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Strategic Issues in College Admissions with Early Decision

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Abstract

In this paper, we consider college admissions with early decision (ED) using a many-to-one matching model with two periods. As in reality, each student commits to only one college in the ED period and agrees to enroll if admitted. Under responsive and consistent preferences for both colleges and students, we show that there exists no stable matching system, consisting of ED and regular decision (RD) matching rules, which is nonmanipulable via ED quotas by colleges or ED preferences by colleges or students. We also show that when colleges or students have common preferences and each student applies early only to the top-ranked college with respect to her RD preference, then no college has a strict incentive to offer a single-choice ED program. On the other hand, if students compromise in the ED market and make early application to colleges that are not top-ranked, then colleges may become better off when they offer ED programs than when they do not.

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1. Introduction

Around the world, countries have adopted a wide variety of centralized and decentralized systems in the admission of high school graduates to colleges. While in the USA, colleges admit incoming students independently via a fully-decentralized system, Turkey employs a fully-centralized placement mechanism, where students submit a limited ranked list of programs and are ranked according to their national test scores (Balinski and Sönmez, 1999). In between these two extremes lies somewhat hybrid systems as in Brazil where mostly public colleges admit via a centralized system (Machado and Szerman, 2016) and as in South Korea where students apply colleges individually, yet the college-specific screening test date is centrally determined (Avery et al., 2014).

The recent advances in matching theory led the way to a burgeoning research that study the strengths and flaws of different assignment systems used in college admissions. The research on centralized systems has focused on the certain desirable characteristics of the mechanisms used such as stability, fairness, efficiency, non-wastefulness, and strategy-proofness (Hafalır et al., 2014; Chen and Kesten, 2017; Yenmez, 2018). On the other hand, in the case of decentralized systems, the focus has been on the screening (Lee, 2009; Kim, 2010; Chen and Kao, 2014; Che and Koh, 2016; Chen et al., 2018; Murra-Anton, 2020), signaling (Avery and Levine, 2010), competitive effects (Avery at al., 2014) and welfare effects (studied by all of the aforementioned studies) of a variety of tools used in college admissions such as early admissions, simultaneous screening and limiting choice.[1]

Although there is no consensus on which mechanism is the most efficient way to select students, albeit scant, a shift from decentralized to centralized system has been observed.[2] In 1952, China switched to a centralized college admission from a decentralized one (Chen and Kesten, 2107). In 2010, Brazil underwent from a decentralized to a partially-centralized one (Machado and Szerman, 2016). In the case of the USA, while acknowledging the need to reform its decentralized college admission, mostly prompted by the unfair treatment of poor students under early admission, Avery et al. (2003) pointed out the importance of financial aid in students' decision as a hindering factor to achieve the transition to a centralized system deservingly. Recently, Yenmez (2018) proposed a centralized clearinghouse system using a matching with contracts framework that accommodates early admissions and handles the financial aid as part of the matching system. While his model improves the existing decentralized system with respect to fairness and unraveling, it does not address the issue of strategy-proofness.

In this paper we address the strategic aspects of early admissions in a centralized college admission problem. In particular, we study the existence of stable and strategy-proof mechanisms. The early admissions programs are used in the USA over the last

[1] Simultaneous screening (Chen and Kao, 2014) and limiting choice problems (Che and Koh, 2016; Chen et al., 2018; Hafalır et al., 2014), which practically limit the students' choice set and prevent multiple application, are similar to early admission programs.

[2] There have been ample incidences of shifts in school choice (Abdülkadiroğlu et al., 2017).

five decades and in South Korea since 1994 (Avery et al. 2014). In the US, there are two types of programs Early Action (EA) and Early Decision (ED), which mainly differ with respect to the commitment expected from students. EA programs are non-binding; students do not have to enroll to EA programs which accept them (and they can submit to an accepting program their decision of enrollment until May 1, the national response date). Because of this lack of commitment, students may in general apply to multiple EA programs, unless the institution they apply has a a Single Choice or Restrictive EA policy (used by Yale, Princeton, and Stanford, for example). ED programs are binding; students are required to enroll if admitted. Therefore, students can apply to only one ED program. Both EA and ED programs usually require high school seniors to apply near November with a decision by late December. Regular Decision (RD) programs offer a later application deadline (January 1) and time to decide whether to matriculate or not until May 1.[3] In South Korea, early admission is implemented as ED only.

The literature on decentralized college admissions offers various arguments to explain why colleges use ED programs and students prefer to apply to it.[4] Students prefer to apply early if the chance to be admitted is higher at ED than RD.[5] Lee (2009) shows that ED results in lower admission standards than in RD and argues that colleges may use ED programs as a screening device to avoid the winner's curse.[6] In a similar vein, according to Kim (2010), a need-blind school uses ED admissions as a screening mechanism to indirectly identify a student's ability-to-pay, while superficially maintaining a need-blind policy. Murra-Anton (2020) shows that a budget-constraint college can use ED along with weaker admission standards for early applicants, to turn the early admissions process into a profitable wealth-screening device. According to Avery and Levine (2010) ED programs enable the student to signal her enthusiasm about a particular college. Avery et al. (2014) show that lower-ranked colleges may gain in competition with higher-ranked colleges by limiting the number of possible applications in ED programs.[7] Che and Koh (2016) conclude that restricting the number of applications as in ED alleviates enrollment uncertainty, but the outcomes are inefficient and unfair. In short, the use of early admissions programs serves the purpose of attracting a better pool of students either by alleviating the informational asymmetry or by avoiding head-on competition.

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[3]Refer to the 2018-2019 Admission Trends Survey of National Association for College Admission Counseling (NACAC), available at http://www.nacacnet.org, for a detailed description of each type of early admissions program.

[4]*The Early Admissions Game* (Avery, et al., 2003) is the seminal empirical study of the effects of early admissions policies in the US.

[5]The 2018-2019 Admission Trends Survey of NACAC reports that between Fall 2017 and Fall 2018, there was an average increase of 11 percent in the number of ED applicants.

[6]Using the data from two liberal arts school, Chapman and Dickert-Conlin (2012) finds the evidence that applying early decision raises the probability of acceptance by 40 percentage points.

[7]According to the 2018-2019 Admission Trends Survey of NACAC, colleges with lower total yield rates tended to admit a greater percentage of their ED applicants compared to those with higher yield rates.

Despite its advantages, colleges have had an unsteady engagement with ED, which can also be seen as a corroborating evidence of the so-called early admission game.[8] In April of 2002, following the announcement of Yale University's president Richard Levine to drop their ED policy, the University of North Carolina at Chapel Hill became the first major selective college to abandon ED admissions. By November 2002, Yale and Stanford switched from ED to EA programs. In 2007-2008, Harvard and Princeton eliminated the early admissions programs entirely. Nevertheless, after 2011, Harvard, Princeton, Stanford and Yale resumed single-choice EA. The gaming via early admissions quotas was also observed in South Korean higher education (Avery et al., 2014). Given these observations, we wonder whether similar policy shifts by colleges would still arise if the college admissions system were centralized. In particular, we ask whether colleges' decisions to open or terminate ED programs can be obtained as (Nash) equilibrium strategies of an early admissions game and to what extent these decisions are affected by the preferences of colleges or students.

In more detail, we aim to study both strategic and stability issues in regard to ED in a centralized college admissions model, and to achieve this we extend the one-period many-to-one matching model of Gale and Shapley (1962) to a two-period model with early and regular decision markets. Our model involves two finite and disjoint sets of individuals, colleges, and students. Each college has a finite capacity that limits the number of students that it can accept in two periods, and each student can enroll to at most one school during the whole matching process.[9] In the RD period, each college has a preference relation over the possible subsets of students and this relation is responsive to its preference over the set of students. On the other hand, each student has a preference relation over the set of colleges and being unmatched. The capacities of colleges together with the preference profiles of colleges and students in the RD period constitute a regular RD market.

In the ED period, each college announces out of its total capacity an ED quota, which it aims to fill with respect to its ED preference ordering.[10] This ordering is responsive to some restriction of its RD preference ordering on a subset of students. On the other side of the market, each student has an ED preference ordering, which restricts her RD preference ordering on a singleton subset of colleges. The quotas of colleges together with the preference profiles of colleges and students in the ED period define an ED market. Clearly, for each RD market, there is a set of induced ED markets.

An allocation in the ED market is a many-to-one ED matching, where no college

[8]Another plausible explanation for the observed instability in the early admission system may be that colleges might have been experimenting in the past to learn the stable market configuration of today where the top colleges use early action and the bottom ones use early decisions (Murra-Anton, 2020).

[9]Many colleges have priority categories for athletes, alumni children, and minorities. We assume that the capacity of each college in our model is net of its priority quota.

[10]Although it is uncommon to announce quotas in a decentralized system –South Korea being the only exception to our knowledge (Avery et al., 2014)– it is essential in a centralized system.

is assigned more students than its ED quota and no student is assigned more than one college. Given a binding ED matching, an allocation in the RD market is a many-to-one RD matching, where all the assignments realized in the ED market are preserved, no college is assigned more students than its overall capacity, and no student is assigned more than one college. We assume that any student rejected from a college in the ED market can still apply to the same college in the RD market.[11]

A matching in the ED market is stable if no student prefers remaining unassigned to her assignment, no college prefers having a student slot vacant rather than filling it with one of its assignments, and there exists no unmatched college-student pair such that the college prefers the student to one of its assignments or keeping a vacant slot (if any) or the student prefers the college to her assignment. Given a matching realized in the ED market, a matching in the RD market is stable if no student having a regular assignment prefers remaining unassigned to her assignment, no college prefers having a regularly assigned student slot vacant rather than filling it with one of its regular assignments, and there exists no unmatched college-student pair such that the college prefers the student to one of its regular assignments or keeping a vacant slot (if any) or the student prefers the college to her regular assignment.

An ED matching rule selects a matching for every ED market, and is stable if it always selects a stable matching. Similarly, a RD matching rule selects a matching for every RD market, given any matching in any ED market induced by the associated RD market. We say that a RD matching rule is stable at an ED matching rule if it always selects a stable matching, given any realization of the ED matching rule applied to any ED market that is induced by the RD market.

An ED matching rule and an RD matching rule as an ordered pair form a matching system. A matching system is stable if it involves a stable ED matching rule at which the RD matching rule within the system is also stable.

We study manipulation of a matching system via ED quotas and preferences, and show that there is no matching system that is stable and nonmanipulable by colleges or students. We also study whether colleges' decision to open/terminate ED programs can arise as an equilibrium of an early admission game where colleges non-cooperatively determine their ED quotas. Our findings show that if students compromise in the ED market and make an early application to colleges that are not top-ranked, then colleges may become better off when they offer ED programs than when they do not.[12] On the other hand, if colleges or students have common preferences and each student applies early only to the top-ranked college with respect to her RD preference, then no college has a strict incentive to offer a single-choice ED program.

Our results on preference manipulation can be related to those in the literature

[11]As pointed out by Avery et al. (2003, pp. 188-189), "...historically most colleges rejected 5 percent or fewer of their early applicants in December. Some, such as Cornell, Georgetown, MIT, and Tufts, have automatically deferred to the regular pool all early applicants who are not admitted in December."

[12]Such compromising behavior by students are confirmed by the experimental results in Chen et al. (2018).

dealing with manipulation of preferences under two-sided matching in a single-period. Roth (1982) shows that there is no stable matching rule which is immune to preference manipulation. Mongell and Roth (1991) report a high percentage of truncated preference profiles (single alternative preference) submitted in sorority rush. Roth and Vande Vate (1991) show that in a decentralized one-to-one matching with random matching process, for any strategies of the other players, each player will always have a truncation strategy as a best response. Roth and Rothblum (1999) introduce the truncation of the true preferences as a potentially profitable strategic behavior, instead of changing the order of true preferences, in a low information environment in one-to-one matchings. Sönmez (1999) shows that there is no stable matching rule in hospital-intern markets which is immune to manipulation via early contracting (unraveling) between a hospital and a single intern.[13]

The paper most related to our study is that by Mumcu and Saglam (2009), studying a similar problem between hospitals and interns, though with some significant differences. Like ours, their model considers two periods of matching, involving a regular market followed by an aftermarket. Although the regular market can be treated as the ED period in our model, students are not restricted to apply to one college (or to any number of colleges for that matter) like in our model with ED. Therefore, the negative result in Mumcu and Saglam (2009) about the nonmanipulability of the matching systems by colleges through their quotas does not imply ours. Moreover, the focus of Mumcu and Saglam (2009) is only manipulation (and strategic games) in quotas, while our paper considers in addition manipulation in preferences.

The organization of the paper is as follows: Section 2 introduces the model. Section 3 gives results on manipulability of matching systems. Finally Section 4 concludes.

2. Model

We consider a college admission problem involving an early decision (ED) market and a regular decision (RD) market. Formally, this problem is denoted by the list (C, S, q, q^E, P^R, P^E) while the pairs (q^E, P^E) and (q, P^R) denote the ED market and the RD market, respectively. The first two components of a college admission problem are non-empty, finite and disjoint sets of colleges C and students S. The third component is a list of positive natural numbers $q = (q_c)_{c\in C}$, where q_c is the total capacity of college c. The fourth component is a list of nonnegative natural numbers $q^E = (q^E_c)_{c\in C}$, where $q^E_c \leq q_c$ denotes the quota of college c in the ED market. We define for all $q \in N^n_+$, the sets $\mathcal{Q}^E_c(q) = \{0, 1, ..., q_c\}$ and $\mathcal{Q}^E(q) = \times_{c\in C}\mathcal{Q}^E_c(q)$. Let $\mathcal{Q}^E = \cup_q \mathcal{Q}^E(q)$. The fifth component of a college admission problem is a list of strict preference relations $P^R = (P^R_i)_{i\in C\cup S}$ where P^R_i denotes the strict preference relation of individual i in the

[13]Unraveling was previously studied by Roth and Xing (1994), showing that the instability of matchings realized at the final date of transactions are neither necessary or sufficient for the unraveling to occur. The two potential causes of unraveling are evolving uncertainty and the exercise of market power.

RD market. Finally, the last component P^E denotes a list of strict preference relations for colleges and students in the ED market.

For any $c \in C$, P_c^R is a linear order on $\Sigma_c^R = 2^S$ and P_c^E is a linear order on some $\Sigma_c^E \subseteq \Sigma_c^R$ such that $\emptyset \in \Sigma_c^E$. Also, for any $s \in S$, P_s^R is a linear order on $\Sigma_s^R = \{\{c_1\}, \{c_2\}, \ldots, \{c_m\}, \emptyset\}$ and P_s^E is a linear order on some $\Sigma_s^E \subseteq \Sigma_s^R$ such that $\emptyset \in \Sigma_s^E$ and $|\Sigma_s^E \backslash \{\emptyset\}| \leq 1$; i.e., each student can apply to at most one college in the ED market.

Given any college c with a strict preference relation P_c^R, we can derive its weak preference relation R_c^R, where $s\, R_c^R\, s'$ for any $s, s' \in S$ if and only if $s\, P_c^R\, s'$ or $s = s'$. Analogously, given any student s with a strict preference relation P_s^R, we can derive her weak preference relation. We introduce the notations $(\succ_c^R, \succeq_c^R, \succ_s^R, \succeq_s^R)$ and $(\succ_c^E, \succeq_c^E, \succ_s^E, \succeq_s^E)$ associated with $(P_c^R, R_c^R, P_s^R, R_s^R)$ and $(P_c^E, R_c^E, P_s^E, R_s^E)$ to represent the strict and weak preference of college c and student s over any two alternatives in the ED and RD markets.

We assume that the ED preference P_c^E of any college c is consistent with its RD preference P_c^R; i.e., for any $T, T' \in \Sigma_c^R$, we have $T \succ_c^E T'$ only if $T \succ_c^R T'$.[14] Likewise, we assume that for any $s \in S$, P_s^E is consistent with P_s^R, i.e., for any $c \in C$ we have $c \succ_s^E \emptyset$ only if $c \succ_s^R \emptyset$ and for any $c, c' \in C$ we have $c \succ_s^E c'$ only if $c \succ_s^R c'$.

We also assume that P_c^R is responsive as in Roth (1985). That is, for all $S' \subset S$ it is true that

i) for all $s \in S \backslash S'$, $S' \cup \{s\} \succ_c^R S'$ if and only if $\{s\} \succ_c^R \emptyset$,

ii) for all $s, s' \in S \backslash S'$ such that $s \neq s'$, $S' \cup \{s\} \succ_c^R S' \cup \{s'\}$ if and only if $\{s\} \succ_c^R \{s'\}$.

Obviously, preferences of students over individual colleges are responsive. Also note that preferences of both colleges and students in the ED market become automatically responsive if their preferences are responsive in the RD market, due to our assumption that the ED preferences must be consistent with the RD preferences.

Let $\mathcal{P}_c^R$ and $\mathcal{P}_s^R$ respectively denote the set of all responsive preference relations for college c and for student s in the RD market. Define $\mathcal{P}^R = \times_{k \in C \cup S} \mathcal{P}_k^R$. Also, given any $P_c^R \in \mathcal{P}_c^R$, let $\mathcal{P}_c^E(P_c^R)$ denote for college c the set of all responsive preference relations, in the ED market, which are consistent with P_c^R. Similarly, given any $P_s^R \in \mathcal{P}_s^R$, let $\mathcal{P}_s^E(P_s^R)$ denote for student s the set of all responsive preference relations, in the ED market, which are consistent with P_s^R. For any $P^R \in \mathcal{P}^R$ define $\mathcal{P}^E(P^R) = \times_{k \in C \cup S} \mathcal{P}_k^E(P_k^R)$ and $\mathcal{P}^E = \times_{P^R \in \mathcal{P}^R} \mathcal{P}^E(P^R)$.

Now, we describe matching problems. Let $\mathcal{E}^R = N_+^n \times \left(\times_{k \in C \cup S} \mathcal{P}_k^R\right)$ denote the class of all matching problems in the RD market. For any $(q, P^R) \in \mathcal{E}^R$ and $q^E \in \mathcal{Q}^E(q)$,

[14] Since Σ_c^E can be a proper subset of Σ_c^R, the consistency assumption does not prevent college c from compromising in the ED market. For example, given a college admission problem where $S = \{s_1, s_2\}$, $C = \{c_1\}$, $P_{c_1}^R = s_1, s_2, \emptyset$, and $P_{c_1}^E = s_2, \emptyset$, we should observe that $P_{c_1}^E$ is consistent with $P_{c_1}^R$ even though c_1 compromises in the ED market by not accepting its top-ranked student s_1 with respect to $P_{c_1}^R$.

let us also define $\mathcal{E}^E(q, P^R, q^E) = \{q^E\} \times \mathcal{P}^E(P^R)$, denoting the class of all matching problems in the ED market. Let $\mathcal{E}^E = \cup_{(q,P^R)} \cup_{q^E \in \mathcal{Q}^E(q)} \mathcal{E}^E(q, P^R, q^E)$.

A matching μ^E in the ED market (simply an ED matching) with the quota profile q^E is a function from the set $C \cup S$ into $2^{C \cup S}$ such that

i) for all $s \in S$, $|\mu^E(s)| \leq 1$ and $\mu^E(s) \subseteq C$;

ii) for all $c \in C$, $|\mu^E(c)| \leq q^E_c$ and $\mu^E(c) \subseteq S$;

iii) for all $(c, s) \in C \times S$, $\mu^E(s) = \{c\}$ if and only if $s \in \mu^E(c)$.

We denote the set of all ED matchings at q^E by $\mathcal{M}^E(q^E)$. Let $\mathcal{M}^E = \cup_{q^E} \mathcal{M}^E(q^E)$.

Given any (q^E, P^E) and any two ED matchings $\mu^E_1, \mu^E_2 \in \mathcal{M}^E(q^E)$, we say that student s strictly prefers μ^E_1 to μ^E_2 if and only if $\mu^E_1(s) \succ^E_s \mu^E_2(s)$ and weakly prefers μ^E_1 to μ^E_2 if and only if $\mu^E_1(s) \succeq^E_s \mu^E_2(s)$. We do the same for each college.

Given any ED matching μ^E and any capacity profile q, we define an RD matching μ^R as a function from the set $C \cup S$ into $2^{C \cup S}$ such that

i) for all $s \in S$, $|\mu^R(s)| \leq 1$, and $\mu^E(s) \subseteq \mu^R(s) \subseteq C$;

ii) for all $c \in C$, $|\mu^R(c)| \leq q_c$, and $\mu^E(c) \subseteq \mu^R(c) \subseteq S$;

iii) for all $(c, s) \in C \times S$, $\mu^R(s) = \{c\}$ if and only if $s \in \mu^R(c)$.

The function μ^R preserves the early matchings achieved under μ^E, i.e. early decisions are binding. Given (q, μ^E), we denote the set of all RD matchings by $\mathcal{M}^R(q, \mu^E)$. Let $\mathcal{M}^R = \cup_{(q,\mu^E)} \mathcal{M}^R(q, \mu^E)$.

Given any two RD matchings μ^R_1 and μ^R_2, we say that student s strictly prefers μ^R_1 to μ^R_2 if and only if $\mu^R_1(s) \succ^R_s \mu^R_2(s)$ and weakly prefers μ^R_1 to μ^R_2 if and only if $\mu^R_1(s) \succeq^R_s \mu^R_2(s)$. We do the same for each college. For any $P \in \mathcal{P}^E \cup \mathcal{P}^R$, we let $A(P_c)$ denote the set of all acceptable students for college c at P_c, i.e., $A(P_c) = \{s \in S : s \succ_c \emptyset\}$. Similarly, we let $A(P_s)$ denote the set of all acceptable colleges for student s at P_s, i.e., $A(P_s) = \{c \in C : c \succ_s \emptyset\}$.

The choice of a college c from a group of students $T \subseteq S$ in the ED market (q^E, P^E) is defined as

$$Ch^E_c(P^E_c, q^E_c, T) = \{T' \subseteq T \cap A(P^E_c) : |T'| \leq q^E_c \text{ and } T' \succ^E_c T''$$

$$\text{for all } T'' \subseteq T \cap A(P^E_c) \text{ such that } T'' \neq T' \text{ and } |T''| \leq q^E_c\}.$$

Similarly, given any ED matching μ^E, the choice of a college c from a group of students $T \subseteq S \backslash \mu^E(c)$ available for matching in the RD market (q, P^R) is defined as

$$Ch^R_c(P^R_c, q_c, \mu^E, T) = \{T' \subseteq T \cap A(P^R_c) : |T'| \leq q_c - |\mu^E(c)| \text{ and}$$

$$T' \cup \mu^E(c) \succ^R_c T'' \cup \mu^E(c) \text{ for all } T'' \subseteq T \cap A(P^R_c)$$

$$\text{such that } T'' \neq T' \text{ and } |T''| \leq q_c - |\mu^E(c)|\}.$$

Given any q^E, a matching $\mu^E \in \mathcal{M}^E(q^E)$ is blocked by student $s \in S$ if $\emptyset \succ_s^E \mu^E(s)$, and blocked by college $c \in C$ if $\mu^E(c) \neq Ch_c^E(P_c^E, q_c^E, \mu^E(c))$. We say that μ^E is acceptable to a college, or to a student, that does not block it. Also, μ^E is blocked by a college-student pair $(c, s) \in C \times S$ if $\{c\} \succ_s^E \mu^E(s)$ and $\mu^E(c) \neq Ch_c^E(P_c^E, q_c^E, \mu^E(c) \cup \{s\})$. We say that μ^E is stable if it is not blocked by a student, a college, or a college-student pair. Given an ED market (q^E, P^E), we denote the set of all stable ED matchings by $\mathcal{S}^E(q^E, P^E)$.

Given an ED matching μ^E, an RD matching $\mu^R \in \mathcal{M}^R(q, \mu^E)$ is blocked by student $s \in S$ if $\emptyset \succ_s^R \mu^R(s) \backslash \mu^E(s)$ and blocked by college $c \in C$ if $\mu^R(c) \setminus \mu^E(c) \neq Ch_c^R(P_c^R, q_c, \mu^E, \mu^R(c) \setminus \mu^E(c))$. We say that μ^R is acceptable to a college, or to a student, that does not block it. Also, μ^R is blocked by a college-student pair $(c, s) \in C \times S$ if $\mu^E(s) = \emptyset$, $\{c\} \succ_s^R \mu^R(s)$ and $\mu^R(c) \setminus \mu^E(c) \neq Ch_c^R(P_c, q_c, \mu^E, \{s\} \cup \mu^R(c) \setminus \mu^E(c))$. We say that μ^R is stable if it is not blocked by a student, a college, or a college-student pair. Given an ED matching μ^E and an RD market (q, P^R), we denote the set of all stable RD matchings by $\mathcal{S}^R((q, P^R), \mu^E)$.

We say that college c and student s are achievable for one another in the ED market (q^E, P^E), if there is a stable ED matching in $\mathcal{S}^E(q^E, P^E)$ at which they are matched. Likewise, we define achievability in an RD market.

An ED matching rule is a function $\varphi^E : \mathcal{E}^E \to \mathcal{M}^E$ such that $\varphi^E(q^E, P^E) \in \mathcal{M}^E(q^E)$ for every $(q^E, P^E) \in \mathcal{E}^E$. Let $\bar{\varphi}^E$ denote the set of all ED matching rules. Similarly, an RD matching rule is a function $\varphi^R : \mathcal{E}^R \times \mathcal{M}^E \to \mathcal{M}^R$ such that $\varphi^R((q, P^R), \mu^E) \in \mathcal{M}^R(q, \mu^E)$ for every $(q, P^R) \in \mathcal{E}^R$, $q^E \in \mathcal{Q}^E(q)$, and $\mu^E \in \mathcal{M}^E(q^E)$. Let $\bar{\varphi}^R$ denote the set of all RD matching rules.

An ED matching rule φ^E is stable if $\varphi^E(q^E, P^E) \in \mathcal{S}^E(q^E, P^E)$ for every $(q^E, P^E) \in \mathcal{E}^E$. On the other hand, an RD matching rule φ^R is stable at an ED matching rule φ^E if $\varphi^R((q, P^R), \varphi^E(q^E, P^E)) \in \mathcal{S}^R((q, P^R), \varphi^E(q^E, P^E))$ for every $(q, P^R) \in \mathcal{E}^R$ and $(q^E, P^E) \in \mathcal{E}^E(q, P^R, q^E)$.

Given any ED matching rule $\varphi^E \in \bar{\varphi}^E$ and any RD matching rule $\varphi^R \in \bar{\varphi}^R$, the ordered pair (φ^E, φ^R) is called a matching system. A matching system (φ^E, φ^R) is stable if φ^E is stable and φ^R is stable at φ^E.

A matching system (φ^E, φ^R) cannot be manipulated by individual $k \in C \cup S$ via its ED preference if for all $(q, P^R) \in \mathcal{E}^R$, $(q^E, P^E) \in \mathcal{E}^E(q, P^R, q^E)$, and $\hat{P}_k^E \in \mathcal{P}_k^E(P_k^R)$ it is true that

$$\varphi^R((q, P^R), \varphi^E(q^E, P^E))(k) \quad \succeq_k^R \quad \varphi^R((q, P^R), \varphi^E(q^E, \hat{P}_k^E, P_{-k}^E))(k).$$

If the above holds for all colleges (students), then we say that the matching system (φ^E, φ^R) is nonmanipulable by colleges (students) via ED preferences.

A matching system (φ^E, φ^R) cannot be manipulated by college $c \in C$ via its ED quota if for all $(q, P^R) \in \mathcal{E}^R$, $(q^E, P^E) \in \mathcal{E}^E(q, P^R, q^E)$, and $\hat{q}_c^E \in \mathcal{Q}_c^E(q)$ it is true that

$$\varphi^R((q, P^R), \varphi^E(q^E, P^E))(c) \quad \succeq_c^R \quad \varphi^R((q, P^R), \varphi^E(\hat{q}_c^E, q_{-c}^E, P^E))(c).$$

If the above holds for all colleges, then we say that the matching system (φ^E, φ^R) is nonmanipulable via ED quotas.

3. Results

Proposition 1. *For any college admission problem with at least two colleges and one student, there exists no matching system that is stable and nonmanipulable via ED quotas.*

Proof. See the Appendix.

The proof of Proposition 1 suggests that a college may benefit from admitting students both in the ED market and in the RD market, when the rest of the colleges, or a sufficient number of them, consider admission only in the RD market. Naturally, Proposition 1 is not valid when there exists a unique college in the admission problem. In that case, a unique stable matching exists for the RD market (and for the ED market), and this stable matching is college-optimal (and also student-optimal), i.e., the unique college would be matched to the highest-ranked achievable students allowed by its quota. Thus, a college that faces no rivals cannot improve the quality of its matches in the RD market (which is already optimal), by changing its quota for the ED market (or by allocating/not allocating some of its total capacity to the ED market). The presence of an ED market would offer an unrivaled college only the opportunity to run and complete its admission process at an earlier time than planned for the RD market.

Next, we consider whether colleges have incentives to manipulate their ED preferences.

Proposition 2. *For any college admission problem with at least two colleges and one student, there exists no matching system that is stable and nonmanipulable by colleges via ED preferences.*

Proof. See the Appendix.

Below, we finally consider manipulation of matching systems by students.

Proposition 3. *For any college admission problem with at least two colleges and one student, there exists no matching system that is stable and nonmanipulable by students via ED preferences.*

Proof. See the Appendix.

Neither Proposition 2 nor Proposition 3 are valid when there exists a unique college in the admission problem (for the same reason as we stated after Proposition 1). In that case, a unique stable matching could exist in the RD market, and this matching would be both college-optimal and student-optimal, eliminating any incentives for manipulation by colleges or students. Below, we will obtain further results under several preference restrictions.

A preference profile $P \in \mathcal{P}^E \cup \mathcal{P}^R$ implies *common preferences for colleges over individual students* if and only if for any $c, c' \in C$ and for any $s, s' \in S$ we have $\{s\}\, P_c\, \{s'\} \Leftrightarrow \{s\}\, P_{c'}\, \{s'\}$. Under this kind of preferences, we know from the earlier results of Konishi and Ünver (2006) and Mumcu and Saglam (2009) that there exists a unique stable matching system that can be obtained by a rule called "the serial dictatorship of students" where students, who are ranked from top to bottom in the ED market according to the common ED preferences of colleges and then ranked in the RD market according to their common RD preferences, are allowed to serially dictate to be matched with their favorite acceptable college (up to their quota) among the colleges that are still available for matching. This particular matching system implies that if neither students nor colleges compromise in the ED market, then colleges have no strict incentive to participate in the ED market. To prove this claim, we will consider an ED quota game played by colleges.

Consider for this game an RD market (C, S, R^R, q^R), where R^R and q^R are commonly known. The strategy of college c is to choose an ED quota $q_c^E \in \mathcal{Q}_c^E(q^R)$. We assume that for each possible choice of q^E, the preferences of colleges and students in the ED period, denoted by $R^E(q^E)$, are also common knowledge. Suppose that a matching system $\vec{\varphi} = (\varphi^E, \varphi^R)$ is used to determine the matchings in the ED market and the RD market. College c's preferences over reported ED quotas are represented by a binary relationship $\succeq_c^{\vec{\varphi}}$ over $\mathcal{Q}^E(q^R)$ such that for all q'^E, $q''^E \in \mathcal{Q}^E(q^R)$ we have $q'^E \succeq_c^{\vec{\varphi}} q''^E$ if and only if

$$\varphi^R((q^R, R^R), \varphi^E(q'^E, R^E(q'^E))) \succeq_c^R \varphi^R((q^R, R^R), \varphi^E(q''^E, R^E(q''^E))).$$

An ED quota game under matching system $\vec{\varphi}$ is described by a strategic form game $(C, (\mathcal{Q}_c^E(q^R), \succeq_c^{\vec{\varphi}})_{c\in C})$. Define college c's *best response correspondence* under matching system $\vec{\varphi}$ by $\beta_c^{\vec{\varphi}} : \mathcal{Q}_{-c}^E(q^R) \to \mathcal{Q}_c^E(q^R)$ such that for any $q_{-c}^E \in \mathcal{Q}_{-c}^E(q^R)$,

$$\beta_c^{\vec{\varphi}}(q_{-c}^E) = \{\tilde{q}_c^E \in \mathcal{Q}_c^E(q^R) : (\tilde{q}_c^E, q_{-c}^E) \succeq_c^{\vec{\varphi}} ({q'}_c^E, q_{-c}^E) \text{ for all } {q'}_c^E \in \mathcal{Q}_c^E(q^R)\}.$$

A *pure strategy (Nash) equilibrium* of the game $(C, (\mathcal{Q}_c^E(q^R), \succeq_c^{\vec{\varphi}})_{c\in C})$ is a strategy profile $q^E \in \mathcal{Q}^E(q^R)$ such that $q_c^E \in \beta_c^{\vec{\varphi}}(q_{-c}^E)$ for all $c \in C$.

Proposition 4. *Consider a college admission problem (C, S, q, q^E, P^R, P^E) where colleges have common preferences over individual students and adopt their RD preferences in the ED market. Also assume that each student applies early only to the top-ranked college with respect to her RD preference. Then, for each college it is a weakly-dominant strategy to report the ED quota as zero if the ED quota game is played under a stable matching system.*

We can extend the above result to the case in which students have common preferences. Formally, we say that a preference profile $P \in \mathcal{P}^E \cup \mathcal{P}^R$ satisfies *common* *preferences for students over individual colleges* if and only if for any $s, s' \in S$ and for any $c, c' \in C$ we have $\{c\} \, P_s \, \{c'\} \Leftrightarrow \{c\} \, P_{s'} \, \{c'\}$. Under these preferences, we know again from Konishi and Ünver (2006) and Mumcu and Saglam (2009) that there exists a unique stable matching system that can be obtained by a rule called "the serial dictatorship of colleges" where colleges, who are ranked from top to bottom in the ED market according to the common ED preferences of students and then ranked in the RD market according to their common RD preferences, are allowed to serially dictate to be matched with their favorite acceptable students (up to their quota) among the students that are still available for matching.

Proposition 5. *Consider a college admission problem (C, S, q, q^E, P^R, P^E) where students have common preferences over individual colleges and each student applies early only to the top-ranked college with respect to her RD preference. Also, assume that colleges adopt their RD preferences in the ED market. Then, for each college it is a weakly-dominant strategy to report the ED quota as zero if the ED quota game is played under a stable matching system.*

We should note that both Proposition 4 and Proposition 5 show results in which no college has a strict incentive to offer a single-choice ED program. Clearly, these results are in conflict with the practices observed in the US College Admissions. What is causing this conflict is simply the fact that the assumption in Propositions 4 and 5 about the ED preferences of students, requiring them to apply early only to their top-ranked college with respect to their RD preferences, is too restrictive. In reality, a student who wants to enjoy her senior year in the high school without any pressure may choose to apply early to a college that is not her top alternative, while at the same time believed to be easier to get admitted.[15] At this point, we claim that the existence of some students who compromise in the ED market may explain why all colleges may not desire to terminate their ED programs. Below, we prove this claim by the help of Examples 1 and 2, which deal with the effects of compromising behavior of students under common preferences for colleges and common preferences for students, respectively.

Example 1. Consider the RD market (C, S, q^R, R^R) with $C = \{c_1, c_2\}$, $S = \{s_1, s_2, s_3\}$, $q^R_{c_1} = 2$, $q^R_{c_2} = 2$, and the following regular preferences for colleges and students:

$$P^R_{c_1} = P^R_{c_2} = \{s_1\}, \{s_2\}, \{s_3\}, \emptyset,$$
$$P^R_{s_1} = P^R_{s_2} = \{c_1\}, \{c_2\}, \emptyset,$$
$$P^R_{s_3} = \{c_2\}, \{c_1\}, \emptyset.$$

Let $\mathcal{Q}^E_c(q^R) = \{0, 1, 2\}$ and $P^E_c = P^R_c$ for all $c \in C$. Also, let $P^E_s(q^E) = Top(R^R_s; 1), \emptyset$ for all $s \in \{s_1, s_3\}$ and $P^E_{s_2}(q^E) = Top(R^R_{s_2}; 2), \emptyset$, for all $q^E \in \mathcal{Q}^E(q^R)$. (Here,

[15] Such compromising behavior of students can indeed be more frequently observed in situations where the applicant pool is very large and the information about the true preference profile as well as the matching process is not completely available to all applicants.

$Top(R_s^R; k)$ denotes the k-th best college from top according to the RD preference of student s.)

We will show that the ED quota profile $(0, 0)$ is not a Nash equilibrium. Note that colleges have common preferences over individual students and therefore there exists a unique stable matching system. This system, denoted by $\vec{\varphi} = (\varphi^E, \varphi^R)$, can be obtained by the rule of serial dictatorship of students. It is easy to check that $\varphi^E((0,0), R^E(0,0))(c_2) = \emptyset$, and $\varphi^R(((2,2), R^R), \varphi^E((0,0), R^E(0,0)))(c_2) = \{s_3\}$. College c_2, which is the lowest-ranked college by the majority of students, has an incentive to enter the ED market, since if it unilaterally deviates and announces $q^E_{c_2} = 2$, we have

$$\varphi^R(((2,2), R^R), \varphi^E((0,2), R^E(0,2)))(c_2) = \varphi^E((0,2), R^E(0,2))(c_2) = \{s_2, s_3\}.$$

Thus, it is not true that $(0, 0) \succeq^{\vec{\varphi}}_{c_2} (0, 2)$. Here, one can easily check that the Nash equilibria of this game are $(0, 1)$, $(1, 1)$, $(2, 1)$, $(0, 2)$, $(1, 2)$, and $(2, 2)$, which all yield the same matching outcome in the RD market. ■

Example 2. Consider the RD market (C, S, q^R, R^R) with $C = \{c_1, c_2\}$, $S = \{s_1, s_2, s_3\}$, $q^R_{c_1} = 2$, $q^R_{c_2} = 2$, and the following regular preferences for colleges and students:

$$P^R_{c_1} = \{s_1\}, \{s_2\}, \{s_3\}, \emptyset,$$

$$P^R_{c_2} = \{s_3\}, \{s_2\}, \{s_1\}, \emptyset,$$

$$P^R_s = \{c_1\}, \{c_2\}, \emptyset, \text{ for all } s \in S.$$

We have $\mathcal{Q}^E_c(q^R) = \{0, 1, 2\}$ for all $c \in C$. Let $P^E_c = P^R_c$ for all $c \in C$, and $P^E_s(q^E) = Top(R^R_s; 2), \emptyset$ for all $s \in S$ and for all $q^E \in \mathcal{Q}^E(q^R)$. (Here, we have kept on assuming common ED preferences for students to simply obtain the unique stable matching by the serial dictatorship of colleges.)

We will show that the ED quota profile $(0, 0)$ is not a Nash equilibrium. Note that students have common preferences over individual colleges and therefore there exists a unique stable matching system. This system, denoted by $\vec{\varphi} = (\varphi^E, \varphi^R)$, can be obtained by the rule of serial dictatorship of colleges. It is easy to check that $\varphi^E((0,0), R^E(0,0))(c_2) = \emptyset$, and $\varphi^R(((2,2), R^R), \varphi^E((0,0), R^E(0,0)))(c_2) = \{s_3\}$. College c_2, which is the lowest-ranked college by all students, has an incentive to enter the ED market, since if it unilaterally deviates and announces $q^E_{c_2} = 2$, it can select the set of students $\varphi^R(((2,2), R^R), \varphi^E((0,2), R^E(0,2)))(c_2) = \varphi^E((0,2), R^E(0,2))(c_2) = \{s_2, s_3\}$. Thus, it is not true that $(0, 0) \succeq^{\vec{\varphi}}_{c_2} (0, 2)$. Here, one can easily check that the Nash equilibria of this game are $(0, 2)$, $(1, 2)$, and $(2, 2)$, which all yield the same matching outcome in the RD market. ■

4. Conclusions

Since its adoption in the USA five decades ago, early admissions programs have been under scrutiny. As early admissions have only been observed in decentralized college admission systems, the literature has sought to provide explanations on its adoption to circumvent the flaws of the decentralized college admissions system. It has also been argued that some of these flaws, such as congestion (Che and Koh, 2016), unfairness and unraveling (Yenmez, 2018) can be alleviated by moving to centralized systems.

We have showed that even in centralized systems the intertemporal quota allocation may be an important reason behind the adoption of ED programs. While the existing ED programs have been invented, to some extent, to strategically manipulate the outcome of the regular admission programs, the ED programs, or their combinations with the RD programs, are themselves prone to the manipulation of colleges (for example, via their quotas and preferences) and students (via their preferences), as shown by our results in this paper.

Using a two period matching model with an ED market followed by an RD market, we have simply established that (i) there exists no stable matching system which is nonmanipulable via ED quotas by colleges (Proposition 1) and (ii) there exists no stable matching system which is nonmanipulable via ED preferences by colleges or students (Propositions 2 and 3, respectively). Interestingly, Proposition 1 suggests that it may not (always) be possible to eliminate strategic incentives of colleges to manipulate the existing college admission system by controlling or changing the (stable) matching rules followed in the ED and RD markets.

In our paper, we have also dealt with the stability of the ED program and showed that when colleges or students have common preferences and each student applies early only to the top-ranked college with respect to her RD preference, then no college has a strict incentive to offer a single-choice ED program (Propositions 4 and 5). On the other hand, when students compromise in the ED market and make an early application to colleges that are not top-ranked, then colleges may become better off when they offer ED programs than when they do not (Examples 1 and 2).[16] Our results can be contrasted to the earlier findings in Kim (2010) and Murra-Anton (2020), which also dealt with identifying incentives of colleges to use early admissions. Like in our paper, Kim (2010) focus on ED programs only and shows using a stylized theoretical model of competition that tuition-maximizing and (superficially) need-blind colleges may implement early admissions as a screening device to identify wealthy students. However, unlike in our paper or Murra-Anton (2020), colleges in Kim (2010) are not allowed to decide whether to offer early decision admissions. As a matter of fact, the model in Murra-Anton (2020) is much more general than ours as it allows colleges

[16] Lower-ranked colleges' incentive to manipulate the outcome via early admissions quotas were also pointed out by Chen et al. (2018) and Avery et al. (2014), respectively, for the cases of Taiwan and South Korea, where those colleges set the same screening test date as that of the best one. Moreover, Avery and Levine (2010) show that a lower-ranked college can benefit from ED not just because of sorting effect, but because it brings a competitive benefit.

to consider (Restricted) EA programs, as well as ED and RD programs. Using a game-theoretic model of college admissions, Murra-Anton (2020) basically shows that in equilibrium more prestigious and wealthier colleges can be more selective and offer more generous financial aid policies and non-binding early admissions (using an EA program) whereas inferior colleges need to offer a binding program (such as ED). Our results, which we have obtained in the absence of financial considerations unlike in Kim (2010) and Murra-Anton (2020), mainly suggest that colleges have big incentives to learn about the preferences of students. Even though we have considered in our study a fully-centralized early admission system, we believe that our results may also hint at an argument that the US colleges might have been experimenting in the decentralized college admission system over the past few decades in order to learn the behavior of students and eventually the equilibrium market configuration of today. However, our results in Propositions 1-3 imply that one should not be very optimistic in terms of manipulability issues if one has to move from the decentralized early admission system to a centralized one.

We believe that our results may also help students (or their parents and counselors) better understand the early admissions game. Each year more than a half a million students participate in the US to play this game (against/with many prestigious colleges and other students) without being totally aware of the potential consequences of their actions, as documented in the book by Avery et al. (2009). Our results show that the coexistence of ED programs with RD programs need not improve the well being of students, whereas it may improve the well being of colleges provided that very eligible students compromise to get rid of the exptected pressure of regular admissions and apply early to colleges that are not their top alternatives. Surely, the existence of ED programs do not harm students either, as long as they can apply RD and/or EA programs as well. In fact, ED programs may even improve the chances of some students being admitted to some prestigious colleges that compromise in early admissions fearing not to be able to fill their capacities in regular admissions.

An important question that we leave for future research is the manipulability of the early admissions system, which includes both ED and early action (EA) programs. While this problem may be novel to the best of our knowledge, the stability of an early admissions system using both EA and ED is already studied by Mumcu and Saglam (2007). Using a two-period matching game with observable actions, they show that for each college an EA program is weakly dominating an ED program whenever each student follows a strategy that recommends to apply to an ED program only if the college that offers it is the top college in her early admission list and weakly preferred to the top college in her regular admission list.[17] In fact, this is a strategy strongly recommended to all students by the College Board, counselors, college admission officers, and many college guides.[18] Mumcu and Saglam (2007) also show that irrespective from

[17] According to Murra-Anton (2020), when need-blind financial aid is a tool used to attract students, wealthier and more prestigious colleges prefer EA and the others ED.

[18] A survey reported in Avery et al. (2003, p. 205) provides statistical evidence that this recom-

the type of the early admissions plan, it is a weakly dominant strategy for each college to choose its early quota as its total capacity and to defer all acceptable applicants ranking outside its capacity size to the regular admissions period.

Finally, we should note here that we have modeled the college admission problem using a many-to-one matching setup in two periods, separating the early and regular decision markets in time dimension as in reality. An alternative, and much richer, model was very recently introduced by Yenmez (2018), who showed that college admissions with early and non-early decisions can be operated by a centralized clearinghouse that can deal with stable many-to-many matchings with contracts between colleges and students. We believe that one can fruitfully study whether our negative results as to the nonmanipulability of stable matching rules could also arise in the alternative model of Yenmez (2018).

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Appendix

Proof of Proposition 1. We first consider a college admission problem (C, S, q, q^E, P^R, P^E) with two colleges and one student; i.e., $C = \{c_1, c_2\}$ and $S = \{s_1\}$. Let $q_{c_1} = 1$, $q_{c_2} = 1$, $q^E_{c_1} = 0$, $q^E_{c_2} = 0$, $\hat{q}^E_{c_1} = 1$;

$$P^R_{c_1} = P^E_{c_1} = \{s_1\}, \emptyset;$$

$$P^R_{c_2} = P^E_{c_2} = \{s_1\}, \emptyset;$$

$$P^R_{s_1} = \{c_2\}, \{c_1\}, \emptyset;$$

$$P^E_{s_1} = \{c_1\}, \emptyset.$$

(Note that the strict preference relation of a student or a college is represented by an ordered list of acceptable mates.) We have $S^E(q^E, P^E) = \{\mu_1\}$, $S^R(q, P^R, \mu_1) = \{\mu_2\}$, $S^E(\hat{q}^E_{c_1}, q^E_{c_2}, P^E) = \{\mu_3\}$, and $S^R(q, P^R, \mu_3) = \{\mu_3\}$, where

$$\mu_1 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \emptyset \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \{s_1\} \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} c_1 & c_2 \\ \{s_1\} & \emptyset \end{pmatrix}.$$

Consider any matching system (φ^E, φ^R) that is stable. Then, we must have $\varphi^E(q^E, P^E) = \mu_1$, $\varphi^R((q, P^R), \mu_1) = \mu_2$, $\varphi^E(\hat{q}^E_{c_1}, q^E_{c_2}, P^E) = \mu_3$, and $\varphi^R((q, P^R), \mu_3) = \mu_3$. Hence,

$$\varphi^R((q, P^R), \varphi^E(\hat{q}^E_{c_1}, q^E_{c_2}, P^E))(c_1) \;\; \succ^R_{c_1} \;\; \varphi^R((q, P^R), \varphi^E(q^E, P^E))(c_1).$$

So, college c_1 can manipulate the matching system (φ^E, φ^R) via its ED quota when $q^E_{c_1} = 0$. It can do so by announcing $\hat{q}^E_{c_1} = 1$ and accepting the unique student s_1 in the ED market. This completes the proof for the case of $|C| = 2$ and $|S| = 1$. In order to extend it to the general case of $|C| \geq 2$ and $|S| \geq 1$, we can include, to the above college admission problem, colleges whose top choice is admitting no student and students whose top choice is staying unmatched both in the ED market and in the RD market. ■

Proof of Proposition 2. We first consider a college admission problem (C, S, q, q^E, P^R, P^E) with two colleges and one student; i.e., $C = \{c_1, c_2\}$ and $S = \{s_1\}$. Let $q_{c_1} = 1$, $q_{c_2} = 1$, $q^E_{c_1} = 1$, $q^E_{c_2} = 0$;

$$P^R_{s_1} = \{c_2\}, \{c_1\}, \emptyset; \qquad P^E_{s_1} = \{c_1\}, \emptyset;$$

$$P^R_{c_1} = \{s_1\}, \emptyset; \qquad P^E_{c_1} = \emptyset;$$

$$P^R_{c_2} = P^E_{c_2} = \{s_1\}, \emptyset;$$

$$\hat{P}^E_{c_1} = P^R_{c_1}.$$

Then, we have $S^E(q^E, P^E) = \{\mu_1\}$, $S^R(q, P^R, \mu_1) = \{\mu_2\}$, $S^E(q^E, \hat{P}^E_{c_1}, P^E_{-c_1}) = \{\mu_3\}$, $S^R(q, P^R, \mu_3) = \{\mu_3\}$, where

$$\mu_1 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \emptyset \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \{s_1\} \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} c_1 & c_2 \\ \{s_1\} & \emptyset \end{pmatrix}.$$

Consider any matching system (φ^E, φ^R) that is stable. Then, we must have $\varphi^E(q^E, P^E) = \mu_1$, $\varphi^R((q, P^R), \mu_1) = \mu_2$, $\varphi^E(q^E, \hat{P}^E_{c_1}, P^E_{-c_1}) = \mu_3$, and $\varphi^R((q, P^R), \mu_3) = \mu_3$. Hence,

$$\varphi^R((q, P^R), \varphi^E(q^E, \hat{P}^E_{c_1}, P^E_{-c_1}))(c_1) \;\; \succ^R_{c_1} \;\; \varphi^R((q, P^R), \varphi^E(q^E, P^E))(c_1).$$

So, college c_1 can manipulate the matching system (φ^E, φ^R) via its ED preference, completing the proof for the case of $|C| = 2$ and $|S| = 1$. In order to extend it to the general case of $|C| \geq 2$ and $|S| \geq 1$, we can include, to the above college admission problem, colleges whose top choice is admitting no student and students whose top choice is staying unmatched both in the ED market and in the RD market. ∎

Proof of Proposition 3. We first consider a college admission problem (C, S, q, q^E, P^R, P^E) with two colleges and one student; i.e., $C = \{c_1, c_2\}$ and $S = \{s_1\}$. Let $q_{c_1} = 1$, $q_{c_2} = 1$, $q^E_{c_1} = 0$, $q^E_{c_2} = 1$;

$$P^R_{c_1} = P^E_{c_1} = \{s_1\}, \emptyset;$$

$$P^R_{c_2} = P^E_{c_2} = \{s_1\}, \emptyset;$$

$$P^R_{s_1} = \{c_1\}, \{c_2\}, \emptyset;$$

$$P^E_{s_1} = \{c_2\}, \emptyset;$$

$$\hat{P}^E_{s_1} = \{c_1\}, \emptyset.$$

We have $S^E(q^E, P^E) = \{\mu_1\}$, $S^R(q, P^R, \ \mu_1) = \{\mu_1\}$, $S^E(q^E, \hat{P}^E_{s_1}, P^E_{-s_1}) = \{\mu_2\}$, and $S^R(q, P^R, \mu_2) = \{\mu_3\}$, where

$$\mu_1 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \{s_1\} \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \emptyset \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} c_1 & c_2 \\ \{s_1\} & \emptyset \end{pmatrix}.$$

Consider any matching system (φ^E, φ^R) that is stable. Then, we must have $\varphi^E(q^E, P^E) = \mu_1$, $\varphi^R((q, P^R), \mu_1) = \mu_1$, $\varphi^E(q^E, \hat{P}^E_{s_1}, P^E_{-s_1}) = \mu_2$, and $\varphi^R((q, P^R), \mu_2) = \mu_3$. Hence,

$$\varphi^R((q, P^R), \varphi^E(q^E, \hat{P}^P_{s_1}, P^E_{-s_1}))(s_1) \;\; \succ^R_{s_1} \;\; \varphi^R((q, P^R), \varphi^E(q^E, P^E))(s_1).$$

So, student s_1 can manipulate the matching system (φ^E, φ^R) via her ED preference, completing the proof for the case of $|C| = 2$ and $|S| = 1$. In order to extend it to the general case of $|C| \geq 2$ and $|S| \geq 1$, we can include, to the above college admission problem, colleges whose top choice is admitting no student and students whose top choice is staying unmatched both in the ED market and in the RD market. ∎

Proof of Proposition 4. Consider a college admission problem (C, S, q, q^E, P^R, P^E) where all assumptions in the proposition hold. Since students apply early only to their top-ranked colleges with respect to their RD preferences, any college can be matched in the ED market if and only if a student ranks it at the top. However, any such student, if not already in the list of students that this particular college accepts in the RD market when it announces zero quota for the ED market, must be unattainable for the college at a stable matching system since under the common preferences for colleges the matchings in every market are determined by the serial dictatorship of students, which is the unique stable matching system. Hence, for each college it is a weakly dominant strategy to report the ED quota as zero. ■

Proof of Proposition 5. Consider a college admission problem (C, S, q, q^E, P^R, P^E) where all assumptions in the proposition hold. As the matchings in every market are determined by the serial dictatorship of colleges, which is the unique stable matching system under the common preferences for students, it is true that no college, except for the top-ranked college according to the common RD preferences of students, has any incentive to participate in the ED market. However, the top-ranked college is also indifferent to participate in the ED market, since it already has the first position in selecting students in the RD market. Hence, for each college it is a weakly dominant strategy to report the ED quota as zero. ■