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# A closed-form competitive equilibrium for the deterministic neoclassical growth model

Sergio Salas
*University of Chicago*

## Abstract

I revisit the deterministic neoclassical growth model and derive a closed-form competitive equilibrium under two standard assumptions: log utility and full capital depreciation. Working directly with the decentralized economy, I provide explicit expressions for allocations, prices, and the household's policy rule. The exposition identifies the role of each assumption in delivering the closed form and shows what fails when either is relaxed

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**Contact:** Sergio Salas - sergiosalaslan@gmail.com.
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# 1. Introduction

The deterministic infinite-horizon neoclassical growth model is among the first advanced models taught in modern macroeconomics, yet it admits no closed-form solution under standard assumptions. A closed-form solution arises only when preferences are logarithmic, capital fully depreciates, and technology is Cobb-Douglas. In this case, the central planner's solution is simple, but it exhibits a feature often overlooked: optimal decisions do not depend on future productivity. Why this occurs cannot be fully understood through the planner's approach. Solving the model instead through the market equilibrium reveals precisely how each assumption—log utility and full depreciation—eliminates the influence of future productivity on current behavior.

Without labor disutility, household choices normally depend on the entire future path of interest and wage rates, which respond to productivity in each period. Changes in the interest rate generate income, substitution, and wealth effects that shape consumption and capital accumulation. Under log utility, the income and substitution effects cancel, removing one channel through which future productivity matters. Yet consumption still depends on future interest rates through human wealth, so log utility alone is insufficient for a closed-form solution.

Adding full depreciation removes this remaining dependence. With capital fully depreciated each period, wage changes exactly offset interest-rate effects in human wealth, leaving current decisions dependent only on contemporaneous prices. The resulting solution appears "myopic"—current choices ignore future productivity—but this stems from perfectly forward-looking behavior that anticipates and cancels all opposing effects.

## 1.1. Related Literature

Most studies have examined stochastic versions of this framework within real business cycle (RBC) models. Long & Plosser (1983) derived closed-form results for a multi-sector RBC model under log utility, Cobb–Douglas production, and full depreciation. King et al. (1988) and McCallum (1989) relaxed some of these assumptions to obtain numerical or partially analytical results. Other work has explored alternatives to full depreciation, such as capital vintages (Radner (1966), Antony & Maußner (2012)) or adjustment costs (Hercowitz & Sampson (1991)), to preserve tractability while maintaining realistic dynamics.

The paper also relates to the literature on "news" shocks—anticipated changes in productivity Beaudry & Portier (2007). Under the conditions for closed-form solutions, such news is irrelevant, and understanding why is key to interpreting the model's myopic appearance.

The rest of the paper presents the model, derives the closed-form market solution, and concludes with brief remarks.

# 2. The Model

## 2.1. Households

The economy is populated by a measure one of infinitely lived households. Each maximizes:

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $0 < \beta < 1$ is the discount factor, $c_t$ is the household's consumption and the utility function is assumed to be of the Constant Intertemporal Elasticity of Substitution form $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. The household budget constraint is:

$$c_t + x_t = v_t k_t + w_t \ell_t, \quad k_{t+1} = (1-\delta)k_t + x_t, \tag{2}$$

with $k_0 > 0$ given. $v_t$ is the rental rate of capital and $w_t$ is the wage rate. $k_t$, $\ell_t$, and $x_t$ are capital, labor, and investment, respectively. It is also assumed that households have one unit of time available, $0 \leq \ell_t \leq 1$.

The budget constraint could also have been formulated with bonds as an additional asset. If $r_t$ is the net real interest rate on bonds, the *no-arbitrage* condition implies:

$$R_t \equiv 1 + r_t = v_t + 1 - \delta, \tag{3}$$

where $R_t$ is defined as the gross real interest rate. In this case bonds and capital are perfect substitutes as a means of savings. However, because households are identical, bonds are in zero net supply. I therefore omit bonds in the formulation of the budget constraint, although the no-arbitrage condition (3) will be used later.

### 2.2. Firms

There is a single CRS firm with Cobb-Douglas production function:

$$y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1. \tag{4}$$

where $y_t$ is output, $A_t$ is productivity—which is assumed deterministic—, $K_t$ is capital and $L_t$ labor, both used in production.

The firm's problem is standard and simple; each period the firm aims to maximize:

$$A_t K_t^{\alpha} L_t^{1-\alpha} - v_t K_t - w_t L_t. \tag{5}$$

#### 2.2.1. Equilibrium

**Definition 1.** *A Competitive Equilibrium is a set of prices $\{r_t, v_t, w_t\}_{t=0}^{\infty}$, allocations $\{K_t, L_t, y_t\}_{t=0}^{\infty}$ for the firm, and allocations $\{c_t, k_t, \ell_t\}_{t=0}^{\infty}$ for the household, such that:*

1. *$\{K_t, L_t, y_t\}_{t=0}^{\infty}$ maximize profits given by (5) at stated prices.*
2. *$\{c_t, k_t, \ell_t\}_{t=0}^{\infty}$ maximize utility (1) subject to the constraint (2) at stated prices.*
3. *Markets clear:*
   - *The labor market: $L_t = \ell_t$.*
   - *The capital market: $K_t = k_t$.*
   - *The goods market:*

$$y_t = c_t + k_{t+1} - (1-\delta)k_t. \tag{6}$$

## 3. Solving the Model

### 3.1. The Households

Households maximize utility (1) subject to the budget constraint (2). The budget constraint can be written as:[1]

$$c_t + k_{t+1} = R_t k_t + w_t. \tag{7}$$

I define a relevant state variable for the household's problem to be *total wealth* $\omega_t$. It is comprised of *non-human* wealth ($R_t k_t$) and *human wealth* ($h_t$):

$$\omega_t \equiv R_t k_t + h_t, \quad h_t = \sum_{j=0}^{\infty} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}}, \tag{8}$$

with $\Pi_{s=1}^{0} R_{t+s} = 1$.

We can now use Dynamic Programming to define the Bellman equation as:

$$\mathcal{V}_t(\omega_t) = \max_{c_t, k_{t+1}} \left[ u(c_t) + \beta \mathcal{V}_{t+1}(\omega_{t+1}) \right], \tag{9}$$

subject to:

$$c_t + k_{t+1} = R_t k_t + w_t. \tag{10}$$

Note that the value function is allowed to change in time as current independent movements in the interest or wage rate may provide the household relevant information beyond that provided in $\omega_t$.[2] The FOC of the problem is:

$$u'(c_t) = \beta R_{t+1} \mathcal{V}'_{t+1}(\omega_{t+1}). \tag{11}$$

### 3.2. The Guess-and-Verify Method

I now guess that the value function adopts the form:

$$\mathcal{V}_t(\omega_t) = B_t \frac{\omega_t^{1-\sigma}}{1-\sigma}. \tag{12}$$

where $B_t$ is an unknown variable.[3] After some manipulations, the FOC and the budget constraint give the solutions:

$$c_t = \theta_{t+1} \left( R_t k_t + w_t + \frac{h_{t+1}}{R_{t+1}} \right), \quad k_{t+1} = (1 - \theta_{t+1})(R_t k_t + w_t) - \theta_{t+1} \frac{h_{t+1}}{R_{t+1}}, \tag{13}$$

[1] Here I introduce the result that $\ell_t = 1$, as there is no disutility from labor.

[2] For example, there might be changes in $R_t$ and $w_t$ that are accompanied by future changes in these variables that leave $\omega_t$ unchanged, yet the current values of both variables do influence current resources according to (10).

[3] That $B_t$ is actually allowed to be a variable and not a constant has to do with the fact that $\omega_t$ might not be a sufficient state of the household problem, as discussed in footnote 2.

where:

$$\theta_{t+1} \equiv \frac{1}{1+\left(\beta R_{t+1}^{1-\sigma} B_{t+1}\right)^{\frac{1}{\sigma}}}. \tag{14}$$

These are not complete solutions for the household yet as they depend on the unknown $B_t$. Note that:

$$\frac{h_{t+1}}{R_{t+1}} = \sum_{j=1}^{\infty} \frac{w_{t+j}}{\Pi_{s=2}^{j} R_{t+s}} \frac{1}{R_{t+1}} = \sum_{j=1}^{\infty} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}}, \tag{15}$$

with $\Pi_{s=2}^{1} R_{t+s} = 1$. Adding $w_t$ to this term, we conclude:

$$w_t + \frac{h_{t+1}}{R_{t+1}} = w_t + \sum_{j=1}^{\infty} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \sum_{j=0}^{\infty} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = h_t. \tag{16}$$

With these results, optimal behavior can be summarized from (13):

$$c_t = \theta_{t+1}\omega_t, \quad k_{t+1} = (1-\theta_{t+1})\omega_t + w_t - h_t. \tag{17}$$

We can see that $\theta_{t+1}$ has the interpretation of a propensity to consume out of total wealth. Consumption and capital are forward-looking solutions influenced by the entire path of future interest and wage rates. These future paths influence both $\theta_{t+1}$ and $h_t$.

Income and substitution effects of changes in $R_{t+1}$ are encapsulated in $\theta_{t+1}$, which in turn depends on the parameter $\sigma$. When $\sigma < 1$, increases in $R_{t+1}$ reduce current consumption through $\theta_{t+1}$ and hence the substitution effect is stronger than the income effect. The opposite result holds when $\sigma > 1$. In addition to these effects, there is also a *wealth* effect because $R_{t+1}$ influences human wealth.

With the solutions obtained in (17), it is possible to find a recursion for $\omega_t$. Replacing $k_{t+1}$ from (17) into the definition of next period total wealth we have:

$$\begin{aligned} \omega_{t+1} = R_{t+1}k_{t+1} + h_{t+1} &= R_{t+1}[(1-\theta_{t+1})\omega_t + w_t - h_t] + h_{t+1} \\ &= R_{t+1}\left[(1-\theta_{t+1})\omega_t + w_t - h_t + \frac{h_{t+1}}{R_{t+1}}\right] \\ &= R_{t+1}(1-\theta_{t+1})\omega_t. \end{aligned} \tag{18}$$

where the last equality used the result in (16). With this result in place, we can move on to the verification step in the Guess-and-Verify method. The Bellman equation is now:

$$\begin{aligned} B_t \frac{\omega_t^{1-\sigma}}{1-\sigma} &= \frac{(\theta_{t+1}\omega_t)^{1-\sigma}}{1-\sigma} + \beta B_{t+1} \frac{[(1-\theta_{t+1})R_{t+1}\omega_t]^{1-\sigma}}{1-\sigma} \\ &= \left\{\theta_{t+1}^{1-\sigma} + \beta B_{t+1}[(1-\theta_{t+1})R_{t+1}]^{1-\sigma}\right\} \frac{\omega_t^{1-\sigma}}{1-\sigma}. \end{aligned} \tag{19}$$

Therefore, equating coefficients in the Bellman equation:

$$B_t = \theta_{t+1}^{1-\sigma} + \beta B_{t+1}[(1-\theta_{t+1})R_{t+1}]^{1-\sigma}. \tag{20}$$

For general parameter values, there is no closed-form solution to equation (20). It is a highly non-linear difference equation in $B_t$ whose solution will depend on the entire infinite sequence $R_{t+1}, R_{t+2}, \ldots$. When $\sigma \to 1$ however (the case where income and substitution effects of change in $R_{t+1}$ cancel exactly), equation (20) adopts the simple form:[4]

$$B_t = 1 + \beta B_{t+1}. \tag{21}$$

Iterating forward and imposing the transversality condition:

$$\lim_{j\to\infty} \beta^j B_{t+j} = 0, \tag{22}$$

we obtain $B = 1/(1-\beta)$. And the propensity to consume out of total wealth becomes independent of time and equal to: $\theta = 1 - \beta$. This is the case where income and substitution effects of changes in the entire path of future interest rates cancel each other out for consumption. In the $\sigma \to 1$ case, solutions (17) simply become:

$$c_t = (1-\beta)\omega_t, \quad k_{t+1} = \beta\omega_t + w_t - h_t. \tag{23}$$

Note, however, that these solutions *still* depend on future values of both wage and interest rates, as human wealth depends on those prices.

### 3.3. The Closed-Form Solution

It is evident that, to solve their problem, households need to forecast future values of both wages and interest rates. To solve for the equilibrium path of these prices, let me assume that households expect the aggregate stock of capital to evolve over time according to the conjecture that investment is a fraction $\phi$ of current output, which is the amount of resources available in the economy:[5]

$$K_{t+1+j} - (1-\delta)K_{t+j} = \phi A_{t+j} K_{t+j}^{\alpha}, \quad 0 < \phi < 1. \tag{24}$$

The conjectured evolution of the aggregate stock of capital will determine the conjectured equilibrium prices $\{v_{t+j}, w_{t+j}\}_{j=0}^{\infty}$, according to the firm's optimality conditions:

$$v_{t+j} = \alpha A_{t+j} K_{t+j}^{\alpha-1}, \tag{25a}$$
$$w_{t+j} = (1-\alpha) A_{t+j} K_{t+j}^{\alpha}, \tag{25b}$$

which solve their problem in (5).

Households can compute $h_t$ under conjecture (24) as follows. First, let me work with the term

[4] Note from (14) that $\lim_{\sigma\to 1} \theta_{t+1}$ exists.

[5] The fact that this fraction is constant is verified by the consistency of rational expectations, as explained in subsection 3.4.

$\frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}}$. Note that:

$$\frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \frac{1}{\Pi_{s=1}^{j-1} R_{t+s}} \frac{w_{t+j}}{R_{t+j}} = \frac{1}{\Pi_{s=1}^{j-1} R_{t+s}} \frac{(1-\alpha) A_{t+j} K_{t+j}^{\alpha}}{\alpha A_{t+j} K_{t+j}^{\alpha-1} + 1 - \delta}, \tag{26}$$

where I used the fact that $R_{t+j} = v_{t+j} + 1 - \delta$. It is clear here that under general $\delta$, the ratio $w_{t+j}/R_{t+j}$ will depend on productivity. Then, in general, $h_t$ does depend on future productivity. However, with the assumption $\delta = 1$, I obtain:

$$\frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \frac{1}{\Pi_{s=1}^{j-1} R_{t+s}} \frac{1-\alpha}{\alpha} K_{t+j}. \tag{27}$$

Next, let me use now conjecture (24)—under assumption $\delta = 1$. Equation (27) becomes:

$$\begin{aligned} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} &= \frac{1}{\Pi_{s=1}^{j-2} R_{t+s}} \frac{1-\alpha}{\alpha} \frac{\phi A_{t+j-1} K_{t+j-1}}{R_{t+j-1}} \\ &= \frac{1}{\Pi_{s=1}^{j-2} R_{t+s}} \frac{1-\alpha}{\alpha} \frac{\phi A_{t+j-1} K_{t+j-1}}{\alpha A_{t+j-1} K_{t+j-1}^{\alpha-1}}, \end{aligned} \tag{28}$$

where the second equality uses the result $v_{t+j-1} = \alpha A_{t+j-1} K_{t+j-1}^{\alpha-1}$, when $\delta = 1$. Equation (28) simplifies then to:

$$\frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \frac{1}{\Pi_{s=1}^{j-2} R_{t+s}} \frac{1-\alpha}{\alpha} \frac{\phi}{\alpha} K_{t+j-1}. \tag{29}$$

Using the same procedure once more:

$$\begin{aligned} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \frac{1}{\Pi_{s=1}^{j-3}} \frac{1-\alpha}{\alpha} \frac{\phi}{\alpha} \frac{K_{t+j-1}}{v_{t+j-2}} &= \frac{1}{\Pi_{s=1}^{j-3}} \frac{1-\alpha}{\alpha} \left(\frac{\phi}{\alpha}\right)^2 \frac{A_{t+j-2} K_{t+j-2}^{\alpha}}{A_{t+j-2} K_{t+j-2}^{\alpha-1}} \\ &= \frac{1}{\Pi_{s=1}^{j-3}} \frac{1-\alpha}{\alpha} \left(\frac{\phi}{\alpha}\right)^2 K_{t+j-2}. \end{aligned}$$

The recursion should be clear by now; applying the recursion until period $t$, I obtain:

$$\frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \frac{1-\alpha}{\alpha} \left(\frac{\phi}{\alpha}\right)^{j-1} K_{t+1}. \tag{30}$$

With this result, human wealth is computed as:

$$h_t = \sum_{j=0}^{\infty} \frac{w_{t+j}}{\Pi_{s=1}^{j} R_{t+s}} = \frac{1-\alpha}{\alpha} K_{t+1} \sum_{j=0}^{\infty} \left(\frac{\phi}{\alpha}\right)^{j-1}. \tag{31}$$

Assuming that $\alpha > \phi$, I obtain:

$$h_t = \frac{1-\alpha}{\alpha - \phi} \frac{\alpha}{\phi} K_{t+1} = \frac{1-\alpha}{\alpha - \phi} \frac{\alpha}{\phi} \phi A_t K_t^{\alpha}, \tag{32}$$

where the second equality uses once again conjecture (24). Note also that by using the wage rate in (25) human wealth is:

$$h_t = \frac{\alpha}{\alpha - \phi} w_t. \tag{33}$$

This result states that, under full depreciation of capital and the conjecture (24) for aggregate capital accumulation, human wealth depends only on current wages!

### 3.4. Consistency

Capital from (23) can be written as:

$$k_{t+1} = \beta(R_t k_t + h_t) + w_t - h_t = \beta R_t k_t + w_t - (1-\beta) h_t. \tag{34}$$

Replacing $h_t$ from (33):

$$k_{t+1} = \beta R_t k_t - \frac{\phi - \alpha\beta}{\alpha - \phi} w_t. \tag{35}$$

Substituting out prices:

$$k_{t+1} = \alpha\beta A_t K_t^{\alpha-1} k_t - \frac{\phi - \alpha\beta}{\alpha - \phi}(1-\alpha) A_t K_t^{\alpha}. \tag{36}$$

Consistency requires that aggregate capital supplied by the measure one of households $k_t$ be equal to the conjectured capital $K_t$, and then equation (36) translates to:

$$K_{t+1} = \left[\alpha\beta - \frac{\phi - \alpha\beta}{\alpha - \phi}(1-\alpha)\right] A_t K_t^{\alpha}. \tag{37}$$

Rational expectations require that the "subjective" conjecture (24) for the evolution of the stock of capital be equal to the actual equilibrium evolution of the stock of capital in (37). Therefore, the following equation must be satisfied:

$$\phi = \alpha\beta - \frac{\phi - \alpha\beta}{\alpha - \phi}(1-\alpha). \tag{38}$$

This is a quadratic equation that upon simplification adopts the simple form: $(\phi - \alpha\beta)(1-\phi) = 0$. Therefore, one solution is $\alpha\beta$ and the other is one. The unit solution must be discarded as otherwise (24) would imply that no resources are left for consumption. Note also that the solution $\phi = \alpha\beta$ is less than $\alpha$, an assumption we have used to obtain equation (33). This equation now reads simply $h_t = w_t/(1-\beta)$, and solutions (23) then cease to depend on the future path of prices. This behavior—which seems to be myopic—is not myopic at all. In fact, households are able to compute exactly the present value of labor income with perfect foresight, and that computation leads them to obtain the result that only current prices matter.

With these results, we can finally obtain the equilibrium solutions of the model. Substituting $R_t$ from (25a), and $h_t$ from (33), together with wages from (25b), total wealth in equilibrium is

given by:

$$\omega_t = \frac{1-\alpha\beta}{1-\beta} A_t K_t^{\alpha}. \tag{39}$$

Going back to (23) and replacing all equilibrium values, I obtain:

$$c_t = (1-\alpha\beta) A_t K_t^{\alpha}, \quad K_{t+1} = \alpha\beta A_t K_t^{\alpha}. \tag{40}$$

These solutions are, of course, the well-known solutions obtained using the social planner's approach. They state that a fraction $\alpha\beta$ of current output is devoted to capital accumulation and the remainder to consumption.

## 4. Concluding Remarks

This paper shows that solving the deterministic neoclassical growth model through the market approach clarifies why future productivity is irrelevant in its closed-form solution. The central planner's formulation hides the interaction between log utility and full depreciation that drives this result. Although closed-form solutions need not imply such independence Antony & Maußner (2012), in this model the same assumptions that ensure tractability also produce the counterfactual myopia noted by Beaudry & Portier (2006).

Future work could extend the analysis to include labor disutility. Under log preferences and full depreciation, a closed-form solution still exists, but identifying the precise mechanisms through which the influence of future productivity disappears remains an open question.

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