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A directional technology convergence index

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Abstract

Technology heterogeneity has been highlighted as an important feature in many fields in economics. We suggest a new index that measures technology convergence over time. Our index is flexible and easy to interpret, compute and aggregate. In particular, nonparametric estimation can be used and the practitioners can select the direction of the technology convergence investigation. We illustrate the usefulness of our new index with the case of the technology clubs in macro-empirics.

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1 Modeling technology heterogeneities

Assume that we observe entities, such as firms, plants, countries and regions, partitioned into groups, capturing technology heterogeneity, over several time periods. Assume also that entities use inputs $\mathbf{x}$ to generate outputs $\mathbf{y}$, captured in the netput vector $\mathbf{z} = \begin{bmatrix} \mathbf{y} \\ -\mathbf{x} \end{bmatrix}$.[1]

In a multiple inputs multiple outputs setting, a simple way to characterize technology is the concept of production possibility set. Given our technology heterogeneity setting, we have a set for each group i and period t:

$$T_t^i = \{\mathbf{z} \mid \ \mathbf{z} \text{ is feasible using the technology at time } t \text{ for group } i\}. \tag{1}$$

That is, technology may evolve over time and may differ across groups. Also, we point out that our definition of technology in (1) is general enough to include technology development or phasing out, i.e. the number of groups may evolve over time; and entities can move from one technology to another, i.e. the number of entities in each group can change over time. An equivalent way to define the technologies, which is directly useful to define our technology convergence indexes, is the concept of directional distance function (Chambers et al., 1998):

$$\forall i, \forall t : \mathbf{z} \in T_t^i \Leftrightarrow \vec{D}_t^i(\mathbf{z}; \mathbf{g}) \geq 0,$$

where $\vec{D}_t^i(\mathbf{z}; \mathbf{g}) = \max \left\{ \beta | \, (\mathbf{z} + \beta \mathbf{g}) \in T_t^i \right\}$ and $\mathbf{g}$ is a nonzero vector determining the direction of $\vec{D}_t^i(\mathbf{z}; \mathbf{g})$.[2] It therefore is a measure of the distance (in the direction of $\mathbf{g}$) of an entity, i.e. a point in the netput space, to the frontier of the production possibility set T_t^i. When $\vec{D}_t^i(\mathbf{z}; \mathbf{g}) = 0$, it means that $\mathbf{z}$ lies on the frontier of T_t^i, while larger value indicates greater distance (in the direction of $\mathbf{g}$) to its frontier.

2 Directional technology gaps

The concept of technology gap, which dates at least to Bates and Rao (2002) and O'Donnell et al. (2008), was introduced to measure the gap between a group-level technology and a benchmark or best practice technology.[3] The latter could be defined in terms of the group-specific counterparts (e.g. as an envelopment) or in any others. This choice is left to the practitioners, see Section 6 for an example.

[1] Note that the netput vector can alternatively be defined as $\mathbf{z} = \begin{bmatrix} \mathbf{y} \\ \mathbf{x} \end{bmatrix}$. It is fairly straightforward to adapt the definitions to that possibly.

[2] The simplest directions we may think about are the inputs, i.e. $\mathbf{g} = (\mathbf{0}, -\mathbf{x})$ and the outputs, i.e. $\mathbf{g} = (\mathbf{y}, \mathbf{0})$ as in our illustration in Section 6.

[3] The terms of overall technology and meta-technology have also been used in the literature.

Building on the definition of directional distance functions, we define the concept of directional technology gap. Formally, it is defined at time t in the direction of $\mathbf{g}$ for an entity operating at $\mathbf{z}$ as follows:

$$g_t^i(\mathbf{z};\mathbf{g}) = \frac{1+\vec{D}_t(\mathbf{z};\mathbf{g})}{1+\vec{D}_t^i(\mathbf{z};\mathbf{g})}, \tag{2}$$

where $\vec{D}_t(\mathbf{z};\mathbf{g})$ is the distance of $\mathbf{z}$ to the best practice technology in the direction of $\mathbf{g}$ at time t. $g_t^i(\mathbf{z};\mathbf{g})$ gives us an one-point technology gap measurement, i.e. for a specific point in the netput space. The benchmark value is one indicating that the technology i is the best practice technology at that point. $g_t^i(\mathbf{z};\mathbf{g}) < 1$ implies that technology i is further away from the best practice technology at that point. Indeed, in that case $\vec{D}_t^i(\mathbf{z};\mathbf{g}) < \vec{D}_t(\mathbf{z};\mathbf{g})$ revealing a larger distance with respect to the best practice technology. Note that a value larger than one is impossible since it would imply that $\vec{D}_t^i(\mathbf{z};\mathbf{g}) > \vec{D}_t(\mathbf{z};\mathbf{g})$. Finally, we notice the directional technology gap can be used to compare group-level technologies since the numerator is fixed in (2).

3 Directional technology convergence indexes

We now have all necessary notations to define our index. Without loss of generality, assume one wants to evaluate technology convergence between two time periods named b and c (with $b < c$). Also, we restrict attention to ratio-based indexes.[4] A first candidate to capture technology gap change over time is:

$$\frac{g_c^i(\mathbf{z}_b;\mathbf{g})}{g_b^i(\mathbf{z}_b;\mathbf{g})} \tag{3}$$

This ratio captures how technology gap has changed for an entity in the direction $\mathbf{g}$ between time b and c. Indeed, only technologies have changed over time in (3), the rest is fixed. When this ratio is smaller than one, the technology gap is smaller at c than b. That is, we observe a technology convergence. When this ratio is larger than one, it is a technology divergence. We notice three important features about this ratio. First, its amplitude can be understood as the convergence/divergence speed. Next, it involves a counterfactual measurement ($g_c^i(\mathbf{z}_b;\mathbf{g})$) which may be computationally challenging, see Section 5. Finally, it depends on the time period selected for the entity (here b). A second option, therefore,

[4] An alternative would be to consider difference-based indexes. It is fairly straightforward to adapt our definition to this case. Ratio-based indexes are more popular in applied research.

is to select the netput at time c when defining the ratio:

$$\frac{g_c^i(\mathbf{z}_c;\mathbf{g})}{g_b^i(\mathbf{z}_c;\mathbf{g})} \tag{4}$$

This ratio has to be interpreted in a similar fashion to (3) but when netput at time c is selected. Therefore, equations (3) and (4) do not yield the same results in general. To overcome the path dependence, we adopt the Fisher ideal approach popularized by Caves et al. (1982):

$$TCI^i(\mathbf{z}_b,\mathbf{z}_c;\mathbf{g}) = \left[\frac{g_c^i(\mathbf{z}_b;\mathbf{g})}{g_b^i(\mathbf{z}_b;\mathbf{g})} \times \frac{g_c^i(\mathbf{z}_c;\mathbf{g})}{g_b^i(\mathbf{z}_c;\mathbf{g})}\right]^{1/2}. \tag{5}$$

While initially a tradition in the literature, the Fisher ideal approach is, in fact, the only merged relative score that satisfies the desirable properties of multiplicativity, positive homogeneity, and symmetry (Aczél, 1990). $TCI^i(\mathbf{z}_b,\mathbf{z}_c;\mathbf{g})$ has to be interpreted as (3) and (4), but has the nice feature of being time-independent.

4 Aggregation

Up to now, our different concepts give us one-point measurements, i.e. they are defined for a specific point in the netput space. In a group-level setting, it is important to be able to provide an index for each group. To do so, we first define the group-level directional technology gap as follows:

$$\overline{g}_t^i(\mathbf{Z},\mathbf{g}) = \frac{1+\overline{\vec{D}}_t(\mathbf{Z};\mathbf{g})}{1+\overline{\vec{D}}_t^i(\mathbf{Z};\mathbf{g})}, \tag{6}$$

where $\mathbf{Z}$ is the group-level netput matrix and $\overline{\vec{D}}_t^i(\mathbf{Z};\mathbf{g})$ and $\overline{\vec{D}}_t(\mathbf{Z};\mathbf{g})$ are the arithmetic averages of the entity-level directional distance functions. In the case where technology i is the best practice technology, i.e. $\vec{D}_t^i(\mathbf{z};\mathbf{g}) = \vec{D}_t(\mathbf{z};\mathbf{g})$, for all $\mathbf{z}$, we have $\overline{g}_t^i(\mathbf{Z},\mathbf{g}) = 1$ implying no technology gap for the group. Smaller value mean that technology i is further away from the best practice one. We point out that our aggregation scheme, while probably not the only option, is coherent with previous findings in the literature (e.g. Briec et al., 2003 and Walheer, 2018a) and do not require extra data (e.g. prices). For an overview of other options see Walheer (2018b).

A natural group-level directional technology convergence index is given as follows:

$$\overline{TCI}^{i}(\mathbf{Z}_b,\mathbf{Z}_c,\mathbf{g}) = \left[\frac{\overline{g}^{i}_{c}(\mathbf{Z}_b;\mathbf{g})}{\overline{g}^{i}_{b}(\mathbf{Z}_b;\mathbf{g})} \times \frac{\overline{g}^{i}_{c}(\mathbf{Z}_c;\mathbf{g})}{\overline{g}^{i}_{b}(\mathbf{Z}_c;\mathbf{g})}\right]^{1/2}. \tag{7}$$

$\overline{TCI}^{i}(\mathbf{Z}_b,\mathbf{Z}_c,\mathbf{g})$ has to be interpreted as the one-point counterpart in (5), but at the group level. A value smaller (larger) than one tells us that technology i converge (divergence) through the best practice technology. Again, the amplitude of the index can be used to investigate the convergence/divergence speed between technologies.

5 Nonparametric estimators

A last concern is how to obtain the indexes in practice. This is computationally challenging given the technology heterogeneity and the counterfactual measurements. An easy way to proceed is to adopt a nonparametric approach. We make use of a DEA-estimator after Charnes et al. (1978). Broadly speaking, we estimate the directional distance functions using the information contained in the data and while imposing some regulatory conditions on the production possibility sets. When assuming that the group-level production possibility sets are monotone, convex, and satisfying constant returns-to-scale, we obtain for $\vec{D}^{i}_{b}(\mathbf{z}_c;\mathbf{g})$:

$$\begin{aligned}\vec{D}^{i}_{b}(\mathbf{z}_c;\mathbf{g}) = & \max_{\beta\geq 0;\lambda_s\geq 0(s\in b)} \beta \\ \text{s.t.} \quad & \sum_{s\in b}\lambda_s \mathbf{z}^{i}_{sb} \geq \mathbf{z}_c + \beta\mathbf{g}.\end{aligned} \tag{8}$$

As explained above, different approaches could be used to define the best practice technology. A popular way is to define it as the (non-convex) envelopment of the group-level counterparts. In that case, we obtain: $\vec{D}_{b}(\mathbf{z}_c;\mathbf{g}) = \min_i \vec{D}^{i}_{b}(\mathbf{z}_c;\mathbf{g})$.

It is straightforward to adapt (8) to obtain the other directional distance functions. Once they have been estimated, the directional technology gaps and convergence indexes for the entities and the groups can be computed.

6 Illustration

We illustrate our new concepts with the case of the technology clubs in macro-empirics. Several studies, initiated by Durlauf and Johnson (1995), Bernard and Durlauf (1996), and Galor (1996), have pointed out the existence of technology clubs among countries. While well-known statistical and econometric techniques have been used to investigate different aspects of their growth process, our new indexes offer the advantage of measuring whether clubs convergence/diverge in terms of technology and at what speed.

Several empirical studies have highlighted the existence of three clubs: ‘Advanced’, ‘Followers’, and ‘Marginalized’. In fact, the ‘Advanced’ club is the best in terms of technology and even defines the world technology. We follow the common practice used in the literature to construct our variables (output, capital, and labour) and take our data from the most recent Penn World Table.[5]. We obtain data for 81 countries for the time span 1965–2014. To partition countries into the three clubs, we use a classification and regression tree analysis in two dimensions: their innovative ability and absorptive capacity.[6]

As the ‘Advanced’ club is considered as the best in terms of technology, we investigate how the two other club technologies have moved with respect to this club between 1965 and 2014, i.e. the ‘Advanced’ club defines the best practice technology in (5), 1965 b, and 2014 c. Given our macro-empirics setting, we select the direction of outputs (i.e. $\mathbf{g} = (\mathbf{y}, \mathbf{0})$). Results for the ‘Marginalized’ countries and for the ‘Followers’ are given in Table 1 and 2, respectively.[7]

All marginalized countries, except Burkina Faso and Uganda, have a technology convergence index larger than one, indicating that this club diverges in terms of technology with respect to ‘Advanced’ countries. This is also captured by the aggregated index with a value of 1.07. We highlight the lager index values of Ethiopia, Egypt, and Bangladesh.

Most of the‘Followers’ have a technology convergence index smaller than one, which indicates a technology convergence with respect to the ‘Advanced’ countries. Note that the speed of convergence of this club is larger than the divergence speed of the ‘Marginalized’ countries: 7 points above unity for the ‘Marginalized’ club and 11 points below for the ‘Followers’. We highlight important convergence speeds of the Asian Tigers (China, Hong Kong, Korea, and Taiwan).

[5]Given our previous notations, we have one output contained in $\mathbf{y}$: real GDP at chained PPPs (in mil. 2011US$) and two inputs contained in $\mathbf{x}$: number of persons engaged (in millions) and capital stock (in mil. constant 2011US$)

[6]Innovative ability is measured by the number of patents and the number of scientific articles; absorptive capacity is measured by the level of human capital (literacy rate, total number of years of school, secondary schooling, higher education) and technological infrastructures (fixed telephony, electricity, computers, Internet users). Data are retrieved from the Registered Patent Database of the the United States Patent and Trademark Office, the World Development Indicators, and Barro and Lee (2013).

[7]‘Advanced’ countries are in 2014 ’Australia’, ’Austria’, ’Belgium’, ’Canada’, ’Denmark’, ’Finland’, ’France’, ’Germany’, ’Iceland’, ’Israel’, ’Japan’, ’Netherlands’, ’New Zealand’, ’Norway’, ’Singapore’, ’Sweden’, ’Switzerland’, ’United Kingdom’, ’United States’. The composition of the two other clubs in 2014 can be found in Tables 1 and 2.

Table 1: Technology convergence index: marginalized versus advanced

Country	Index
'Algeria'	1.12
'Bangladesh'	1.13
'Burkina Faso'	1.00
'Côte d''Ivoire'	1.06
'Egypt'	1.18
'Ethiopia'	1.20
'India'	1.09
'Madagascar'	1.13
'Malawi'	1.07
'Mali'	1.06
'Morocco'	1.09
'Mozambique'	1.01
'Niger'	1.08
'Nigeria'	1.04
'Pakistan'	1.05
'Senegal'	1.10
'Uganda'	0.98
Club	**1.07**

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Table 2: Technology convergence index: followers versus advanced

Country	Index
'Argentina'	0.92
Bolivia'	0.90
'Brazil'	0.93
'Cameroon'	0.75
'Chile'	0.94
'China'	0.73
'Colombia'	0.81
Congo'	1.00
'Costa Rica'	0.76
'Cyprus'	1.11
'Dominican Republic'	0.83
'Ecuador'	0.88
'Ghana'	0.83
'Greece'	0.93
Hong Kong'	0.76
'Indonesia'	0.87
Iran'	0.98
'Ireland'	0.88
'Italy'	0.85
'Jamaica'	1.05
'Jordan'	0.92
'Kenya'	0.70
Korea'	0.81
'Luxembourg'	0.86
'Malaysia'	0.90
'Malta'	0.91
'Mexico'	0.95
'Peru'	0.77
'Philippines'	0.75
'Portugal'	0.94
'Romania'	0.94
'South Africa'	0.93
'Spain'	0.85
'Sri Lanka'	0.79
Syria'	0.96
'Taiwan'	0.77
Tanzania'	0.89
'Thailand'	0.88
'Trinidad and Tobago'	1.15
'Tunisia'	0.98
'Turkey'	0.93
'Uruguay'	1.00
Venezuela'	1.24
'Zambia'	0.99
Club 7	**0.89**

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