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Abstract

The paper develops a "Catch-22" model of optimal consumption and portfolio allocations in continuous time in the absence of assets enabling simultaneous insurance against potentially correlated equity returns risk and inflationary news risk. As such, no portfolio choice can attain full insurance, involving trading off exposure to equity risk for exposure to innovations against expected inflation. Based on the formulation of the problem via the Hamilton-Jacobi-Bellman equation and the reduction of the stochastic dynamic optimisation programme to a non-stochastic recursive problem, analytical results from the model highlight a critical dimension to pricing risk at the margin in the model, with implications for an analysis of the Equity premium puzzle relative to standard Mehra-Prescott environments. The paper shows that a potential (partial) answer to the equity premium puzzle, in a Catch-22 environment resulting from a combination of stochastic inflation and incomplete-markets, (i) plausibly comes down to how heavy-tailed is the distribution of inflationary news, and (ii) how heavy-tailed such distribution must be to explain a larger positive premium in turn depends on the nominal-real covariance. A combination of heavy-tailed distributions of innovations or news against the expected inflation rate and a negative nominal-real covariance, in particular, can explain large(r) Equity Premia. The findings also highlight a new nexus between monetary policy regimes and the Premium.

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1. Introduction and Background

The Equity Premium puzzle refers to the difficulty of explaining the discrepancy between stock returns and the returns to bonds; that is, the premium the former have historically commanded over the latter. Since the foundational contribution by Mehra and Prescott (1985), the problem has been in particular the fact that, pinning down the premium in terms of the standard combination of risk-aversion and risk in returns to equity, while qualitatively feasible, quantitatively requires implausible risk-aversion among agents – in the sense that they do not match the values inferred from standard macroeconomic models elsewhere in the field – or variances in the returns to equity. Recently, an interest has developed into the implications of stochastic inflation paths and, as a consequence in a New-Keynesian world, monetary policy for stock returns (Kehre and Lenel, 2022; Boons et al., 2019; Song, 2017). Naturally, this research strand has immediate applications to contributing to explaining the equity premium puzzle.

In this paper, I contribute to this literature and its application to the equity premium puzzle by building a simple model of portfolio selection for risk-averse households in continuous time, kindred in spirit to the earlier continuous-time Mertonian and Fisherian tradition (Chang, 2004), and presenting some preliminary results from the analysis relevant to the presented puzzle. The novelty of the contribution is twofold. First, the core of the model is a "Catch-22" scenario resulting from the absence of assets enabling simultaneous insurance against potentially correlated equity returns risk and risk in the form of inflationary news. As such, no portfolio choice can attain full insurance and always involve trading off exposure to equity risk for exposure to inflationary news. This places the proposed approach in the "Market Failure" strategy to explain the equity risk premium, in particular as it may stem from incomplete or missing markets.

Second, on an innovative methodological end relative to the discrete time approach taken in close works in the literature, the application reaps off the benefits of casting such problem in a continuous time framework, in line with the "continuous time" revolution in macro-modelling. Particularly, subject to finding a suitable recursive representation of the dynamic stochastic optimisation problem in the form of a HJB equation, (i) the problem becomes non-stochastic and (ii) both the problem and the equilibrium condition for the portfolio allocation becomes essentially static. This approach thus has the general advantage of not relying on numerical methods for solving for allocation choices, which are thus in closed form. Second, in the specific model I develop, this comes with the benefit of identifying the set of *equilibrium equity premia* in terms of underlying fundamental parameters: in addition to the variance of the equity returns emphasised in benchmark models of the equity risk premium, the variance of noise or innovations (inflationary news) against expected inflation, and a parameter governing nominal-real covariance similar to Campbell et al. (2017). Consequently, we can analytically and clearly disentangle the effect on admissible equity premia of each such channel, all of which are to some degree emphasised in the recent empirical and discrete-time theoretical literature.

Based on such exercise, the paper shows that a potential (partial) answer to the equity premium puzzle as such is that, with stochastic inflation and a resulting "Catch-22" situation due to the incomplete-markets environment, (i) it plausibly lies in how heavy-tailed is the distribution of inflationary news, and (ii) how heavy-tailed such distribution must be to explain a larger positive premium depends on the nominal-real covariance. The model findings also bear applications to assessing the nexus between monetary policy regimes and Premium. Section 2 briefly reviews the relevant literature, Section 3 presents the model and solution approach, and Section 4 discussed the main results.

2. Related Literature and Contribution

A number of strategies have been proposed to explain at least part of the Equity Premium puzzle. These have included breaking with CRRA-type preferences conflating risk-aversion and elasticity of intertemporal-substitution measures through the use of alternative programmes, specifically Epstein-Zin preferences (Boons et al., 2019). Another one has been to introduce extreme events, such as 1929-style financial panics and stock market crashes, implying that even though such events are unlikely to materialise, the non-zero risk of diluvial losses is still priced into the premium (Julliard and Ghosh, 2012). As outlined in the introduction, an alternative strategy can come from a recent literature renovating an interest in the asset-pricing consequences of stochastic inflation (Campbell et al., 2017; Camba-Mendez and Werner, 2017), and as a consequence on asset pricing as a transmission channel for monetary policy, either as indirect transmission mechanisms alternative to the canonic one in NK models or as secondary targets (intended or otherwise) of monetary policy rules (Kehre and Lenel, 2022; Bernanke and Kuttner, 2004). Naturally, this research line, insofar as it can pinpoint asset pricing consequences of stochastic inflation and inflationary or monetary news, bears applied relevance to the equity premium puzzle. The paper contributes to this literature and bridging it explicitly to the one on explaining part of the equity premium (rather than to the inflation risk premium priced in bonds returned) both substantively, directly exploring the transmission from stochastic inflation to the set of equilibrium equity premia and establishing some new analytical results, and methodologically, developing a model for examining such transmission that enjoys some comparative advantages relative to the literature. On the methodological end, work on the asset pricing implications of stochastic inflation has tended to be either in the context of discrete-time numerical DSGE models (Campbell et al., 2017) or of empirical time series analysis (Camba-Mendez and Werner, 2017). A key benefit of the continuous time environment in which the stochastic dynamic optimisation model is developed is that, as noted, the analysis can proceed largely through analytical means, exploiting the advantage that – contrary to its counterpart in discrete time models – the HJB is non-stochastic. As a consequence, the established propositions are able to effectively disentangle the effects of two separate channels of import to more complex DSGE or macroeconometric analyses of the asset pricing implications of stochastic inflation such as the above ones: the heavy-tailedness or dispersion of news against expected inflation and the sign of the covariance between inflation and risky stock returns. As such, the modelling framework proposed here, reaping off the benefit of the ongoing continuous-time turn in macroeconomics, can be viewed as complementing the perspective of richer DSGE or empirical models with less clear-cut insights into the transmission mechanism of interest. Second, through such alternative methodology, the simple environment novelly stresses how crucial to the transmission might be in particular a Catch-22 situation resulting from incomplete markets, i.e. the inability of households to simultaneously insure against equity and inflation/deflation risk. The transmission mechanism pinned down analytically through the continuous time model, as such, fundamentally boils down to the institutional environment of economies.

Substantively, the paper builds on such methodological change of gear to establish a number of results. With reference to the above literature, the most critical result is that while nominal-real covariance plays a role in shaping the transmission from stochastic inflation to an upward or downward pressure in the Equity Premium, this ceases to be true at sufficiently heavy-tailed distribution of the news/innovations against the expected inflation rate, and is not itself critical as the latter result in larger premia even in environments with zero nominal-real covariance. This is a result of the fact that the analytically derived marginal impact of the latter on the minimum equilibrium equity premium is nonlinear, and particularly quadratic, with nominal-real covariance only affecting the coefficient on the first-order or linear effect. While through the model one obtains, therefore, results sympathetic and in turn corroborated by the emphasis on nominal-real covariance as a key driving factor in the surveyed literature, an innovation relative to such literature is the emphasis on the heavy-tailedness of news against expected inflation as the primary driving factor behind the equity premium (or, alternatively, a weaker or negative bond's relative inflation risk premium), as opposed to the nominal-real covariance.

3. A Catch-22 Model

We assume an institutional structure akin to incomplete markets, in the sense that individuals do not have access to tools allowing to insure against (potentially interacting) inflationary news and equity returns risk simultaneously. In other words, achieved insurance against inflation is itself a risky outcome for which no secondary insurance market exists, and achieved insurance against bad equity returns draws (hedging) is a risky outcome on the grounds of inflationary news, for which no secondary insurance market exists. Absent inflation news (e.g. with "monetary surprises), and up to inflation expectations, bonds could then be safely used to as *hedges* – here, given the presence of a stochastically evolving price path, such role is partly mitigated, to converge to the vernacular in Campbell et al., (2017), by the fact that hedging involves an *inflation-bets*. The kernel of the economy is thus an incomplete or missing asset-markets model, which places the approach in the "Market Failure" tradition and which we now proceed to formalise.

3. Stochastic Diffusion Models for Asset Returns and Prices

In any period, the bond delivers a time-invariant, non-stochastic *nominal* return Q_t^b. This is modelled as the geometric process governed by the (ordinary deterministic) differential equation

$$\frac{dQ_t^b}{Q_t^b} = R_b dt \tag{1}$$

The other asset is an equity, labelled y, delivering a risky real-return q_t^y governed by the Stochastic Differential Equation (SDE) or *Diffusion*:

$$\frac{dq_t^y}{q_t^y} = r_y dt + \sigma_y dZ_t^y, \tag{2}$$

where Z_t^y with $dZ_t^y = Z_{t+\Delta t}^y - Z_t^y$ as $\Delta t \to 0$ is a standard Brownian Motion (or Wiener Process). The equity return is thus modelled by a Geometric Brownian Motion (GBM). R_b, r_y, and σ_y are exogenously given, time-invariant parameters, respectively capturing the *conditionally (expected) nominal rate of return to bonds*, the *conditionally expected real rate of return to equity*, and the *dispersion* of the real return of equity around its conditional mean, i.e. a measure of equity risk. While these are exogenous parameters, because they define the premium, only some combinations will be admissible for an interior optimal solution to the household's problem to exist. We will insist on this strategy (Chang, 2004), to draw out the implications for the equity premium. Based on the above, we also define the *nominal* return to equity as:

$$Q_t^y = P_t q_t^y,$$

Where P_t is the price level in money terms. The price level is also assumed, finally, to evolve stochastically according to the stochastic diffusion:

$$\frac{dP_t}{P_t} = \pi dt + \sigma_P dZ_t^P, \tag{3}$$

Where π is the expected inflation rate, Z_t^P is a Brownian motion modelling noise around the inflation trend, and σ_P represents the dispersion of the price around the expected inflation rate. That is, σ_P represents a measure of "inflation surprises" or "inflation news" around inflation expectations. Higher values of σ_P corresponds to heavier-tailed distributions of inflation news, i.e. to an environment in which large positive or negative monetary surprises, respectively translating into inflationary or deflationary innovations against the expected inflation rate, occur with larger probabilities. The parameter, together with the real-nominal covariance defined later, plays a key role in the Catch-22 model and in understanding the expansionary role of inflation uncertainty on the equity premium. In setting up the dynamic optimisation problem faced by the household and subsequently deriving its recursive representation and solution through the Hamilton-Jacobi-Bellman (HJB) equation, Ito's Lemma is used extensively. It is worth recalling it in the used "differential" form, together with some key notational conventions, but the reader is referred to Chang (2004) and Pham (2009) for a respectively pedagogical and rigorous treatment of stochastic processes, Ito calculus, and applications to the recursive treatment of continuous time stochastic optimisation problems, as well as for some earlier Mehra-Style precursors to the model developed here. Ito's lemma says that, given a one-dimensional Ito process X_t defined by the SDE:

$$dX_t = \mu(X_t, t)dt + \sigma(X_t, t)dB_t$$

A sufficiently smooth (twice-differentiable) function $y = f(X_t)$ also evolves according to the SDE:

$$df(X_t) = f'(X_t)dX_t + \frac{1}{2}f''(X_t)d[X]_t$$

Where $d[X_t] = dX_t dX_t$ is the quadratic variation of the process. The latter is sometimes denoted, in short-cut derivations, e.g. with reference to the Taylor Expansion approach to deriving the formula, as $(dX_t)^2$ (cf. Chang, 2004). When y is a function $f(X_t, Y_t)$ of multiple Ito processes the multidimensional Ito's lemma also includes the quadratic covariation term $d[X, Y]_t = d[Y, X]_t = dX_t dY_t$. Putting this to use immediately, the above *nominal return* to equity can be shown, through Ito's Lemma, to follow in turn the SDE:

$$\frac{dQ_y}{Q_y} = (r_y + \pi + \lambda\sigma_P\sigma_y)dt + \sigma_P dZ_t^P + \sigma_y dZ_t^y \tag{4}$$

Where λ is the "instantaneous" nominal-real covariance rate, satisfying (Chang, 2004)

$$\lambda = \lim_{dt\to 0}\frac{1}{dt}E_t\{d[Z^P, Z^y]_t\}.$$

Equations (1)-(4) form the stochastic core of the incomplete markets model, once the constraint on the optimising household is introduced.

3. Household Stochastic Dynamic Optimisation Problem constrained by a SDE for Real Wealth

The household's programme, starting at some time $t_0 \in \mathbb{R}_+$ is assumed to consist of the standard continuous-time stochastic dynamic optimisation problem:

$$\max_{\{c_s\}_{s\in[t_0,\infty]},\{b_s\}_{s\in[t_0,\infty]},\{y_s\}_{s\in[t_0,\infty]}} \mathbb{E}_{t_0}\int_{t_0}^{\infty} e^{-\rho(s-t_0)}u(c_s)ds \tag{5}$$

Where, for some arbitrary period $s = t$, $(c_t, b_t, y_t) \in \mathbb{R}^3_{++}$ are the period t-controls in terms of respectively consumption, bond position, and equity at the end of that period given by the control process $c(\omega, s), b(\omega, s), y(\omega, s)$ at time s=t. The time-invariant function $u(\cdot)$ that models the instantaneous return in any period is assumed to be of the CRRA type, hence continuous and twice differentiable, while $e^{-\rho\Delta t}$ for $\rho \in (0, 1)$ is the factor by which the utility of consumption Δt-time units ahead is discounted.Given *nominal wealth* W_t, assumed strictly positive, and the price level P_t, the control processes are assumed to display continuous sample functions and to be *admissible* – technical details on the definition of admissible controls given process modelling the state, assumed to be càd-làg, are in Chang (2004) or Pham (2009).[1] The relevant state of the model, in addition to the price level P_t, is nominal wealth. We assume that, over any small time interval $[t, t + \Delta t]$ where $\Delta t \to 0$, over which "stock-holdings" controls b_t, y_t remain fixed and "flows" control c_t remain constant, nominal wealth evolves accoridng to the SDE:

$$dW_t = P_t(h - c_t)dt + b_t dQ_t^b + y_t dQ_t^y \tag{6}$$

[1]Technically, akin to a budget constraint in static optimisation problems, this is needed to ensure that that the controlled diffusion process governing the state, here dW_t, has a unique solution given an initial condition $W = W_0 \in \mathbb{R}_{++}$. Formally, the below dynamic optimisation problem is thus with respect to the choice of processes from the set of admissible controls, i.e. such that (c_t, b_t, y_t) and W_t, P_t uniquely solve the SDE for nominal wealth.

Where $h \in \mathbb{R}_+$ models the non-financial real income flow $h_t = hdt$, assumed constant for simplicity. Plugging-in the SDEs modelling stochastic asset returns (1) and (3):

$$dW_t = P_t(h - c_t)dt + b_t \left[Q_t^b R_b dt\right] + y_t \left[Q_t^y (r_y + \pi + \lambda\sigma_P\sigma_y)dt + Q_t^y \sigma_P dZ_t^P + Q_t^y \sigma_y dZ_t^y\right] \quad (7)$$

As standard in portfolio optimisation models, we can reduce the number of controls by defining the time-t share of an asset z in time-t nominal as $s_{z,t} = Q_t^z z_t / W_t \in [0,1]$, and noting such shares sum to one. Letting the share of equity $s_{y,t} = Q_t^y y_t / W_t \in [0,1]$, implying the share of wealth held in bonds is $s_{b,t} = 1 - s_{y,t}$, the SDE for nominal wealth constraining the household's optimisation problem can be conveniently rewritten as:

$$dW_t = P_t(h - c_t)dt + W_t R_b dt + s_{y,t} W_t \left[\Psi dt + \lambda\sigma_P\sigma_y dt + \sigma_P dZ_t^P + \sigma_e dZ_t^y\right]. \quad (8)$$

Where we have made explicit the equity premium $\Psi = r_e + \pi - R_b$, i.e. the (equilibrium) difference between the returns paid by the two assets. The interpretation of the above SDE is standard, i.e. over a small time-interval, nominal wealth behaves as a diffusion: the conditionally expected increment is given by the nominal return to a portfolio consisting of only safe assets (bonds) plus, depending on the share allocated to the risky asset, the premium in excess of the safe nominal rate Ψ payable on average to wealth held as equity. To make further progress, we formulate the above problem in such a way to reduce the number of state variables involved, and simplify the recursive formulation and analysis of the associated HJB equation, by working with real wealth and re-defining the controls to portfolio shares. To obtain the SDE for real wealth satisfied by admissible processes, we can exploit Ito's Lemma for (multivariate) twice-differentiable functions of diffusion processes. Let real wealth be defined as a function of nominal and real wealth via $w_t = W_t / P_t := g(W_t, P_t)$. Then by Ito's Lemma, over any small time-interval the diffusion $w_t = g(W_t, P_t)$ is modelled by the SDE:

$$dw_t = g'_{W_t}(W_t, P_t)dW_t + g'_{P_t}(W_t, P_t)dP_t + \frac{1}{2}g''_{W_tW_t}(W_t, P_t)d[W]_t + g''_{P_tP_t}(W_t, P_t)d[P]_t +$$

$$\frac{1}{2}g''_{W_tP_t}d[W,P]_t + \frac{1}{2}g''_{P_tW_t}d[P,W]t,$$

and hence

$$dw_t = \frac{1}{P_t}dW_t - \frac{W_t}{P_t}\frac{dP_t}{P_t} + \frac{W_t}{P_t}\frac{d[P]_t}{P_t^2} - \frac{1}{P_t^2}d[W,P]_t. \quad (9)$$

Recalling the notational convention for quadratic variation $d[X]_t = dX_t dX_t$ and quadratic covariation $d[X,Y]_t = dX_t dY_t = d[Y,X]_t$, and plugging-in the definition of real wealth, equation (1), (3), (4), and (6), under the rules of stochastic calculus we arrive at the SDE for real wealth that admissible pairs of controls $(c, s_y)(\omega, t)$ and endogenous state $W(\omega, t)$, given the price P_t process, must therefore satisfy:

$$dw_t = [(h - c_t) + w_t(R_b - \pi + \sigma_P^2) + s_{y,t}w_t(\Psi - \sigma_P^2)]dt + s_{y,t}w_t(\sigma_y dZ_t^y + \sigma_P dZ_t^P) - w_t\sigma_P dZ_t^P \quad (10)$$

In particular, to obtain the above, note that under the rules of stochastic calculus:

$$\frac{1}{P_t^2}dP_t dW_t = \left(\pi dt + \sigma_P dZ_t^P\right)\left(s_{y,t}\frac{W_t}{P_t}(\sigma_y dZ_t^y + \sigma_P dZ_t^P)\right) + o(dt) = \sigma_P^2 s_{y,t} w_t dt + \sigma_P\sigma_y\lambda s_{y,t} w_t dt$$

And λ is the instantaneous nominal-real covariance defined earlier. Now that the problem has been reformulated in terms of *real wealth* as the single state variable, we can reformulate the household problem recursively, leading to the (stationary) HJB equation satisfied at a (global) solution in terms of a value function and associated admissible optimal control policies c^*, s_y^*:[2]

$$0 = u(c^*) + [A^{(c_t^*, s_{y,t}^*)}V](w_t) - \rho V(w_t)$$

where the backward operator $A^{c_t^*, s_{y,t}^*}$ on smooth functions of the state, for (optimal) admissible control processes is defined as:

$$[A^{(c_t^*, s_{y,t}^*)}V](w_t) = \lim_{dt \to 0} \frac{1}{dt} E_t\{V'(w_t)dw_t + \frac{1}{2}V''(w_t)d[w]_t\}.$$

Hence, plugging-in the SDE for the controlled diffusion of real wealth w_t:

$$0 = u(c) + V'(w_t)[(h - c_t) + w_t(R_b - \pi + \sigma_P^2) + s_{y,t}w_t(\Psi - \sigma_P^2)] +$$

$$\frac{1}{2}V''(w_t)[s_{y,t}^2 w_t^2 \sigma_y^2 + (1 - s_{y,t})^2 w_t^2 \sigma_P^2 - 2s_{y,t}w_t^2\sigma_y\sigma_P\lambda + 2s_{y,t}^2 w_t^2 \sigma_y \sigma_P \lambda] - \rho V(w_t). \tag{11}$$

Under the standard transversality condition $\lim_{s\to\infty} e^{-\rho s}V(w_s) = 0$, the value function solving the above HJB (with the associated optimal controls) is also a solution the original sequential problem[3] This *Verification Theorem* is also covered more rigorously in Pham (2009). Based on this, the paper now derives and discusses the main result implied by the above solution. Namely, the implications for the Equity Premium Ψ under an assumption of interiority for the optimal control processes.

4. Main Results and Discussion

At an interior solution, such that $0 < s_{y,t}^* < 1$ and $c \in \mathbb{R}_++$ at each point in the state space, the share of equity satisfies the following, fully static, first order condition:

$$V'(w_t)w_t(\Psi - \sigma_P^2) + V''(w_t)[w_t^2\sigma_y^2 s_{y,t} - (1 - s_{y,t})w_t^2\sigma_P^2 - w_t^2\sigma_y\sigma_P\lambda + 2s_{y,t}w_t^2\sigma_y\sigma_P\lambda] = 0$$

Recalling, with CRRA utility, that the Arrow-Pratt measure of risk aversion is given by $\theta = -\frac{V''(w_t)w_t}{V'(w_t)}$ (Chang, 2004), then we can rewrite the above (under the maintained assumption $w_t > 0$): as:

$$-\frac{1}{\theta}(\Psi - \sigma_P^2) + [\sigma_y^2 s_{y,t} - (1 - s_{y,t})\sigma_P^2 - \sigma_y\sigma_P\lambda + 2s_{y,t}\sigma_y\sigma_P\lambda] = 0.$$

We can thus solve for the optimal share of equity in the portfolio (when $w_t > 0$), which is evidently static so that we can suppress the time subscript, as:

$$s_{y,t} = \frac{(\Psi - \sigma_P^2) + \theta(\sigma_P^2 + \sigma_y\sigma_P\lambda)}{\theta(\sigma_y^2 + \sigma_P^2 + 2\sigma_y\sigma_P\lambda)} \tag{12}$$

The solution for the share of equity, together with the assumption of an interior solution, pins down the set of Equity Premia consistent with an interior equilibrium. Specifically, we must have that:

$$0 < \Psi - \sigma_P^2 + \theta(\sigma_P^2 + \sigma_y\sigma_P\lambda) < \theta(\sigma_y^2 + \sigma_P^2 + 2\sigma_y\sigma_P\lambda)$$

$$(1 - \theta)\sigma_P^2 - \theta\lambda\sigma_P\sigma_y < \Psi < \theta\sigma_y^2 + \theta\lambda\sigma_P\sigma_y + \sigma_P^2$$

We can establish a number of results on the relevance of inflationary news for the equilibrium equity premium $\Psi \in \mathcal{E}$ in the outlined incomplete markets economy through comparative stat-

[2]The derivation is standard and omitted for the sake of brevity. Again, the main idea is in the limit of the time interval shrinking to zero and under Ito's lemma to describe the gain in the value of the problem, passing the expectation operator results in a deterministic problem.

[3]cf. Chang (2004) for a proof based on an application of Ito's Lemma and Dynkin's formula obtain the diffusion equation for the present value of the solution to (11).

ics on the equilibrium set $\mathcal{E}$. We focus, conservatively, on the minimum equity premium Ψ_{min}, corresponding to the weak lower boundary of the set. Clearly, by construction $\Psi > \Psi_{min}$ strictly. After presenting these, I elaborate on the implications for asset pricing and particularly the equity premium puzzle – particularly, the conditions under which more volatile prices or "heavier-tailed" inflation news, corresponding to larger values of σ_P, transmit to an increase in the minimum equilibrium equity premium.

Proposition 1.*(Strict Positivity of the Minimum Equity Premium).* With stochastic inflation, so that $\sigma_P > 0$, the minimum equilibrium equity premium is strictly positive whenever,

$$\lambda < \lambda^* = \frac{1-\theta}{\theta}\frac{\sigma_P}{\sigma_y}.$$

With non-stochastic inflation, so that $\sigma_P = 0$, the minimum equity premium is zero as standard. By contrast, whenever $\lambda > \lambda^*$, negative equity premia become possible equilibria.

Proposition 2.*(Nonlinear Transmission of Inflationary News to the Equity Premium).* With CRRA preferences and stochastic inflation, dispersion in inflation news σ_P, i.e. a heavier-tailed the distribution of innovations against expected inflation, has a *non-linear* impact on the minimum equity premium. In particular, the non-linear transmission is quadratic, with the marginal response of the minimum premium to an increase in σ_P linear and given by:

$$\frac{\partial \Psi_{min}}{\partial \sigma_P} = 2(1-\theta)\sigma_P - \theta\lambda\sigma_y.$$

The marginal impact has a zero which is a function of the nominal-real covariance:

$$\sigma_P^*(\lambda) = \frac{\theta}{2(1-\theta)}\lambda\sigma_y,$$

and is negative and positive respectively for $\sigma_P < \sigma_P^*(\lambda)$ and $\sigma_P > \sigma_P^*(\lambda)$. It follows that, with negative instantaneous nominal-real covariance $\lambda < 0$, an increase in heavy-tailedness of inflationary news σ_P monotonically increases the minimum equity premium away from zero. With *positive* nominal-real covariance $\lambda > 0$, this is only true past threshold σ_P^*. Three corollaries follow:

1. Independently of whether the nominal-real covariance is positive or negative, more heavier-tailed inflationary news always end up raising the minimum equity premium: for any bounded λ, there is always some $\sigma_P > \sigma_P^*(\lambda)$ such that $\partial\Psi_{min}/\partial\sigma_P > 0$ for $\sigma_P > \sigma_P^*(\lambda)$.

2. Hence, larger positive equity premia can always be explained, independently of λ, by a sufficiently heavy-tailed distribution of inflationary news.

3. The (cross-partial) derivative of the marginal effect with respect is positive whenever $\lambda < 0$. In other words, the positive impact of the presence of inflationary news in a negative nominal-real covariance environments on the equity premium is amplified by riskier returns to equity.

Proposition 3. *(Comparative Statics relative to Equity Risk and Nominal-Real Covariance)* The marginal impact of an increase in the instantaneous nominal-real covariance and in the riskness of returns to equity are, respectively:

$$\frac{\partial \Psi_{min}}{\partial \lambda} = -\theta\sigma_P\sigma_y \qquad \frac{\partial \Psi_{min}}{\partial \sigma_e} = -\theta\lambda\sigma_p$$

The former is always negative. However, the second is positive whenever the real-nominal covariance is negative. A corollary of this is that, with negative nominal-real covariance, increases in equity risk and the heavy-tailedness of inflationary news have mutually reinforcing partial positive impacts on the minimum equity premium.

I concentrate on the key propositions 1-2 to begin with. Proposition 1 and 2 tells us that, as long as the correlation λ between inflationary news and equity risk is weakly negative, the presence of stochastic or uncertain inflation, i.e. of inflationary or deflationary news around the expected inflation rate must result in a strictly positive equilibrium equity premium. This holds, importantly, in a zero nominal-real covariance environment – weakening the conditions under which uncertainty on the inflation path carries asset pricing consequences relative to the emphasis on nominal-real covariances (and its sign) in the recent literature on asset pricing in similarly "incomplete markets" environments arising from stochastic inflation (Campbell et al., 2017; Camba-Mendez and Werner, 2017). Further, under the same circumstances, the heavier-tailed the distribution of inflationary news against expected inflation, the larger the resulting equilibrium equity premium. Finally, the contrasting case positive nominal-real covariance only flips the result insofar as the "heavy-tailedness" of inflationary news in contained below some threshold. This both mitigates the emphasis on the role played by the sign of the nominal-real covariance stressed in the recent literature for asset pricing, i.e. saying it matters insofar as uncertainty or heavy-tailedness remains sufficiently bounded, and by the same coin sharpens the answer as to how inflation uncertainty can explain part of the equity premium puzzle. In fact, in the proposed model, (more) heavy-tailedness in inflationary (and, symmetrically, deflationary) news past a threshold σ_P^* can always explain larger equity premia Ψ as conventionally defined. A potential (partial) answer to the equity premium puzzle as such is that, with stochastic inflation and a resulting "Catch-22" or incomplete-markets environment, (i) it lies in how heavy-tailed is the distribution of inflationary news, and (ii) how heavy-tailed such distribution must be to explain a larger positive premium depends on the nominal-real covariance. A combination of heavy-tailed inflation news and a negative nominal-real covariance, in particular, can explain larger equity premia than expected from standard Mehra-Prescott style models (cf. Chang, 2004; Mehra and Prescott, 1985).

To elaborate on the economic intuition underlying the above results in the developed "Catch-22" environment, we might begin with a benchmark zero nominal-real covariance case and note that it seems *counter-intuitive* that an increase in the dispersion of inflationary news increases rather than decreases the critical minimum equity premium – in other words, it increases the bonds' inflation risk premium they command over inflation-hedges such as equity. Because in environments marked by heavy-tailed inflationary news or more extreme price level shock events there is an "extra" source of demand for equity (i.e. on precautionary grounds), their price should increase and hence relative premium commanded by equity should fall. The model says that the exact opposite occurs. Why is this the case? An alternative idea is that in the outlined Catch-22 environments, where incomplete markets prevent simultaneous insurance against inflationary news and equity risk (whether or not covariant), cannot be achieved, there is an implicit *malus* to transferring wealth through equity as opposed through bonds relative to when the inflation path is purely deterministic and bonds are safe assets. This comes, in particular, from the fact that with increasing heavy-tailedness and uncertainty surrounding the expected inflation path, deflationary risk heightens and, in a rational expectations environment, this gets priced into the minimum return to be commanded by equity in order for agents to be willing to trade at an interior equilibrium. This logic is particularly clear in the light of the first order condition derived from the problem, where the gain from allocating/transferring wealth through equity at the margin (at a first-order approximation) is penalised by the variance of prices, which is where the lower bound comes from in the equation. Rather than equity becoming a marginally "safer" asset in heavier-tailed inflationary news environments, it becomes riskier to hold, as the chance of foregoing real gains from transferring wealth bonds when deflation risk increases. In other words, because bonds, in this Catch-22 model, are not just pure risk-hedgers inflation bets but, borrowing the terminology in Campbell et al. (2017) also deflation-hedges. Note this explanation, in part, also contributing to the literature on the role of extreme events (heavytails) albeit through a different logic, and placing emphasis on inflationary risk rather than just (perhaps begging the question) equity risk, bridging such "tails-driven" approach to the puzzle with the recent literature on the role of uncertain inflation paths via deflationary expectations.

To strengthen the interpretation, consider the case in which the covariance is, further, negative rather than just non-positive, which in our framework constitutes the most conservative way (i.e. requiring lower implied inflation uncertainty σ_P) to address the equity premium puzzles. Propositions (2), particularly the third corollary, and Proposition (3) can be similarly rationalised and consolidate the above logic. In particular, in $\lambda < 0$ environments a heightened risk of larger than expected returns to equity goes hand-in-hand with increasing the risk of larger deflationary news against expected inflation. In such contexts, through the above economic logic, the malus carried by equity becomes greater as the risk heightens to forego gains from deflationary news when allocating the portfolio to equities, and households must be accordingly rewarded more for them to choose to trade at an interior equilibrium. The effects of heavier tails in inflation news and in equity risk in the presence of a negative nominal-real covariance can be expected, if the outlined logic is true, to interact in a complementary manner: as Proposition 3 demonstrates, this is precisely the case in the developed framework. This explanation of the results supports and is corroborated by evidence on how very low and even negative inflation risk premia commanded by bonds, which can result in a positive premium commanded by assets, like equity in the present Catch-22 model, that are not sensitive to inflation, have been primarily associated to increased deflation risk (Camba-Mendez and Werner, 2017). Our model indicates that this is entirely to be expected as an asset pricing implication of the inability to simultaneously insure against stochastic inflation (or better, deflation) and equity risk, and heightens in environments marked by a negative nominal-real covariance. The secular decline in the equity risk premium, particularly after 2000s, can be accordingly understood through the presented model in a way that is both consistent with, and fleshes out the asset-pricing implications, of a switch in the sign of the empirical nominal-real covariance since the 2000s (cf. Ibid.; Campbell et al., 2017). To sum up, on the back of the outlined logic underlying the derived results, part of the explanation of the historical equity premium might plausibly lie in a combination of heavy-tailed distribution of innovations against expected inflation, which comes with crucially heightened uninsurable deflationary risk, and – even though this ceases to matter with sufficiently heavy-tailed news – a negative nominal-real covariance. Changes to the premium, particularly its decline, might as such be reflective of either a weaker negative or positive nominal-real covariance, or the slimming down of the tails of the distribution in inflationary news relative to in the past.

Before concluding, it is also worth pointing out the implications of the results for monetary policy. In particular, concerning the effects of implementing monetary policies that achieve a stronger stabilisation of prices, i.e. such to slim down the tails of inflationary news against expected inflation, modelled as a reduction in σ_P. What the model says, is that the asset pricing implications of such monetary policy critically for the equity premium, given by has a non-linear impact on the equity premium given by $-\partial\Psi_{min}/\partial\sigma_P$, depend on the nominal-real covariance. Slimming down the tails of the distribution of inflationary news: with negative nominal-real covariance, a reduction of the premium is always achieved, while with positive nominal-real covariance, this is only true until σ_P falls weakly below $\sigma_P^*(\lambda)$. As such, in the latter environments, a trade-off between stabilising the path of prices and trimming-down the minimum equity premium only exists at large values of σ_P; conversely, after σ_P falls sufficiently (i.e. below $\sigma_P^*(\lambda)$, further stabilisation comes with an increase in the minimum premium.

5. Conclusion

In this note, I develop a simple "Catch-22" model of optimal consumption and portfolio allocations in continuous time in the absence of assets enabling simultaneous insurance against potentially correlated equity returns risk and risk in the form of inflationary news. As such, no portfolio choice can attain full insurance and always involve trading off exposure to equity risk for exposure to inflationary news. Based on the formulation of the problem via the Hamilton-Jacobi-

Bellman equation and the achieved reduction of the stochastic dynamic optimisation programme to a non-stochastic recursive problem, preliminary analytical results from the model highlight a critical dimension to pricing risk at the margin in the model, with implications for an analysis of the Equity premium puzzle. In particular, the marginal impact of higher inflation volatility on equity premia is non-linear, and the inflationary news-equity returns risk covariance plays a key a role in shaping the set of Equilibrium Premia in a way consistent with the Equity Premium Puzzle. A potential (partial) answer to the equity premium puzzle as such is that, with stochastic inflation and a resulting "Catch-22" or incomplete-markets environment, (i) it lies in how heavy-tailed is the distribution of inflationary news, and (ii) how heavy-tailed such distribution must be to explain a larger positive premium depends on the nominal-real covariance. A combination of heavy-tailed inflation news and a negative nominal-real covariance, in particular, can explain larger equity premia than expected from standard Mehra-Prescott style models.

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