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Advantageous defensive efforts in contests

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Abstract

Understandably, defensive sabotage or destructive efforts devoted to reducing the opponent's score are an option used by contestants who endeavor to enhance their objective of winning the contest. This paper focuses on two questions. First, does this option necessarily reduce the effective productive efforts of the contestants? Second, can such efforts also help to attain an objective that is affected by the contestants' effective efforts and by the gap in their performance? The positive answer to these questions is our novel contribution to the literature on contests with sabotage.

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1. Introduction

The assumption of two contestants who engage in destructive activities of defensive nature is very plausible in the context of war and conflict, sports, political campaigning, and litigation. Such activities are intended to reduce the opponent's score, whereas standard activities are devoted to raising one's own score. Three recent studies that invoke this assumption are Kimbrough et al. (2020), Doğan et al. (2019), Bernhardt and Ghosh (2020), and Baharad et al. (2021).

A common assumption in the literature is that a contest designer wishes to induce high effort (as in business organizations and in sports). Applying the extended contest setting of Baharad et al. (2021), which allows for productive (offensive) and destructive (defensive) efforts, we focus on the contest outcome in terms of the exerted *effective efforts* and *equality*. Effective efforts disregard direct defensive (sabotage) efforts but take into account their decaying effect on the productive (offensive) efforts. Equality is defined as the contestants' relative winning probabilities. If a designer is concerned about these variables, he may be able to affect them by controlling the marginal cost of the defensive efforts and invoke a variant of an affirmative-action policy. The purpose of this paper is to establish the reducing effect of defensive efforts on exerted effective efforts and clarify the potential advantageous consequences of defensive (sabotage) activities for an objective that hinges on both effective efforts and equality. The latter goal yields a novel rationalization for engaging in defensive (sabotage) activities.

2. The extended contest

Our setup is based on an extended version of Tullock's two-player contest, in which sabotage is modeled as in Doğan et al. (2019) and Baharad et al. (2021).[1] A real-world example of the extended contest is litigation. Gathering evidence, summoning witnesses, paying lawyers, and any other imaginable action that a plaintiff and a defendant take throughout a civil trial may constitute defensive (sabotage) measures provided they are invoked for the sake of damaging the

[1] The focus in Doğan et al. (2019) is on free riding in teams; therefore, they apply a stylized two-member two-team setting with four players. What we adopt from their model is the specific assumption regarding the way sabotage affects productive efforts. The focus in Baharad et al. (2021) is on the conditions ensuring (i) the existence of sabotage and excessive sabotage, (ii) larger sabotage efforts by the favorite player, and (iii) widening or narrowing the gap between winning probabilities relative to a contest without sabotage.

adversary. Assuming complete information, V_i, $i=1,2$ denotes the risk-neutral players' valuations of winning the contest. Each contestant invests in offensive (productive) and defensive (destructive or sabotage) efforts in amounts x_i and s_i. Sabotage is the exertion of destructive effort against rivals with the intention of reducing their likelihood of winning. A significant variable in our setting is x_{ie}, the effective effort of Contestant i, which depends on his offensive effort and its effectiveness, γ_i (productivity per unit of productive effort) and on the defensive effort of his rival, j, $j \neq i$,

$$(1)\, x_{ie} = \frac{\gamma_i}{1+\gamma_j s_j} x_i$$

The winning probability of i is given by the extended lottery proposed by Tullock:

$$(2) \qquad p_i(x_1, x_2, s_1, s_2) = \frac{\frac{\gamma_i}{1+\gamma_j s_j} x_i}{\left(\frac{\gamma_i}{1+\gamma_j s_j} x_i\right) + \left(\frac{\gamma_j}{1+\gamma_i s_i} x_j\right)}, i=1,2$$

The linear cost of offensive and defensive effort is *assumed* ¿ *be equal* ¿ x_i and $\mu_i s_i$, *respectively*, where 1 (μ_i ¿ is the common positive marginal cost of i's unit of offensive (defensive) effort. The relationship between μ_1 and μ_2 is unclear. For instance, in the litigation example, a weaker case of the defendant requires fewer resources for sabotage (defending the plaintiff's stronger case). The plaintiff's stronger case, however, is typically more comprehensive, (persuasive on several issues). This means that the defendant may have more alternatives for sabotage activities, among which she can select the least costly. With no loss of generality, we normalize the effectiveness of the weaker Contestant 2 to 1 and denote by $\gamma > 1$ the effectiveness of the strong Contestant 1. The payoff functions are:

$$(3)\, \pi_1 = V_1 \frac{\frac{\gamma}{1+s_2} x_1}{\left(\frac{\gamma}{1+s_2} x_1\right) + \left(\frac{1}{1+\gamma s_1} x_2\right)} - x_1 - \mu_1 s_1$$

$$(4)\ \pi_2 = V_2 \frac{\frac{1}{1+\gamma s_1}x_2}{\left(\frac{\gamma}{1+s_2}x_1\right)+\left(\frac{1}{1+\gamma s_1}x_2\right)} - x_2 - \mu_2 s_2$$

In an interior equilibrium, the productive and destructive efforts are:[2]

$$(5)\ x_1^{¿}=A V_1, \quad x_2^{¿}=A V_2, \quad s_1^{¿}=B \mu_2 V_1-\frac{1}{\gamma} \quad \text{and} \quad s_2^{¿}=B \mu_1 V_2-1$$

where, after normalization of the players' effectivity, $\gamma_1=\gamma, \gamma_2=1$, $A=\frac{\gamma^2 V_1^2 V_2^2 \mu_1 \mu_2}{(\mu ¿¿ 2 V_1^2 \gamma^2+\mu_1 V_2^2)^2 ¿}$,

and $B=\frac{\gamma^2 V_1^2 V_2^2}{(\mu ¿¿ 2 V_1^2 \gamma^2+\mu_1 V_2^2)^2 ¿}$. By (5), the non-negative sabotage efforts of the contestants require that

$$(6)\ s_1=\frac{\gamma^2 \mu_2 V_1^3 V_2^2}{(\mu ¿¿ 2 V_1^2 \gamma^2+\mu_1 V_2^2)^2-\frac{1}{\gamma} \geq 0 ¿} \quad \text{or} \quad \mu_1 \leq \frac{V_2 \sqrt{\gamma^3 \mu_2 V_1^3}-\gamma^3 \mu_2 V_1^2}{V_2^2} \Leftrightarrow \mu_2 \leq \frac{V_2^2}{\gamma V_1}$$

and

$$(7)\ s_2=\frac{\gamma^2 \mu_1 V_2^3 V_1^2}{(\mu ¿¿ 2 V_1^2 \gamma^2+\mu_1 V_2^2)^2-1 \geq 0 ¿} \quad \text{or} \quad \mu_2 \leq \frac{\gamma V_1 \sqrt{\mu_1 V_2^3}-\mu_1 V_2^2}{V_1^2 \gamma^2} \Leftrightarrow \mu_1 \leq \frac{\gamma^2 V_1^2}{V_2}$$

By (1) and (5), the contestants' effective efforts are:

$$(8)\ x_{1e}=\frac{V_1 \gamma \mu_2}{V_2} \quad \text{and} \quad x_{2e}=\frac{V_2 \mu_1}{\gamma V_1}$$

So the total effective efforts are:

$$(9)\quad T x_e=\frac{V_1^2 \gamma^2 \mu_2+\mu_1 V_2^2}{V_2 \gamma V_1}$$

[2]These efforts are obtained from the first-order conditions. It can be verified that the second-order conditions for the maximization of the players' expected payoffs are satisfied.

By (2) and (8), the equilibrium odds ratio of winning is:

(10) $$\frac{p_{1e}}{p_{2e}}=\frac{\mu_2\gamma^2 V_1^2}{\mu_1 V_2^2}$$

For equality of winning probabilities to exist, $\frac{p_{1e}}{p_{2e}}=1$, *the following must obtain*:

(11) $$\mu_1=\frac{\gamma^2 V_1^2}{V_2^2}\mu_2$$

In the benchmark contest without sabotage, the contestants' efforts are equal to:

(12) $$x_1^b=\frac{\gamma V_1^2 V_2}{(\gamma V_1+V_2)^2} \quad \text{and} \quad x_2^b=\frac{\gamma V_2^2 V_1}{(\gamma V_1+V_2)^2}$$

Hence, their effective efforts are (note that only 1's effort changes):

(13) $$x_{1e}^b=\frac{\gamma^2 V_1^2 V_2}{(\gamma V_1+V_2)^2} \quad \text{and} \quad x_{2e}^b=\frac{\gamma V_2^2 V_1}{(\gamma V_1+V_2)^2}$$

and, therefore, the total effective effort is:

(14) $$T x_e^b=\frac{\gamma V_1 V_2(V_1+V_2)}{(\gamma V_1+V_2)^2}$$

By (2) and (13), the odds ratio in the benchmark case is:

(15) $$\frac{p_{1e}^b}{p_{2e}^b}=\frac{\gamma x_{1e}^b}{x_{2e}^b}=\frac{\gamma V_1}{V_2}$$

3. Advantageous defensive activities (sabotage)

In the absence of the option of defensive activities (sabotage), the results obtained in the classical Tullock lottery serve as our benchmark. We assume the desirability (undesirability)(of high effective effort, as in business organizations and in sports (litigation) and the desirability of equality of the contestants' performance. Increased equality (contest tightness) and large effective efforts may be desirable for political or economic reason—e.g., to foster voters' participation in an election, or attract more spectators to a competition. In litigation, equality may be desirable because it increases the likelihood of out of court settlement. However, reduced effective efforts may be preferred because they imply more refined arguments that facilitate the task of the judge's ruling.

Let us turn then to demonstrating the potential advantage of defensive activities (sabotage).

Given γ, V_1 and V_2, with no loss of generality, we assume that $\frac{\gamma V_1}{V_2} \geq 1$. That is, Contestant 1 has an advantage over Contestant 2. Since we focus on effective efforts and equality, we note that,

by (9) and (14), $T x_e \geq T x_e^b \Leftrightarrow \mu_1 \geq \frac{T x_e^b}{V_2} - \frac{V_1^2 \gamma^2}{V_2^2} \mu_2$ and by (10) and (15), $\frac{p_{1e}}{p_{2e}} \leq \frac{p_{1e}^b}{p_{2e}^b} \Leftrightarrow$

$\mu_1 \geq \frac{\gamma V_1}{V_2} \mu_2$. As will be shown, both can be satisfied simultaneously only with equality. Our main result presents the upper bound of the effective efforts and clarifies the potential advantage of defensive activities (sabotage).

Proposition : Given γ, V_1 and V_2, and degree of equality *t*, $\frac{\gamma V_1}{V_2} \geq t \geq 1$,

$$T x_e^b - T x_e = \frac{\gamma V_1 V_2}{(\gamma V_1 + V_2)} - \frac{t V_2}{(t+1)} = \frac{V_2 (\gamma V_1 - t V_2)}{(t+1)(\gamma V_1 + V_2)}$$

and

$$T x_e^b - T x_e \geq 0.$$

Proof:

By (10), equality of degree t means that $\frac{p_{1e}}{p_{2e}}=\frac{\mu_2\gamma^2 V_1^2}{\mu_1 V_2^2}=t$ or

$$(16) \qquad \mu_2=\frac{t\mu_1 V_2^2}{\gamma^2 V_1^2}$$

and, therefore, by substitution into (9), we get:

$$(17) \qquad T\,x_e=\frac{V_1^2\gamma^2\mu_2+\frac{\mu_2\gamma^2 V_1^2}{t\,V_2^2}V_2^2}{V_2\gamma V_1}=\frac{V_1^2\gamma^2\mu_2(1+t)}{t\,V_2\gamma V_1}=\frac{(1+t)\gamma V_1\mu_2}{tV_2}$$

Since $\frac{\frac{\partial T\,x_e}{\partial\mu_1}\wedge\partial T\,x_e}{\partial\mu_2}$ are positive, maximization of $T\,x_e$ requires that μ_1 and μ_2 are as large as possible under constraints $s_i\geq 0$, (6) and (7). This, in turn, implies that the constraint on s_1 is binding. Substituting μ_2 in (16) into the binding constraint (6), we get:

$$(18) \qquad \mu_1=\frac{V_2\sqrt{\gamma^3\frac{t\,\mu_1 V_2^2}{\gamma^2 V_1^2}V_1^3}-\gamma^2\frac{t\,\mu_1 V_2^2}{\gamma^2 V_1^2}V_1^2}{V_2^2}$$

or, $\quad \mu_1=\sqrt{t\gamma\mu_1 V_1}-t\mu_1$ or $\qquad \sqrt{\mu_1}(1+t)=\sqrt{t\gamma\,V_1}$

Hence, $\mu_1=\frac{t\gamma\,V_1}{(1+t)^2}$. Substituting this μ_1 into (16) means that $\mu_2=\frac{t^2 V_2^2}{(1+t)^2\gamma\,V_1}$.

Finally, substituting this μ_2 *into* (17) *gives that* $\arg\max_{\mu_1,\mu_2} T\,x_e=\frac{t\,V_2}{(t+1)}$. *Hence,*

$$T\,x_e^b-T\,x_e=\frac{\gamma V_1 V_2}{(\gamma V_1+V_2)}-\frac{t\,V_2}{(t+1)}=\frac{V_2(\gamma V_1-t\,V_2)}{(t+1)(\gamma V_1+V_2)}$$

Since, by assumption, $\frac{\gamma V_1}{V_2}\geq t\geq 1$, $T\,x_e^b-T\,x_e\geq 0$.

Q.E.D.

The proposition has three interesting implications.

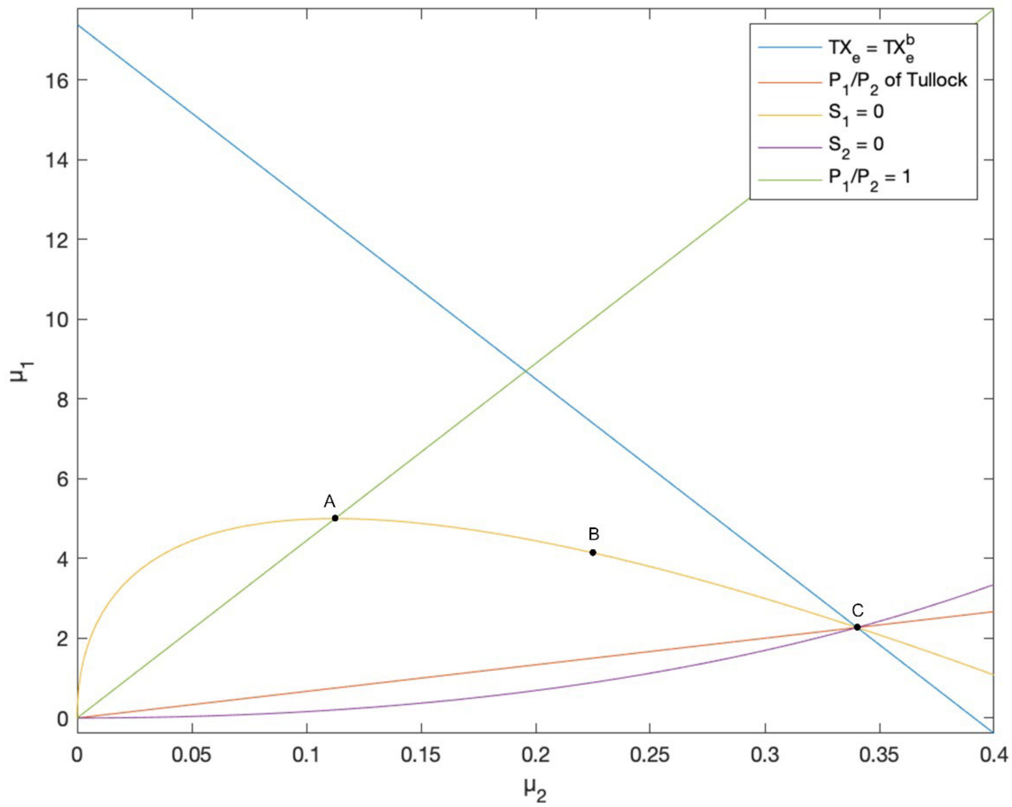
(i) Any desired degree of equality that involves the largest possible effective efforts requires that only Contestant 2 engages in sabotage; the constraint $s_1 \geq 0$ is *binding*.

(ii) *The maximal effective effort* increases with *t* and its upper bound is that obtained in the benchmark contest without defensive efforts. That is, when $t = \frac{\gamma V_1}{V_2}$, $T x_e = T x_e^b$; the costs of defensive activity eliminate the incentives to engage in sabotage.

(iii) *The* tradeoff between the enhancement of equality and the foregone effective efforts is given in the first part of the proposition.

In the litigation example, it makes sense to assume that the likelihood of settlement is positively related to equality in performance; closer cases would usually be settled. If the parties' likelihood of reaching an out-of-court settlement is of central importance, then the option of defensive activities may be advantageous because it fosters this objective. This conclusion is further strengthened when the designer prefers lower effective efforts.

Figure 1: Constraints ensuring potentially advantageous non-negative sabotage in terms of effective efforts and relative performance for $V_1 = 10, V_2 = 3 \wedge \gamma = 2$



Five constraints are depicted in Figure 1: $Tx_e =$¿ Tx_e^b (blue), $\frac{p_{1e}}{p_{2e}} = \frac{p_{1e}^b}{p_{2e}^b}$ (orange), $\frac{p_{1e}}{p_{2e}} = 1$ (green), $s_1 = 0$(¿)$, \wedge s_2 = 0$(¿). The first constraint can be satisfied only at$\mu_1 =$¿2.27 and

Table 1: Numerical illustration corresponding to Figure 1

Point	μ_1	μ_2	Tx_e	Tx_e^b	$\frac{p_{1e}}{p_{2e}}$	$\frac{p_{1e}^b}{p_{2e}^b}$
A	5	0.11	1.5	2.61	1	6.66
B	4.14	0.23	2.12	2.61	2.41	6.66
C	2.27	0.34	2.61	2.61	6.66	6.66

$\mu_2 =$¿0.34; see Table 1 and point C in the figure. At this point, the blue line intersects the graphs that represent three other constraints: the boundary of the non-negativity constraints of the defensive efforts and the line indicating that the odds ratio of winning is equal to that of the benchmark contest, 6.66. In turn, the effective effort, 2.61, is equal to that of the no-sabotage contest. The other extreme case is represented by Point A, the intersection of the graph representing maximal equality, $\frac{p_{1e}}{p_{2e}}=1$, with the graph representing the boundary of the constraint $s_1 \geq 0$. Now $\mu_1=5 \wedge \mu_2=0.11 \wedge \mathit{the\ constraint}\ \mu_2 \geq 0\ \mathit{is\ not\ binding}$.To attain perfect equality, the effective efforts must be reduced by (2.61 – 1.5)=1.11 relative to the benchmark contest. The intermediate case, Point B on the boundary of the constraint $s_1 \geq 0$,represents an increase in equality to $\frac{p_{1e}}{p_{2e}}=2.41$. At this point, $\mu_1=4.14 \wedge \mu_2=0.23 \wedge, \mathit{again}, \mathit{the\ constraint}\ \mu_2 \geq 0\ \mathit{is\ not\ binding}$. To attain less equality (a larger t), a smaller sacrifice of (2.61 – 2.12)=0.39 in the effective efforts relative to the benchmark contest is needed. As implied by the Proposition, there is necessarily a trade-off between increased equality and total effective efforts. In the litigation example, defensive activities are necessarily advantageous, see first paragraph of this section. In general, if the significance assigned to increasing equality is sufficiently high relative to the significance of large effective efforts, defensive activities (sabotage) will be advantageous.

4. Conclusion

The higher productivity of Contestant 1 and the difference in winning valuations capture the asymmetry between the contestants. Instead of common affirmative action, which resorts to multiplicative bias or head starts (Mealem and Nitzan, 2016), in a contest with defensive activities the different marginal costs of sabotage are the means by which equality and effective efforts can be affected. Four real-world examples, corresponding to the aforementioned applications of the model, in which the contest designer is able to control the marginal costs of sabotage, are briefly discussed below. In all of them, the defensive activities (sabotage) are observable. In the war and conflict example, the marginal cost of sabotage can be affected by sanctions imposed by the UN or any intervening state. The economic incentives of such sanctions are usually direct. In soccer, the designer of a match can control the marginal cost of sabotage by determining sanctions commensurate with misconduct of the teams. In this context, yellow and red cards are often used as proxies for sabotage. Finally, in a litigation contest or a political campaign, the designer of the legal system can establish punishments corresponding to different degrees of sabotage, thereby controlling the marginal cost of sabotage.

Our main contribution is the clarification of the possible advantage of defensive activities (sabotage) in promoting a dual objective that depends on equality between the contestants and on their effective efforts.

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