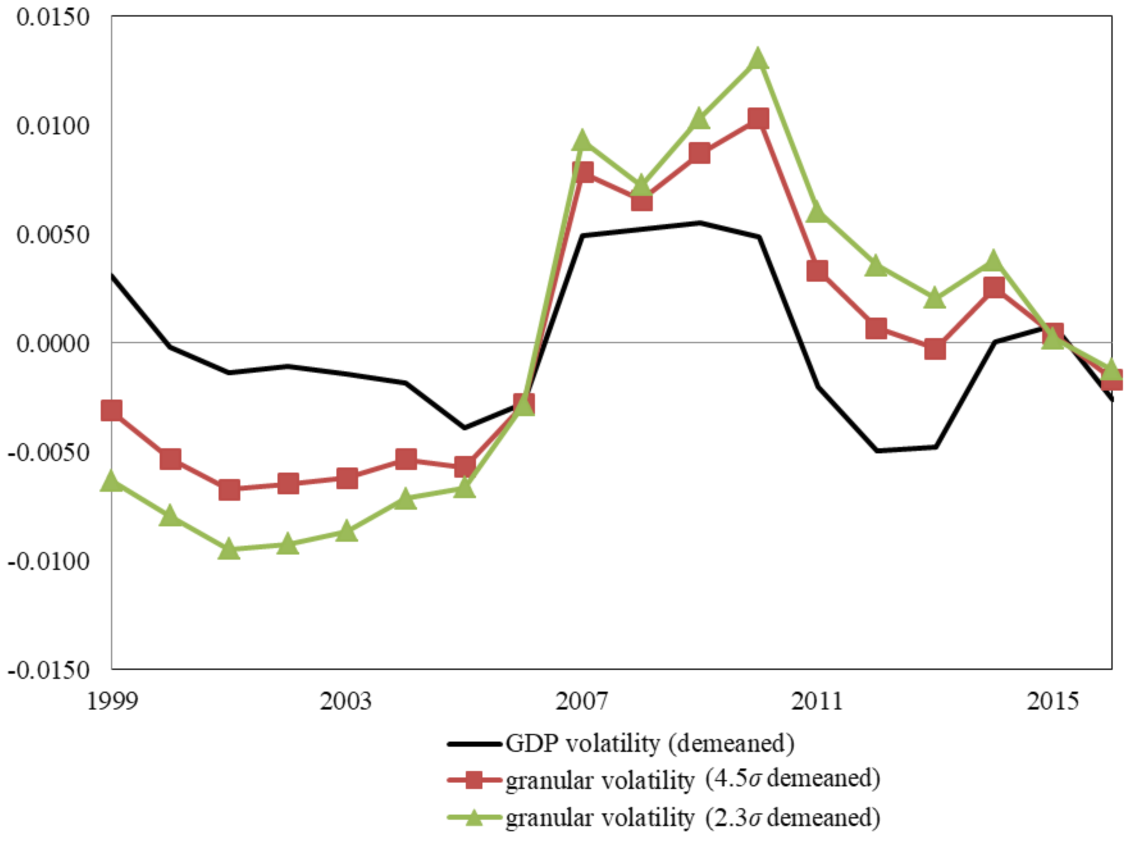

Figure 2. Brazilian GDP and granular volatilities, 1999-2016

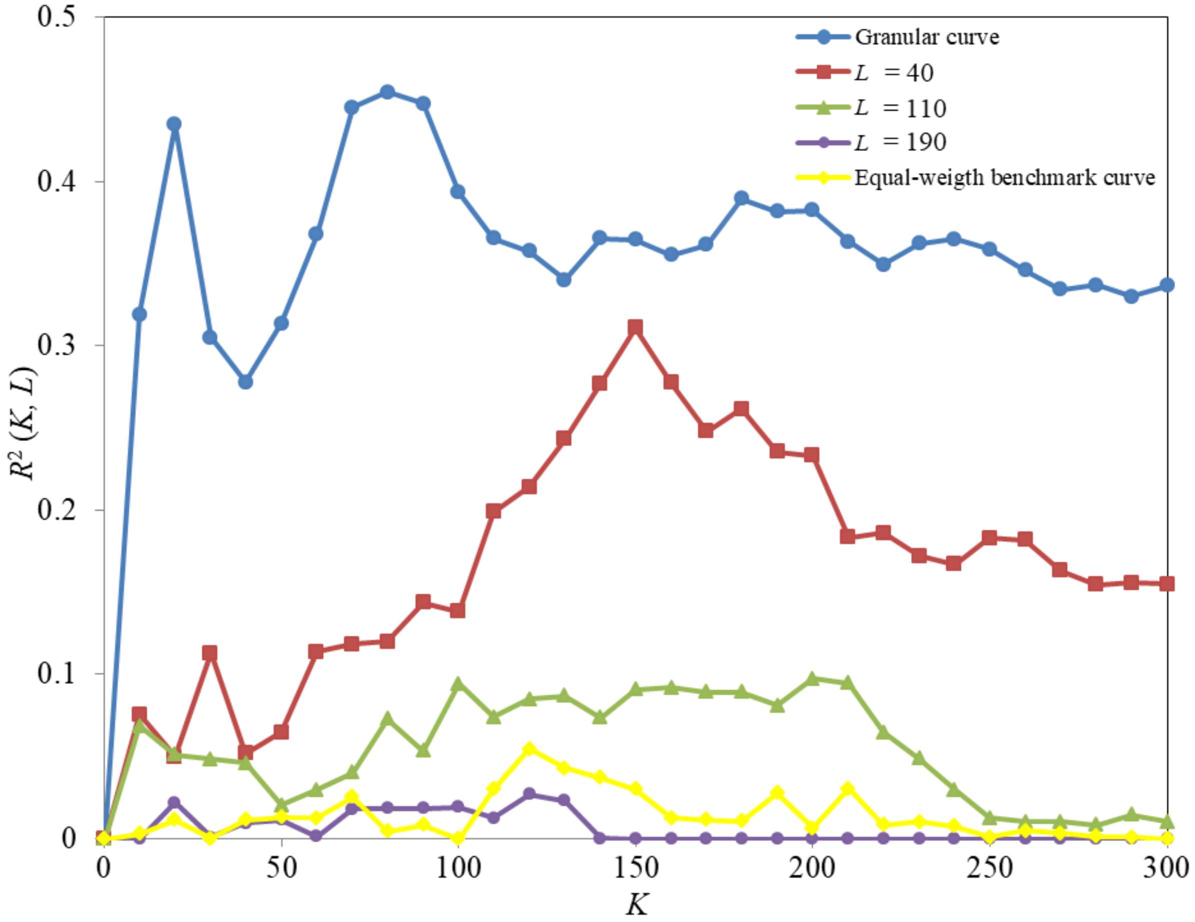

Figure 3. Explanatory power of the regression in equation (5) as a function of increasing K and different L in the computation of the granular residual. The incremental step is $\Delta K = 10$

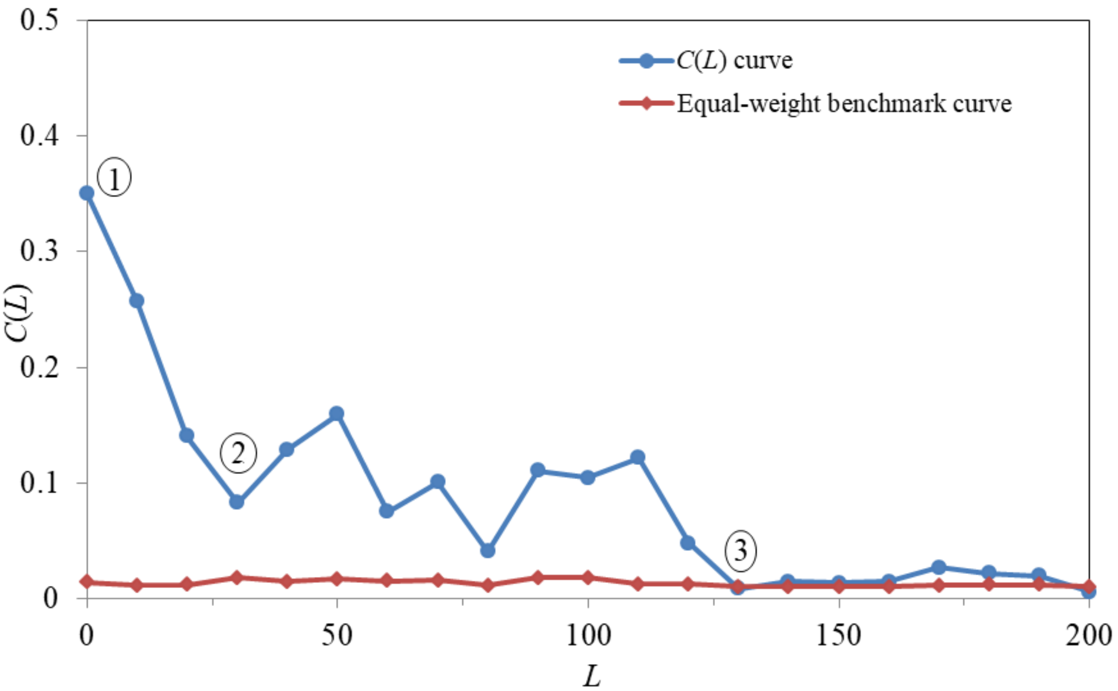

Figure 4. Evolution of the $C(L)$ curve and the equal-weight benchmark curve

Table 3. Quarterly data for the top 100 firms: Explanatory power of the granular residual

	GDP growth$_t$
Γ_t	0.806***
	(0.164)
Intercept	0.015***
	(0.004)
N	87
R^2	0.222
Adj. R^2	0.213

From 1997:q2 to 2018:q4, GDP growth is regressed on the granular residual Γ_t for the top 100 firms using equation (1). Firms are the largest by net revenues from the previous quarter. Standard errors are in parentheses. ***$p < 0.01$.

Table 4. Quarterly data for the top 100 firms: Explanatory power of the granular residual, industry demeaned

	GDP growth$_t$
$\bar{\Gamma}_t$	0.418***
	(0.133)
Intercept	0.013***
	(0.004)
N	87
R^2	0.104
Adj. R^2	0.093

From 1997:q2 to 2018:q4, GDP growth is regressed on the granular residual $\bar{\Gamma}_t$ for the top 100 firms using equation (4). Firms are the largest by net revenues from the previous quarter. Standard errors are in parentheses. ***$p < 0.01$.

4. Time-varying contribution of the granular residual

Prior to the 2008 U.S. financial crisis, the Brazilian GDP presented consistent growth. Then, GDP plummeted in 2009 but showed a massive recovery in 2010. This was followed by sluggish growth, a deep recession in 2015-2016, and subsequent slow growth until 2018. Thus, we should consider the hypothesis that 2008 is a turning point for the largest Brazilian firms, as seems to have occurred elsewhere (Fornaro and Luomaranta, 2018).

To check this, we take two subsamples of the quarterly data: 1997:q2 to 2007:q4 and 2008:q1 to 2018:q4. (Results do not change a great deal had we split the sample at 2008:q4 to 2018:q4, which was in line with the date of the Lehman Brothers collapse.) To filter out the effects of the 2015-2016 Brazilian recession, we also consider a subsample for the period 1997:q2 to 2014:q4. Tables 5 and 6 suggest the granular hypothesis breaks down after the 2008 crisis. While the granular residual still accounts for a large portion of GDP changes until 2008—the adjusted R^2s are consistently around 0.5 and the coefficients associated with Γ_t are highly significant and positive—its explanatory power is greatly diminished thereafter. This phenomenon also seems to have occurred elsewhere (Fornaro and Luomaranta, 2018). Apparently for the post-2008 period, the role played by aggregate demand in aggregate business cycle fluctuations came to the fore, along the lines suggested by Dosi et al. (2019). Finally, the 2015-2016 domestic recession also adds to weaken the explanatory power. (In our Supplemental Material, Table S6, we show that our results are robust to controlling for commodities and exchange rate shocks as well as monetary and fiscal shocks. This is available at https://doi.org/10.6084/m9.figshare.11823792.v1.)

Table 5. Explanatory power of the quarterly granular residual for three subsamples

	GDP growth$_t$		
	Pre-2008	Pre-2015	Post-2008
Γ_t	1.064***	0.899***	-0.397
	(0.138)	(0.164)	(0.584)
Intercept	0.028***	0.020***	0.005
	(0.004)	(0.004)	(0.005)
N	43	72	44
R^2	0.589	0.303	0.011
Adj. R^2	0.579	0.293	-0.012

From 1997:q2 to 2018:q4, GDP growth is regressed on the granular residual Γ_t for the top 100 firms using equation (1). Firms are the largest by net revenues from the previous quarter. Standard errors are in parentheses. ***$p < 0.01$.

Table 6. Explanatory power of the quarterly granular residual (industry demeaned) for three subsamples

	GDP growth$_t$		
	Pre-2008	Pre-2015	Post-2008
$\bar{\Gamma}_t$	0.848***	0.563***	-0.318
	(0.136)	(0.138)	(0.242)
Intercept	0.027***	0.018***	0.006
	(0.004)	(0.005)	(0.006)
N	43	72	44
R^2	0.486	0.193	0.039
Adj. R^2	0.474	0.182	0.016

From 1997:q2 to 2018:q4, GDP growth is regressed on the granular residual $\bar{\Gamma}_t$ for the top 100 firms using equation (4). Firms are the largest by net revenues from the previous quarter. Standard errors are in parentheses. ***$p < 0.01$.

5. Conclusion

In line with most international evidence, our findings suggest we cannot dismiss that the Brazilian business cycle is granular. The explanatory power of the granular residual indicates that idiosyncratic shocks to net revenues of the 100 largest firms explain about one-third of GDP fluctuations for annual data. Similar findings also hold at the industry-level. Moreover, the granular volatility seems to track the swings in GDP volatility, and the granular size of the Brazilian economy is defined by its 130 largest firms. Quarterly data cannot dismiss the granular hypothesis either. However, the granular hypothesis seems to break down after the 2008 financial crisis.

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