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On protocol robustness and extreme tail risk: Fréchet vs. Gumbel domains across cryptocurrencies and traditional assets, 2022-2024

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Abstract

This paper applies Generalized Extreme Value (GEV) theory to identify the distributional tail domain of daily losses across a seven-asset portfolio—three cryptocurrencies (Bitcoin, Ethereum, Solana), two G10 currency pairs (EUR/USD, CAD/USD), an emerging-market currency (MXN/USD), and Gold—over N = 757 synchronized trading days spanning January 2022 to November 2024. Block-maxima estimation over 36 monthly blocks yields GEV shape parameters ξ consistently above zero for all three cryptocurrencies (BTC: 0.171; ETH:0.281; SOL: 0.310), placing their loss distributions in the Fréchet (heavy-tail) domain; AIC confirms the GEV specification over the Gumbel for Ethereum and Solana. Traditional assets are consistent with the lighter-tailed Gumbel domain. The Normal distribution systematically understates the empirical 99% Value at Risk of cryptocurrencies by up to 2.45 percentage points, while closely approximating VaR for all currency and Gold positions. A Chow structural break test applied to the AR(1) mean equation detects crypto-specific mean instability at the FTX exchange collapse (November 2022)—not transmitted to currency or commodity markets—while the 2024 U.S. electoral cycle produces no structural break in any of the seven assets. These findings carry direct implications for EVT-based capital adequacy frameworks for digital asset portfolios.

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1. INTRODUCTION

The period from January 2022 to November 2024 constitutes an unusually rich laboratory for studying extreme financial risk. It encompasses the collapse of the FTX exchange (November 2022)—the largest idiosyncratic shock to cryptocurrency markets since Mt. Gox—the subsequent recovery of 2023, and the 2024 U.S. presidential election, whose outcome materially reshaped macroeconomic expectations for monetary policy and digital-asset regulation. A fundamental question for risk managers and regulators is whether standard Gaussian models adequately capture the extreme loss behavior of digital assets over this cycle, and, if not, to which parametric tail family their loss distributions belong.

This question is particularly relevant in light of recent advances in blockchain protocol design. Conley's (2025) Proof of Honesty protocol—achieving 99% Byzantine Fault Tolerance and substantially improving consensus efficiency—demonstrates that cryptographic networks can be made highly resistant to adversarial manipulation. Yet, as Conley (2017) argues, protocol robustness does not insulate a token's market price from macroeconomic shocks. Even a cryptographically secure network issues a token whose loss distribution may exhibit heavy tails during periods of systemic stress. Accurately characterizing this tail behavior requires moving beyond Gaussian VaR (Urquhart 2016; Bouri et al. 2018) toward the Generalized Extreme Value framework, which nests the Gumbel, Fréchet, and Weibull families under a single shape parameter (Embrechts, Klüppelberg and Mikosch 1997).

Initial exploratory research (Arizmendi 2025) documented heavy-tailed behavior for cryptocurrencies over a shorter 2023–2024 window. The present paper extends that analysis to the full 2022–2024 market cycle, adds four traditional assets for cross-asset benchmarking, and critically estimates the GEV shape parameter ξ to formally identify the distributional domain of each asset's tail. The cryptocurrency literature documents persistent return anomalies (Caporale et al. 2018), asymmetric volatility clustering (Baur and Dimpfl 2018; Gkillas and Katsiampa 2018), and conditional tail risk (Borri 2019)—stylized facts that collectively motivate a GEV approach as the rigorous framework for tail domain identification.

This paper makes three contributions. First, it provides a systematic cross-asset GEV domain identification over a full market cycle, finding that cryptocurrency losses belong to the Fréchet domain ($\xi > 0$ for BTC, ETH, and SOL) while currency and Gold losses are consistent with the Gumbel domain. Second, it documents that the Normal distribution systematically understates the 99% daily VaR of Bitcoin and Ethereum by 2.19 and 2.45 percentage points respectively, while accurately approximating VaR for all traditional assets. Third, Chow structural break tests reveal a crypto-specific mean-equation shift at the FTX collapse without transmission to currency or commodity markets, while the 2024 U.S. electoral cycle produces no structural break in any asset.

2. METHODOLOGY AND DATA

2.1 Data Construction and Econometric Diagnostics. The dataset comprises daily closing prices for seven assets: Bitcoin (BTC), Ethereum (ETH), Solana (SOL), EUR/USD, CAD/USD, MXN/USD, and Gold (XAU/USD), sourced from Investing.com. Prices are synchronized to the intersection of all seven trading-day calendars, yielding $N = 757$ daily log-return observations per asset spanning

3 January 2022 to 29 November 2024.[1] Let $P_{i,t}$ denote the closing price of asset i on day t. Daily log-returns are $r_{i,t} = \ln(P_{i,t}/P_{i,t-1})$. USD/MXN is inverted to MXN/USD before computing returns, so that the loss series $L_{i,t} = -r_{i,t}$ is uniformly positive when asset i depreciates in USD terms. All VaR figures in Table I are positive quantities representing the maximum expected daily loss per unit invested.

Augmented Dickey-Fuller (ADF) tests confirm stationarity of all seven return series ($p < 0.001$). ARCH-LM tests with ten lags reveal significant volatility clustering in five assets: ETH ($p = 0.009$), SOL ($p < 0.001$), EUR/USD ($p < 0.001$), CAD/USD ($p < 0.001$), and MXN/USD ($p < 0.001$). Bitcoin is a notable exception ($p = 0.29$)—consistent with the smoothing of idiosyncratic volatility clustering as institutional participation has deepened in BTC markets (Corbet et al. 2019)—and Gold is borderline ($p = 0.08$).

A Chow structural break test is applied to the AR(1) mean equation $r_{i,t} = c_i + \varphi_i r_{i,t-1} + \varepsilon_{i,t}$ at two candidate break dates. At the FTX collapse (11 November 2022), the test detects significant mean-equation instability for BTC ($F = 3.83$, $p = 0.022$) and SOL ($F = 4.20$, $p = 0.015$), and a marginal signal for ETH ($F = 2.50$, $p = 0.083$). All four traditional assets show no break ($p > 0.38$), confirming that the FTX shock was a crypto-specific event with no systemic transmission to currency or commodity markets. At the 2024 U.S. electoral cycle break date (31 July 2024), no asset exhibits a significant break ($p > 0.31$ for all seven), confirming structural stability in mean dynamics throughout the political uncertainty period.

2.2 Generalized Extreme Value Framework. The Fisher-Tippett-Gnedenko theorem (Fisher and Tippett 1928; Gnedenko 1943) establishes that the normalized maximum of a large i.i.d. sample converges to the Generalized Extreme Value (GEV) distribution (Embrechts, Klüppelberg and Mikosch 1997; Gumbel 1958). For $\xi \neq 0$:

$$F(x;\xi,\mu,\sigma) \;=\; \exp\left\{-\left(1+\xi\,\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right\}, \qquad 1+\xi\,\frac{x-\mu}{\sigma} > 0 \tag{1}$$

where $\mu \in \mathbb{R}$ is the location parameter, $\sigma > 0$ the scale, and $\xi \in \mathbb{R}$ the shape parameter. Three families are nested: $\xi > 0$ (Fréchet, power-law tail), $\xi = 0$ (Gumbel, exponential tail; equation (1) converges to $\exp\{-\exp[-(x-\mu)/\sigma]\}$ as $\xi \to 0$), and $\xi < 0$ (Weibull, bounded tail). Under the Fréchet domain, the k-th moment of X is infinite for $k \geq 1/\xi$, directly undermining variance-based risk models.

We apply the block maxima method: the loss series $\{L_{i,t}\}$ is partitioned into 36 non-overlapping monthly blocks of 21 trading days each, and the block-maximum loss $X_{i,k} = \max\{L_{i,t} : t \in \text{block } k\}$ is extracted per block. GEV parameters (ξ,μ,σ) are estimated by Maximum Likelihood on the 36 block maxima. Model selection between the unrestricted GEV (3 parameters) and the Gumbel ($\xi = 0$, 2 parameters) uses AIC: $\Delta\text{AIC} = \text{AIC}_{\text{Gumbel}} - \text{AIC}_{\text{GEV}}$, where $\Delta\text{AIC} > 0$ favors GEV.

For direct comparison, we report two daily VaR metrics estimated from the full loss series

[1] Prices are synchronized to the intersection of trading-day calendars across all seven series, yielding 758 common dates. After first-differencing, $N = 757$ daily log-return observations are available per asset. Cryptocurrency series (BTC, ETH, SOL) include seven-day trading; traditional assets operate on standard weekday calendars. Synchronization removes weekend cryptocurrency observations to ensure strict cross-asset comparability.

$\{L_{i,t}\}$: the Historical VaR (empirical 99th percentile) and the Normal VaR:

$$\mathrm{VaR}_{\alpha}^{\mathrm{N}} = \mu_L + z_{\alpha} \cdot \sigma_L \tag{2}$$

where μ_L and σ_L are the sample mean and standard deviation of $\{L_{i,t}\}$, and $z_{0.99} \approx 2.326$. The GEV shape parameter ξ is estimated from block maxima to identify the asymptotic tail domain; it does not produce a daily VaR figure. We acknowledge that ARCH effects in five series formally violate the i.i.d. assumption; following McNeil and Frey (2000), GARCH-filtered EVT remains an important direction for future research.

3. EMPIRICAL RESULTS

Table I presents 99% VaR estimates and GEV domain identification for all seven assets. All VaR figures use the $L_{i,t} = -r_{i,t}$ sign convention (positive = loss). The shape parameter $\hat{\xi}$ is estimated from 36 monthly block maxima; $\Delta\mathrm{AIC} = \mathrm{AIC}_{\mathrm{Gumbel}} - \mathrm{AIC}_{\mathrm{GEV}}$ captures model preference on those block maxima.

Table I. 99% VaR Estimates, GEV Shape Parameters, and Model Selection

Asset	Hist VaR	Normal VaR	$\hat{\xi}$ (GEV)	ΔAIC	EVT Domain
Bitcoin (BTC)	10.31%	8.12%	0.171	+0.1	Fréchet (GEV)
Ethereum (ETH)	12.67%	10.22%	0.281	+3.2	Fréchet (GEV)
Solana (SOL)	15.14%	14.84%	0.310	+7.4	Fréchet (GEV)
EUR/USD	1.26%	1.17%	0.797†	−17.0	Gumbel
CAD/USD	1.02%	0.97%	0.790†	−29.9	Gumbel
MXN/USD	1.96%	1.69%	0.761†	−16.1	Gumbel
Gold (XAU/USD)	2.43%	2.05%	0.057	−1.8	Gumbel

Notes: Historical VaR = empirical 99th percentile of daily losses $\{L_{i,t} = -r_{i,t}\}$. Normal VaR from equation (2). $\hat{\xi}$ estimated by MLE on 36 monthly block maxima (21 trading days per block). $\Delta\mathrm{AIC} = \mathrm{AIC}_{\mathrm{Gumbel}} - \mathrm{AIC}_{\mathrm{GEV}}$: positive values indicate GEV preference. † $\hat{\xi}$ for currency pairs is unreliable in short samples; AIC strongly favors the parsimonious Gumbel ($\Delta\mathrm{AIC} < -16$ in all three cases). Sample: $N = 757$, 3 January 2022–29 November 2024.

The GEV domain identification is the central result. All three cryptocurrencies yield shape parameters $\hat{\xi} > 0$: BTC (0.171), ETH (0.281), and SOL (0.310). AIC confirms the GEV specification over the Gumbel for ETH ($\Delta\mathrm{AIC} = +3.2$) and SOL ($\Delta\mathrm{AIC} = +7.4$); for BTC the margin is negligible ($\Delta\mathrm{AIC} = +0.1$), though $\hat{\xi} > 0$ places it tentatively in the Fréchet domain. Fréchet tail structure implies power-law rather than exponentially decaying losses: for ETH ($\hat{\xi} = 0.281$), moments of block-maximum losses are theoretically infinite for $k > 3.6$, and for SOL ($\hat{\xi} = 0.310$) for $k > 3.2$. These conditions directly undermine variance-based risk measures and Gaussian capital models for positions in these assets.

For traditional assets, AIC strongly favors Gumbel over GEV in all four cases: ΔAIC ranges from −16.1 (MXN/USD) to −29.9 (CAD/USD) for currency pairs, and −1.8 for Gold. Gold is the most instructive benchmark: $\hat{\xi} = 0.057 \approx 0$ is confirmed by both the point estimate and model selection, placing it squarely in the Gumbel domain. For the three currency pairs, $\hat{\xi}$ estimates are numerically large (0.76–0.80) but statistically unreliable—an artifact of attempting to estimate a

three-parameter GEV from 36 block maxima of a low-volatility series—and the AIC correctly penalizes the extra parameter. The result is a clean cross-asset partition: the Fréchet domain is a cryptocurrency-specific phenomenon in this sample.

The VaR comparison confirms the practical consequences of this partition. For Bitcoin, the Normal VaR (8.12%) falls 2.19 percentage points below the Historical VaR (10.31%), a 21% relative understatement. For Ethereum, the gap is 2.45 percentage points (10.22% vs. 12.67%, a 19% understatement). Solana is a partial exception: the Normal VaR (14.84%) nearly matches the Historical VaR (15.14%), a near-coincidence attributable to the extreme positive skewness of its loss distribution (skewness $= 0.84$), which causes the Normal to fortuitously approximate the empirical 99th percentile despite an underlying Fréchet tail structure. For all four traditional assets, the Normal VaR understatement is below 0.4 percentage points in absolute terms and economically negligible, consistent with their Gumbel-domain structure and low excess kurtosis (1.2–1.9 vs. 6.9–8.8 for cryptocurrencies).

Two additional results merit emphasis. Bitcoin's absence of significant ARCH effects ($p = 0.29$) contrasts markedly with the strong volatility clustering documented in earlier BTC studies (Katsiampa 2017), consistent with the maturation of institutional market participation over 2022–2024. And the cross-asset comparison of the two G10 currency pairs is instructive: CAD/USD and EUR/USD exhibit nearly identical VaR profiles (Historical VaR of 1.02% and 1.26% respectively), Gumbel-domain tail structure, and Normal VaR approximation accuracy within 0.1 percentage points. The Mexican peso (MXN/USD), as an emerging-market currency, carries a higher Historical VaR of 1.96%—nearly double the CAD/USD level—but remains in the Gumbel domain, demonstrating that tail heaviness in the GEV sense is a distinct property from mere volatility magnitude.

Figure 1 illustrates the cross-asset contrast for three representative assets. For Bitcoin and Ethereum, the Gumbel fit produces a heavier right tail than the Normal but both models underestimate the empirical tail density in the extreme right; the Historical VaR marker lies beyond the Normal VaR in both crypto panels, confirming the documented understatement. For EUR/USD, the Normal and Gumbel fits are nearly indistinguishable and both closely track the empirical distribution, confirming that Gaussian models are adequate for this asset class.

4. CONCLUSION AND PRACTICAL IMPLICATIONS

This paper applies block-maxima GEV estimation to a seven-asset cross-section spanning three major cryptocurrencies, two G10 currencies, an emerging-market currency, and Gold over the 2022–2024 market cycle. Three findings stand out. First, cryptocurrency loss distributions belong to the Fréchet domain ($\xi > 0$; GEV preferred by AIC for Ethereum and Solana), implying power-law tail decay and theoretically infinite higher moments. Traditional assets are consistent with the lighter-tailed Gumbel domain. This cross-asset partition constitutes the paper's central empirical contribution. Second, the Normal distribution systematically understates the 99% daily VaR of Bitcoin and Ethereum by approximately 2.2–2.5 percentage points, a material shortfall for risk management. Third, the FTX collapse caused crypto-specific mean-equation structural breaks without systemic transmission to traditional assets, while the 2024 U.S. electoral cycle left all seven assets structurally stable.

From a risk management perspective, the Fréchet characterization of cryptocurrency loss

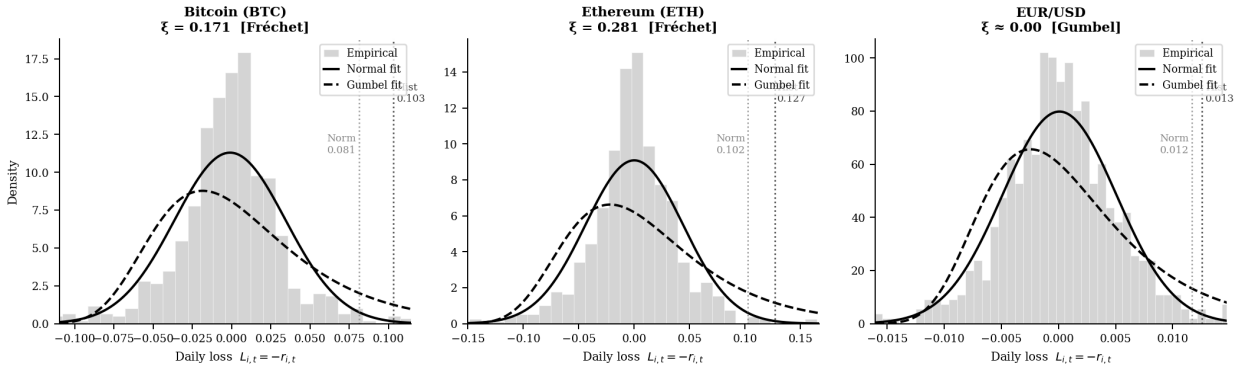

Figure 1. *Fitted Loss Distributions: Normal and Gumbel Benchmarks. Empirical daily loss distributions* $\{L_{i,t}\}$ *(grey histogram) overlaid with fitted Normal (solid) and Gumbel (dashed) curves for Bitcoin (left), Ethereum (centre), and EUR/USD (right). Dotted vertical lines mark the empirical Historical 99% VaR (dark) and Normal 99% VaR (grey). GEV shape parameters (block-maxima MLE, 36 blocks) are shown in panel subtitles. Sample:* $N = 757$*, January 2022–November 2024.*

distributions implies that Gaussian VaR frameworks may materially underestimate tail exposure for digital asset portfolios. EVT-based approaches—specifically GEV block-maxima or Peaks-Over-Threshold—are warranted for these positions, and should be recalibrated as market structure evolves. The absence of significant ARCH effects in Bitcoin over 2022–2024 suggests that risk models for BTC may require different volatility-dynamics assumptions than in earlier periods, and that the GARCH-EVT pipeline (McNeil and Frey 2000) merits separate calibration across assets within the cryptocurrency class.

From a regulatory perspective, the documented cross-asset partition in GEV domains provides an empirical basis for applying elevated capital charges to cryptocurrency positions relative to traditional currency or commodity exposures of equivalent nominal size, within frameworks such as the EU's MiCA regulation and evolving U.S. digital asset guidelines. Homogeneous risk weighting across these asset classes is not supported by the tail evidence.

Finally, the results reinforce a distinction that is central to the economics of digital assets. Even as blockchain protocols achieve high levels of cryptographic fault tolerance—as in Conley's (2025) Proof of Honesty architecture—the corresponding tokens exhibit Fréchet tail behavior during market stress. Protocol robustness and distributional tail heaviness operate on orthogonal dimensions: the former addresses network integrity; the latter reflects the market's aggregation of macroeconomic uncertainty, liquidity dynamics, and speculative demand. Capital adequacy frameworks must formally embed this distinction.

Data Availability Statement

Daily price data are sourced from Investing.com for all seven assets. The synchronized dataset and Python estimation scripts are available from the corresponding author upon reasonable request.

Disclosure Statement

No potential conflict of interest was reported by the author.

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