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Deterrence, disruption, and nonlinear price dynamics at energy chokepoints

Masaaki Yoshimori
McCourt School of Public Policy, Georgetown University

Abstract

This paper develops a tractable sequential game of strategic interaction at critical maritime energy chokepoints, where geographic asymmetries generate disproportionate effects on global oil prices. We model a sequential game between a challenger and an incumbent in which oil prices are determined endogenously by a nonlinear function of disruption and deterrence. The interaction between diminishing returns to deterrence and convex deterrence costs generates threshold equilibria with three possible outcomes: effective deterrence, active disruption, and an intermediate regime in which disruption persists despite positive deterrence. The intermediate equilibrium provides a theoretical explanation for sustained price pressure and incomplete stabilization during periods of geopolitical tension. More broadly, the framework helps explain nonlinear oil price responses to geopolitical shocks and provides a tractable theoretical foundation for future empirical research on energy market dynamics.

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Contact: Masaaki Yoshimori - my612@georgetown.edu.
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1. Introduction

Global energy markets are highly sensitive to geopolitical risk, yet recent developments surrounding tensions involving Iran have underscored the fragility of oil supply chains and the disproportionate influence of strategic chokepoints. The Strait of Hormuz—through which approximately 20% of global oil supply transits—represents a critical node where even limited disruptions can generate outsized global effects. Episodes of heightened tension have produced sharp and nonlinear movements in oil prices, characterized by rapid escalation, volatility, and incomplete reversals, reflecting both physical supply constraints and shifting expectations.

Recent geopolitical developments surrounding the Iran conflict highlight several stylized facts. First, even limited disruptions or credible threats to maritime transit can trigger substantial price increases, with crude oil prices exceeding $100 per barrel and, in extreme cases, approaching $150 under severe supply constraints (International Energy Agency, 2023; World Bank, 2023). Second, price responses are highly nonlinear: initial disruptions generate large price effects, while further escalation produces diminishing marginal increases as markets adjust through rerouting, inventory drawdowns, and demand responses (Baumeister and Hamilton, 2019). Third, deterrence efforts—such as military deployments and diplomatic interventions—reduce price pressures but exhibit diminishing effectiveness, as uncertainty and geopolitical risk premia persist even under partial stabilization (Caldara and Iacoviello, 2022). This persistence is consistent with insights from Akerlof (1970), where incomplete information can generate sustained distortions even when stabilizing mechanisms are present.

These patterns are difficult to reconcile with linear models of supply shocks or symmetric models of conflict. Existing literature in energy economics has emphasized the role of supply-demand fundamentals (Kilian, 2009), while recent work highlights the macroeconomic importance of geopolitical risk (Ready, 2018). However, these approaches generally abstract from the strategic interaction between disruption and deterrence, particularly in environments where geographic constraints amplify market power.
This paper develops a theoretical framework to address this gap. It models strategic interaction between a challenger and an incumbent in a sequential game of disruption and deterrence, where oil prices are determined endogenously through a nonlinear function of disruption and deterrence intensity. The central mechanism is that deterrence reduces the price impact of disruption, but at a diminishing rate, while the challenger's incentives depend on the residual price effect after deterrence.

The model generates threshold behavior in equilibrium. Small changes in deterrence intensity or disruption incentives can induce discrete shifts between regimes of effective deterrence and active disruption. As a result, equilibrium outcomes are not characterized by smooth adjustment, but by regime changes and persistent intermediate states. In particular, the framework identifies an intermediate region in which deterrence is positive yet insufficient to prevent disruption, leading to sustained price pressure and incomplete stabilization. This

mechanism helps explain observed oil market dynamics in the context of Iran-related tensions. Price spikes, partial reversals, and persistent volatility emerge as equilibrium outcomes of strategic interaction under nonlinear constraints. More broadly, the framework provides a tractable approach to analyzing how localized geopolitical shocks propagate through global energy markets. By linking strategic behavior at chokepoints to price formation, it offers a unified explanation for asymmetric influence, nonlinear volatility, and the limits of deterrence in contemporary energy security environments.

2. Literature Review

Recent literature has increasingly emphasized the role of nonlinear dynamics and strategic interaction in shaping macroeconomic and financial outcomes under uncertainty. In energy markets, models incorporating uncertainty and expectations highlight how volatility arises from forward-looking behavior and incomplete information (Pindyck, 2004; Alquist and Kilian, 2010). Empirical work also suggests that oil prices respond asymmetrically to shocks, with nonlinear adjustment mechanisms driven by constraints in supply chains and inventory dynamics (Kilian and Murphy, 2014; Baumeister and Kilian, 2016). These findings point to the importance of moving beyond linear frameworks in analyzing energy price formation.

From a methodological perspective, this paper relates to the theory of dynamic and sequential games under incomplete information. Foundational contributions by Fudenberg and Tirole (1991) and Osborne and Rubinstein (1994) provide the basis for analyzing Subgame Perfect Nash Equilibria in strategic environments. The model also connects to the global games literature, where small changes in fundamentals can generate discrete shifts in equilibrium behavior (Carlsson and van Damme, 1993; Morris and Shin, 1998, 2003). These frameworks highlight the importance of threshold effects and equilibrium selection in environments characterized by strategic uncertainty. In addition, the analysis relates to models of strategic complementarities and tipping points (Cooper and John, 1988; Angeletos and Werning, 2006), where nonlinearities give rise to multiple regimes and discontinuous transitions. Applications in macroeconomics and financial markets further demonstrate how such mechanisms generate persistent volatility and regime switching (Brunnermeier and Oehmke, 2013; Gennaioli *et al*., 2018).

Building on these strands, this paper integrates nonlinear price formation with strategic interaction, providing a tractable framework in which deterrence and disruption jointly determine equilibrium outcomes through threshold dynamics and regime shifts.

3. Theoretical Framework

This section develops a sequential game of deterrence and disruption at a critical maritime energy chokepoint. The objective is to explain how localized strategic interactions generate nonlinear oil price dynamics through endogenous deterrence decisions. The framework focuses on the Strait of Hormuz as a representative chokepoint, although the analysis applies more generally to geographically constrained energy transportation routes.

The game consists of two players: an incumbent, denoted by I, and a challenger, denoted by E. The incumbent represents the coalition seeking to maintain freedom of navigation and uninterrupted oil transportation through the Strait of Hormuz. In practice, this coalition may correspond to the United States and its regional partners, although the model is intentionally formulated in a general manner. The incumbent possesses conventional military superiority and can allocate costly deterrence resources to reduce the likelihood and effectiveness of

maritime disruption. The challenger represents a regional actor capable of disrupting maritime traffic through attacks on commercial shipping, mining operations, or credible threats against vessels transiting the Strait. Iran provides a natural empirical example, although the model is intended to characterize strategic interactions at critical maritime chokepoints more generally.

The strategic interaction is sequential. In the first stage, the incumbent chooses the level of deterrence, $\alpha_I \in [0, \infty)$, where larger values represent greater military deployment, surveillance, diplomatic pressure, or other stabilization efforts. After observing the incumbent's deterrence decision, the challenger chooses an action $\alpha_E \in \{0,1\}$, where $a_E = 1$ denotes disruption and $a_E = 0$ denotes no disruption. This timing reflects the forward-looking nature of deterrence. The incumbent commits resources before any disruption occurs, while the challenger conditions its decision on the observed level of deterrence.

Oil prices are determined endogenously according to

$$P = P_0 + \frac{\phi a_E}{1 + \kappa a_I} \quad (1)$$

where P_0 is the benchmark oil price, $\phi > 0$ represents the maximum price impact of a successful disruption, and $\kappa > 0$ measures the effectiveness of deterrence in reducing disruption-induced price increases.

When no disruption occurs ($a_E = 0$), the oil price remains at its benchmark level, $P = P_0$. If disruption occurs ($a_E = 1$), deterrence reduces the resulting price increase. The price function satisfies $\frac{\partial P}{\partial a_I} < 0$ and $\frac{\partial^2 P}{\partial a_I^2} > 0$. Consequently, deterrence lowers the price effect of disruption, although its marginal effectiveness declines as deterrence becomes more intensive. This specification captures the idea that initial deterrence substantially reduces geopolitical risk, whereas further military escalation yields progressively smaller stabilization benefits. The model therefore formalizes the geographic asymmetry of maritime chokepoints: localized disruption generates globally significant price responses because alternative transportation routes are limited in the short run.

The challenger's payoff is

$$\pi_E = a_E\left[\theta_E\left(P - P_0\right) - F_E\right] \quad (2)$$

where $\theta_E > 0$ represents the economic or strategic benefit derived from higher oil prices, and $F_E > 0$ denotes the fixed cost of initiating a disruption.

The incumbent's payoff is

$$\pi_I = -\lambda\left(P - P_0\right)^2 - k a_I^2 - \delta a_E \quad (3)$$

where $\lambda > 0$ measures the welfare losses associated with oil price deviations, $k > 0$ represents the convex cost of deterrence, and $\delta > 0$ captures the direct strategic cost incurred when disruption is successful.

The incumbent is interpreted as a welfare-maximizing security coalition rather than an oil-exporting country. Consequently, both unusually high and unusually low oil prices are

undesirable because they create macroeconomic instability, increase uncertainty, and distort investment decisions. The quadratic loss function therefore penalizes deviations from the benchmark price regardless of direction, while deterrence expenditures are assumed to exhibit increasing marginal costs.

Substituting Equation (1) into Equation (2) yields

$$\pi_E = a_E \left[\frac{\theta_E \phi a_E}{1+\kappa a_I} - F_E \right]$$

Since $\alpha_E \in [0,1]$, it follows that $a_E^2 = a_E$. Therefore, $\pi_E = a_E \left[\frac{\theta_E \phi}{1+\kappa a_I} - F_E \right]$. The challenger compares the payoff from disruption ($a_E = 1$) with the payoff from remaining inactive ($a_E = 0$). Disruption is optimal if and only if

$$\frac{\theta_E \phi}{1+\kappa a_I} \geq F_E \quad (4)$$

Equation (4) shows that disruption becomes less attractive as deterrence increases. Conversely, larger supply shocks or higher disruption gains strengthen incentives to disrupt.

Solving Equation (4) for the incumbent's deterrence level yields the critical threshold

$$\bar{a}_I = \frac{1}{\kappa}\left(\frac{\theta_E \phi}{F_E} - 1 \right) \quad (5)$$

To focus on the strategically relevant case, the analysis maintains the assumption $\theta_E \phi > F_E$. Thus, in the absence of deterrence, disruption is profitable for the challenger, and the critical deterrence threshold satisfies $\hat{a}_I > 0$. If instead $\theta_E \phi \leq F_E$, the challenger does not disrupt even at $a_I = 0$, and the incumbent optimally chooses zero deterrence. This limiting case is immediate and is excluded from the subsequent analysis.

Hence,

$$\begin{cases} a_I < \bar{a}_I, & \text{Disruption occurs} \\ a_I \geq \bar{a}_I, & \text{Disruption is deterred} \end{cases}$$

The threshold exhibits intuitive comparative statics, $\frac{\partial \bar{a}_I}{\partial \theta_E} > 0, \frac{\partial \bar{a}_I}{\partial \phi} > 0, \frac{\partial \bar{a}_I}{\partial F_E} < 0$. Higher disruption gains or larger potential supply shocks require stronger deterrence to prevent disruption, whereas larger disruption costs reduce the deterrence required for maintaining stability. The threshold mechanism is the central feature of the model. Rather than generating smooth changes in equilibrium behavior, small changes in deterrence may trigger discrete transitions between stable shipping conditions and active disruption. This threshold property provides the foundation for the equilibrium analysis developed in the following section.

4. Method

The game is solved by backward induction. Because the challenger observes the incumbent's deterrence choice before acting, the solution proceeds by first deriving the challenger's optimal response and then solving the incumbent's optimization problem.

Equation (4) implies that the challenger adopts a threshold strategy. When the challenger is indifferent at $a_I = \hat{a}_I$, we adopt the conventional tie-breaking rule that it refrains from disruption.

$$a_E^*(a_I) = \begin{cases} 1, & a_I < \bar{a}_I \\ 0, & a_I \geq \bar{a}_I \end{cases} \quad (6)$$

Thus, disruption occurs whenever deterrence falls below the critical threshold derived in Equation (5). Otherwise, disruption is deterred. The incumbent anticipates the challenger's best response and therefore solves a piecewise optimization problem. Two regimes arise naturally.

(i) Disruption Regime

If $a_I < \bar{a}_I$, it follows that the challenger chooses disruption ($a_E = 1$).

The incumbent therefore solves

$$\max_{a_I < \bar{a}_I} -\lambda \left(\frac{\phi}{1 + \kappa a_I} \right)^2 - k a_I^2 - \delta \quad (7)$$

The first-order condition is

$$\frac{2\lambda \phi^2 \kappa}{(1 + \kappa a_I)^3} - 2k a_I = 0 \quad ¿(8)$$

The lower-bound condition must also be considered because $a_I \in [0, \infty)$. Evaluating the marginal payoff at $a_I = 0$ gives

¿Hence, zero deterrence cannot maximize the incumbent's payoff in the disruption regime.

Moreover, $\frac{\partial^2 \pi_I^D}{\partial a_I^2} = \frac{-6\lambda \phi^2 \kappa^2}{(1 + \kappa a_I)^4} - 2k < 0$, so the payoff is strictly concave. Since the marginal payoff is positive at the lower bound and becomes negative for sufficiently large a_I, Equation (8) has a unique positive solution, denoted by $a_I^D > 0$. This solution is the incumbent's candidate optimum in the disruption regime, provided that it lies below the deterrence threshold $\hat{a}_I$.

(ii) Successful Deterrence Regime

Suppose that $a_I \geq \bar{a}_I$. Then, no disruption occurs ($a_E = 0$), and the oil price remains at its benchmark level, $P = P_0$.

The incumbent therefore solves

$$\max_{a_I \geq \bar{a}_I} -k a_I^2 \quad (10)$$

Since deterrence costs are strictly increasing, the incumbent chooses the smallest deterrence level consistent with preventing disruption,

$$a_I^{ND} = \bar{a}_I \quad (11)$$

The incumbent compares its payoffs under the two regimes.

Under successful deterrence,

$$\pi_I^{ND} = -k \bar{a}_I^2 \quad (12)$$

Under disruption,

$$\pi_I^D = -\lambda\left(\frac{\phi}{1+\kappa a_I^D}\right)^2 - k\left(a_I^D\right)^2 - \delta \quad (13)$$

The equilibrium depends on which payoff is larger.

5. Results

The equilibrium analysis demonstrates that the sequential game admits a pure-strategy Subgame Perfect Nash Equilibrium characterized by a deterrence threshold. As established in Proposition 1, the challenger follows a cutoff strategy: disruption occurs whenever deterrence falls below the critical threshold, whereas sufficiently strong deterrence prevents disruption. Consequently, equilibrium behavior is discontinuous rather than smooth. Small changes in deterrence around the threshold can generate discrete shifts between active disruption and successful deterrence, producing regime switching in equilibrium.

The equilibrium reflects two opposing economic forces. On the one hand, deterrence reduces the price impact of disruption through the nonlinear price function specified in Equation (1). The marginal effectiveness of deterrence, however, declines as deterrence intensity increases, implying diminishing returns to additional military or strategic investment. On the other hand, deterrence itself is costly. Because these costs are assumed to be convex, progressively higher levels of deterrence require disproportionately larger expenditures. The incumbent therefore faces a trade-off between reducing oil price volatility and minimizing the cost of maintaining security.

Comparative statics further illustrate how model parameters influence equilibrium outcomes. As shown in Proposition 2, the deterrence threshold increases with the potential price impact of disruption (ϕ) and with the challenger's gains from higher oil prices (θ_E). In contrast, higher disruption costs (F_E) reduce the deterrence required to prevent disruption. These results imply that stronger economic incentives for disruption or greater market sensitivity require more intensive deterrence to maintain stability, whereas costly disruption naturally discourages hostile actions.

A particularly important implication of the model is the existence of an intermediate equilibrium regime in which deterrence is positive but insufficient to eliminate disruption,

$$a_E = 1, \; 0 < a_I^* < \bar{a}_I \quad (13)$$

In this regime, the incumbent invests in deterrence because completely abandoning deterrence would generate even greater welfare losses, yet the chosen level remains below the threshold required to prevent disruption. Consequently, equilibrium is characterized by persistent upward pressure on oil prices together with partial stabilization. This outcome emerges endogenously from diminishing returns to deterrence and convex deterrence costs rather than from exogenous institutional or technological constraints. The existence of this intermediate regime helps explain why periods of sustained geopolitical tension are often associated with elevated but not explosive oil prices despite substantial security expenditures. The model also has broader policy implications. Because deterrence exhibits diminishing marginal effectiveness, increasing military deployments alone may not be sufficient to eliminate geopolitical price risk. Beyond a certain point, additional deterrence generates relatively small reductions in expected oil price volatility while imposing increasingly large economic costs. This finding suggests that efficient energy security requires a broader portfolio of policies, including strategic petroleum reserves, diversification of supply routes, expanded liquefied natural gas infrastructure, and alternative transportation corridors. These

complementary measures effectively increase the cost of disruption or reduce its price impact, thereby lowering the deterrence threshold required to maintain market stability.

Finally, although the model is developed as a static sequential game, the threshold mechanism provides a natural explanation for persistent volatility observed in global energy markets. Repeated episodes of geopolitical tension may generate recurrent transitions between disruption and deterrence regimes, producing clustered volatility and nonlinear price dynamics. A fully dynamic analysis based on a Markov Perfect Equilibrium is beyond the scope of the present paper and is therefore left for future research.

6. Conclusion

This paper develops a sequential game to analyze strategic interaction at critical energy chokepoints. The model demonstrates how endogenous oil price formation and costly deterrence generate threshold equilibria, implying that small changes in deterrence may produce discrete shifts between disruption and stability. The analysis therefore provides a simple theoretical explanation for the nonlinear relationship between geopolitical risk and energy price volatility.

The model identifies three equilibrium regimes: effective deterrence, active disruption, and an intermediate equilibrium regime in which disruption persists despite positive deterrence. This intermediate regime shows that deterrence may be strategically valuable without being sufficient to eliminate disruption, reflecting the interaction between diminishing returns to deterrence and convex deterrence costs.

The results also have important policy implications. Because additional deterrence becomes progressively less effective, military measures alone may not be sufficient to ensure energy security. Complementary policies, including strategic petroleum reserves, diversified transportation routes, and alternative energy infrastructure, can reduce the economic consequences of disruption. Although the analysis is developed as a static sequential game, the framework provides a useful foundation for future empirical research on geopolitical risk, maritime security, and global energy markets.

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Appendix A. Proofs of the Main Results: Proof of Proposition 1

The game is a finite sequential game with perfect information. The incumbent first chooses a deterrence level, $\alpha_1 \in ¿$, after which the challenger observes this choice and selects $\alpha_E \in [0,1]$.

The challenger's payoff is given by

$$\pi_E = a_E\left[\theta_E\left(P - P_0\right) - F_E\right] \quad \text{(A1)where oil prices satisfy}$$

$$P = P_0 + \frac{\phi a_E}{1+\kappa a_I} \quad \text{(A2)}$$

Substituting (A2) into (A1) yields

$$\pi_E = a_E\left[\frac{\theta_E \phi a_E}{1+\kappa a_I} - F_E\right] \quad \text{(A3)}$$

Since $a_E \in [0,1]$, it follows that $a_E^2 = a_E$.

Therefore,

$$\pi_E = a_E\left[\frac{\theta_E \phi}{1+\kappa a_I} - F_E\right] \quad \text{(A4)}$$

The challenger chooses disruption whenever

$$\frac{\theta_E \phi}{1+\kappa a_I} \geq F_E \quad \text{(A5)}$$

which establishes the threshold strategy described in Equation (6). Anticipating this response, the incumbent solves the optimization problems defined by Equations (7)-(10). Anticipating the challenger's threshold response, the incumbent solves the optimization problem separately in the disruption and deterrence regimes. In the disruption regime, the incumbent's marginal payoff at the lower bound is ¿¿ Therefore, $a_I = 0$ cannot be optimal when disruption is profitable in the absence of deterrence. Moreover, the incumbent's payoff is strictly concave in a_I, and its marginal payoff eventually becomes negative as deterrence increases. It follows that the first-order condition admits a unique positive solution $a_I^D > 0$. If $a_I^D < \hat{a}_I$, it is the unique optimum within the disruption regime; otherwise, the incumbent's equilibrium choice is determined by comparison with the successful-deterrence candidate $a_I = \hat{a}_I$.

Appendix B. Proofs of the Main Results: Proof of Proposition 2

The critical deterrence threshold is obtained from Equation (A5)

$$\bar{a}_I = \frac{1}{\kappa}\left(\frac{\theta_E \phi}{F_E} - 1\right) \quad \text{(B1)}$$

Differentiating (B1) with respect to the model parameters yields

$$\frac{\partial \bar{a}_I}{\partial \theta_E} = \frac{\phi}{\kappa F_E} > 0 \quad \text{¿(B2)} \quad \frac{\partial \bar{a}_I}{\partial \phi} = \frac{\theta_E}{\kappa F_E} > 0 \quad \text{(B3)}$$

and

$$\frac{\partial \bar{a}_I}{\partial F_E} = \frac{-\theta_E \phi}{\kappa F_E^2} < 0 \quad \text{(B4)}$$

Equations (B2)-(B4) establish the comparative statics of the deterrence threshold. A larger potential price impact of disruption (ϕ) or greater gains from disruption (θ_E) increase the

level of deterrence required to prevent disruption. Conversely, higher disruption costs (F_E) reduce the deterrence threshold because disruption becomes less attractive to the challenger. The incumbent's optimization problem in the disruption regime is

$$\max_{a_I<\bar{a}_I}\left\{-\lambda\left(\frac{\phi}{1+\kappa a_I}\right)^2-k a_I^2-\delta\right\} \quad \text{(B5)}$$

The first-order condition is

$$\frac{2\lambda\phi^2\kappa}{\left(1+\kappa a_I\right)^3}-2k a_I=0 \quad \text{(B6)}$$

Differentiating (B6) with respect to a_I gives the second-order derivative

$$\frac{d^2\pi_I}{da_I^2}=\frac{-6\lambda\phi^2\kappa^2}{\left(1+\kappa a_I\right)^4}-2k \quad \text{(B7)}$$

Since $\lambda>0,\phi>0,\kappa>0,k>0$, it follows immediately that

$$\frac{d^2\pi_I}{da_I^2}<0 \quad \text{(B8)}$$

Furthermore, evaluating Equation (B6) at the lower bound gives ¿¿ Because the marginal payoff is positive at $a_I=0$, becomes negative for sufficiently large a_I, and is strictly decreasing by Equation (B7), Equation (B6) has exactly one positive solution. Thus, the incumbent’s disruption-regime candidate satisfies $a_I^D>0$ and is unique.

Together, Equations (B2)-(B8) establish the comparative statics and uniqueness of the incumbent's optimal deterrence strategy, thereby proving Proposition 2.