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### Time-varying tail risk in BRIC stock markets under global shocks: An ADCC-based approach

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#### Abstract

This study develops a robust market risk framework for BRIC portfolios navigating heightened global uncertainty. By applying the Bai-Perron test, multiple breakpoints emerged within the volatility series, particularly during the COVID-19 pandemic. The inclusion of breakpoint dummy variables into a rolling-window GARCH model was found to be statistically insignificant, implying that rolling estimation sufficiently captures regime shifts autonomously. Using a standard ADCC-GARCH estimation, the results show that BRIC markets exhibit strong volatility persistence, heavy tails, and stable long-run correlations. The portfolio optimization for the subsequent period after 2022 reveals China and India as dominant time-varying allocation drivers during the Russia-Ukraine conflict. Finally, dynamic VaR and ES forecasts, validated by conservative yet statistically reliable backtest results, indicate that the proposed ADCC-GARCH framework accurately quantifies severe tail risk in shock-prone emerging markets.

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# 1. Introduction

The global financial markets are facing periods of extreme global uncertainty such as the COVID-19 pandemic, financial crises, and geopolitical conflicts (Russia-Ukraine conflict, Iran conflict, etc.) Thus, the literature has highlighted the growing importance of dynamic correlation models in managing global portfolios strategy and risk management. The stylized facts in global financial markets are such as volatility clustering, structural changes, and asymmetric dependence, promote the application of multivariate models such as the ADCC-GARCH model (Cappiello et al., 2006) for quantifying market risk and portfolio optimization. Sahiner (2022) conducted rolling-window GARCH approaches that effectively capture time-varying volatility without requiring explicit regime-switching specifications. Similarly, Bundoo and Ramlukun (2022) found strong volatility persistence and market interdependence within BRIC markets, particularly when studying the sensitivity of emerging markets to external shocks.

In a study by Agyei et al. (2022), they found evidence that of volatility regime shifts intensified during the COVID-19 crisis, while return series often remained comparatively stable and stationary. Their findings support the view that multiple structural breaks are primarily concentrated in volatility rather than returns. Furthermore, studies using ADCC-GARCH models reveal heavy-tailed distributions and time-varying correlations across financial assets, especially during crisis periods (Jana et al., 2023). Another study of portfolio allocation by Wang et al. (2022) suggested that diversification benefits shift over time, with certain markets becoming dominant drivers during geopolitical disruptions such as the Russia–Ukraine conflict.

In terms of risk forecasting, recent research confirms that rolling Value-at-Risk (VaR) and Expected Shortfall (ES) or Conditional VaR (CVaR) models are effective in capturing downside risk during turbulent periods (Alexander & Dakos, 2023) while remaining statistically reliable under backtesting procedures. Consequently, the ADCC-GARCH framework provides a robust methodology for tail-risk quantification and dynamic portfolio risk management in the BRIC equity markets.

In this study, we conduct a market risk analysis for the BRIC stock markets under global uncertainty caused by pandemics, financial crises, and geopolitical conflicts. A seven-step procedure is created for this purpose. The procedure focuses on structural changes, dynamic correlation, dynamic portfolio weight allocation and rolling VaR and ES forecasts evaluations.

## 2. Methodology

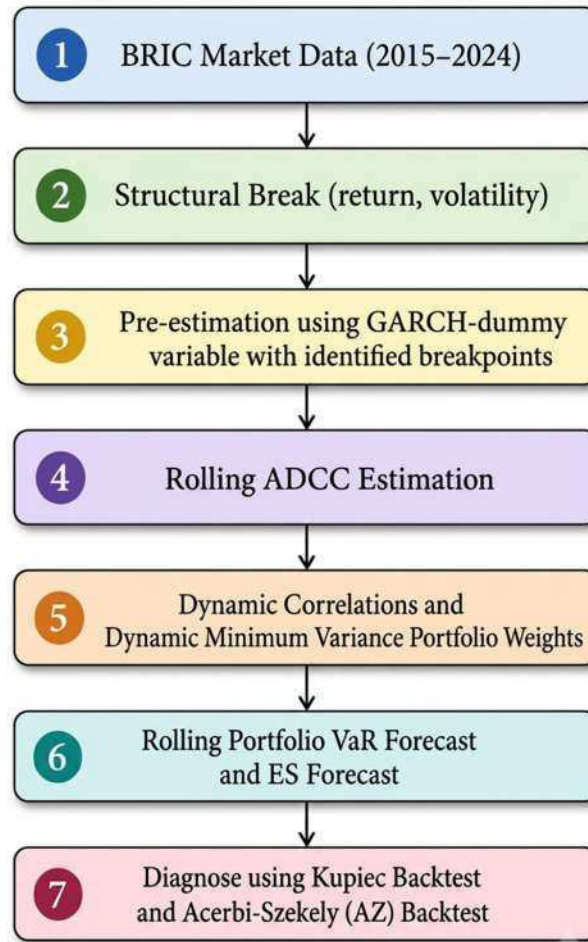


Figure 1: The Seven-step Procedure

Figure 1 provides a complete empirical framework for analyzing the BRIC stock markets. Firstly, daily adjusted close datasets are collected for the BRIC countries such as Brazil, Russia, India and China from Yahoo Finance. The overall dataset is divided into an in-sample period (2015 to 2020) and an out-of-sample period (2021 to 12 June 2024). Although the Russian stock market did not stop trading in June 2024, the availability of this dataset became unreliable from Yahoo's international data feed after the U.S. imposed sanctions on the Moscow Exchange. For out-of-sample forecasts, a rolling estimation of the ADCC-GARCH model is implemented with the window lengths of 750 and 1,000 trading days. The 750-day window is expected to obtain better responsiveness while still providing enough data for the four countries, whereas the 1,000-day

window should provide more stable parameter estimates, albeit with slower adaptation to regime changes. Both window sizes are used in the market and tail risk analysis as a robustness check. The estimated time-varying correlations are used to determine the dynamic VaR and ES (or CVaR). The sample size for the out-of-sample forecasts is 729 observations, although there are 1,259 trading days from 1 January 2021 through 12 June 2024. This is because the data merges four different trading calendars and removes missing dates.

Secondly, to identify the possibility of sudden regime changes caused by crises or global events, the return series and volatility series are checked using the Bai-Perron multiple breakpoint test (Bai and Perron, 2003). Ignoring structural breaks may cause biased volatility and correlation estimates. Thirdly, after the identification of breakpoints, a pre-estimation using rolling GARCH with dummy variables to account for regime shifts in volatility is performed where

$$D_t = \begin{cases} 1 & \text{break point} \\ 0 & \text{otherwise} \end{cases}. \text{ If the dummy variables are not statistically significant at 5\% level, they}$$

will be omitted from the GARCH model. For the multiple breakpoints analysis,

$$\sigma_t^2 = \omega + \sum_{j=1}^m \delta_j D_{j,t} + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2, \text{ where } \delta > 0, \text{ volatility increases after the break while } \delta < 0$$

, volatility decreases after the break. The dummy variables for multiple breakpoints are defined

$$\text{as } D_{j,t} = \begin{cases} 1 & t \geq T_j \\ 0 & \text{otherwise} \end{cases}, \text{ where } T_1, \dots, T_m \text{ are the times when the structural break occurred.}$$

Fourthly, the Cappiello et al. (2006) ADCC (Asymmetric Dynamic Conditional Correlation) model is used to estimate the time-varying correlations among the BRIC stock markets. Rolling estimation (time window of 750 and 1,000 observations, approximately three years of data) is implemented in order to smooth the volatility shocks and update the correlations dynamically. The rolling estimation approach for ADCC approach has been proven to partially mitigate the impacts of structural breaks in financial data analysis (Bauwens, et al., 2006; Starica & Granger, 2005). Assuming that each of the BRIC stock returns follows a GARCH (1,1) model, the continuously compounded for log returns series  $\mathbf{r}_t = (r_{1t}, r_{2t}, r_{3t}, r_{4t})'$  follows the multivariate equation  $\mathbf{r}_t = \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t$ ,

$$\text{where } \boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \\ \varepsilon_{4,t} \end{pmatrix} \text{ and the conditional covariance matrix, } \mathbf{H}_t = \text{Cov}(\boldsymbol{\varepsilon}_t). \text{ The decomposition of the}$$

ADCC matrix is  $\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t$ , with  $\mathbf{D}_t = \text{diag}(\sigma_{1t}, \sigma_{2t}, \sigma_{3t}, \sigma_{4t})$  where  $\sigma_t$  is the individual conditional volatility series from four separate GARCH models. In addition, assume that the individual innovation at the margin is Student-t distributed with  $\varepsilon_{i,t} = \sigma_{i,t} z_{i,t}$  and  $z_{i,t} \sim t(0, 1, \nu_i)$  where  $\nu_i$  is the degrees of freedom standardized to mean 0 and variance 1, and the standardized residuals form the series  $\mathbf{z}_t = (z_{1t}, z_{2t}, z_{3t}, z_{4t})'$ . The ADCC model, defines bad news (negative

shocks) as  $\mathbf{n}_t = \mathbf{z}_t \odot \mathbf{I}(\mathbf{z}_t < 0)$  where  $\odot$  is the Hadamard product and the correlation dynamics are driven by the equation

$$\mathbf{Q}_t = (1 - a - b - g) \bar{\mathbf{Q}} + a(\mathbf{z}_{t-1} \mathbf{z}_{t-1}') + b\mathbf{Q}_{t-1} + g\mathbf{n}_{t-1} \mathbf{n}_{t-1}' \quad (1)$$

where  $a$  is the impact of shocks on correlation,  $b$  is the volatility persistence and  $g$  is the bad news impact or leverage effect. After that the value can then be converted to a correlation matrix,  $\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2}$ . Finally, the 4×4 covariance matrix can be obtained as  $\mathbf{H}_t$ .

Fifthly, the dynamic minimum variance portfolio weights are determined using the covariance matrix from the ADCC model. In this way, the portfolio allocation becomes adaptive rather than static in BRIC markets. The optimal time-varying portfolio weight denotes the proportion of total capital that should be invested in each asset to achieve a global minimum-variance portfolio at each point in time. For global minimum variance portfolio, the optimal weights are defined as

$$\mathbf{w}_t = \frac{\mathbf{H}_t^{-1} \mathbf{1}}{\mathbf{1}' \mathbf{H}_t^{-1} \mathbf{1}} \text{ where } \sum \omega_i = 1 \text{ with } \omega_i = \frac{\sum_{j=1}^4 (\Sigma^{-1})_{ij}}{\sum_{i=1}^4 \sum_{j=1}^4 (\Sigma^{-1})_{ij}}.$$

Sixthly, the 95% and 99% Value-at-Risk (VaR) and 95% and 99% Expected Shortfall (ES) are computed to forecast portfolio tail risk. For the VaR formulation, the portfolio return is  $r_{p,t} = \mathbf{w}_t' \mathbf{r}_t$  with variance  $\sigma_{p,t}^2 = \mathbf{w}_t' \mathbf{H}_t \mathbf{w}_t$ . Under the Student-t assumption, the standardized residuals are  $\mathbf{z}_t \sim t_v(0, \mathbf{R}_t, v)$ . Thus, the portfolio daily VaR and ES are written as

$$\begin{aligned} VaR_{\alpha,t} &= -r_{p,t} - \sigma_{p,t} \cdot t_v^{-1}(\alpha) \\ ES_{\alpha,t} &= -E[r_{p,t} | r_{p,t} \leq -r_{p,t} - \sigma_{p,t} \cdot t_v^{-1}(\alpha)] \\ &= -r_{p,t} + \sigma_{p,t} \cdot \frac{f_{t_v}(t_v^{-1}(\alpha))}{\alpha} \cdot \frac{\left(v + (t_v^{-1}(\alpha))^2\right)}{v-1} \end{aligned} \quad (2)$$

where  $f_{t_v}(\cdot)$  is the Student-t density function, and  $t_v^{-1}(\alpha)$  is the standardized  $\alpha$ -quantile of the Student-t distribution with  $v$  degrees of freedom. ES is in general a better risk measurement because it captures extreme co-movements and is more sensitive to fat tails. The rolling forecast allows for the continuous updating of risk estimates and provides real-time risk monitoring.

Finally, we validate the VaR and ES forecasts using the Kupiec backtest (1995) and the Acerbi-Szekely backtest (2014).

### 3. Empirical Study

Table 1: Descriptive Statistics for return series

|        | Mean      | SD       | Min       | Max      |
|--------|-----------|----------|-----------|----------|
| Brazil | 0.000459  | 0.016589 | −0.159930 | 0.130223 |
| Russia | 0.000401  | 0.016804 | −0.404674 | 0.182619 |
| India  | 0.000506  | 0.011559 | −0.141017 | 0.085947 |
| China  | −0.000050 | 0.014028 | −0.108324 | 0.100453 |

Table 1 reports the descriptive statistics of the BRIC stock market returns from 2015 to 2025, highlighting various market performance and risk characteristics. Figure 2 and Figure 3 illustrate the price indices and return series. Among the BRIC countries, India recorded the highest average daily return (0.000506) along with the lowest standard deviation (0.011559). This suggests a relatively favorable risk-return trade-off compared to the other BRIC markets. Brazil and Russia showed positive average returns with relatively higher volatility, reflecting greater investment risk. Russia experienced the highest standard deviation (0.016804) and the most extreme return fluctuations, with a minimum daily return of −40.47% and a maximum daily return of 18.26% which may be due to geopolitical and economic shocks. China displayed a negative mean return (−0.000050) with moderate volatility, implying comparatively weaker market performance during the sample period.

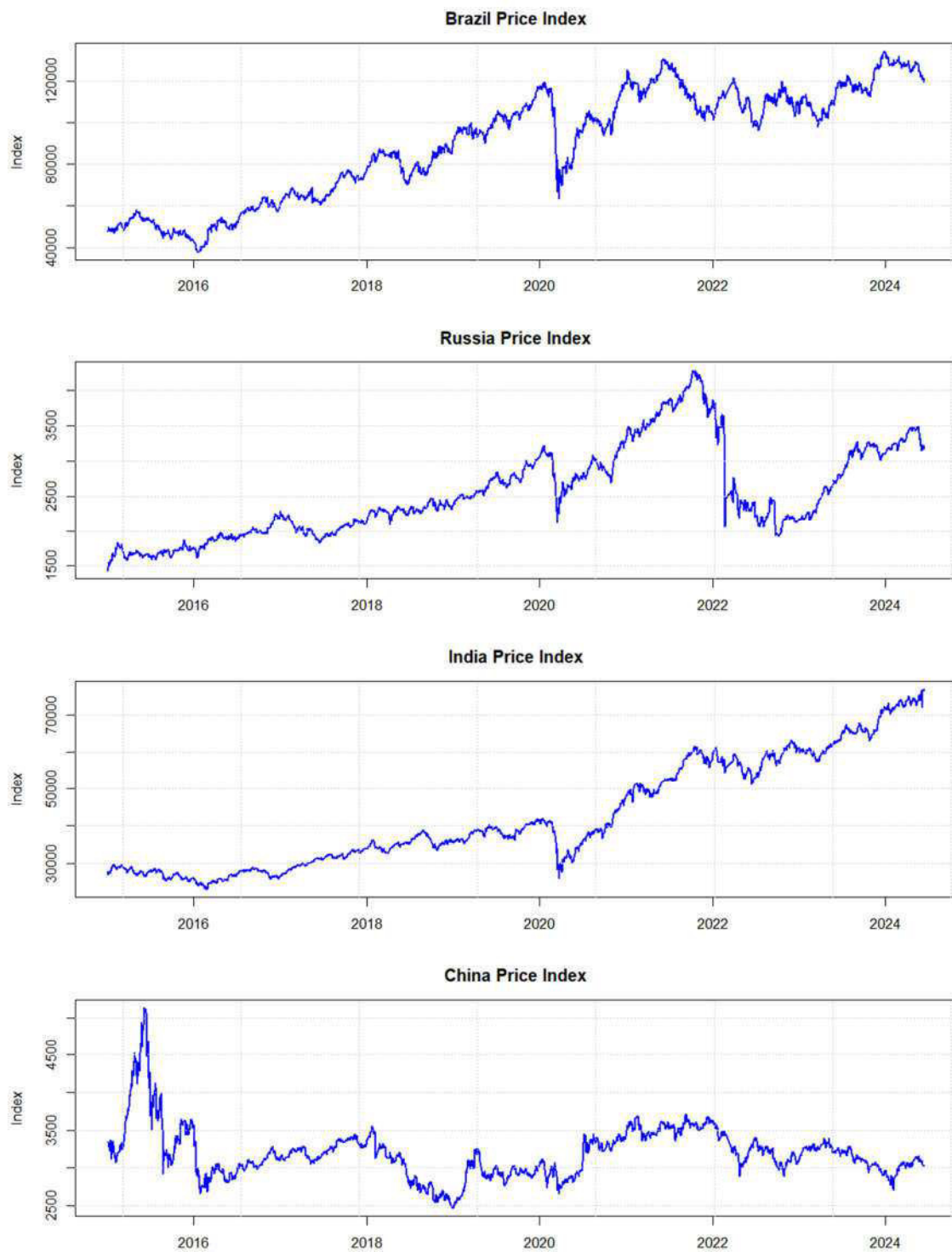


Figure 2: BRIC Price Indices

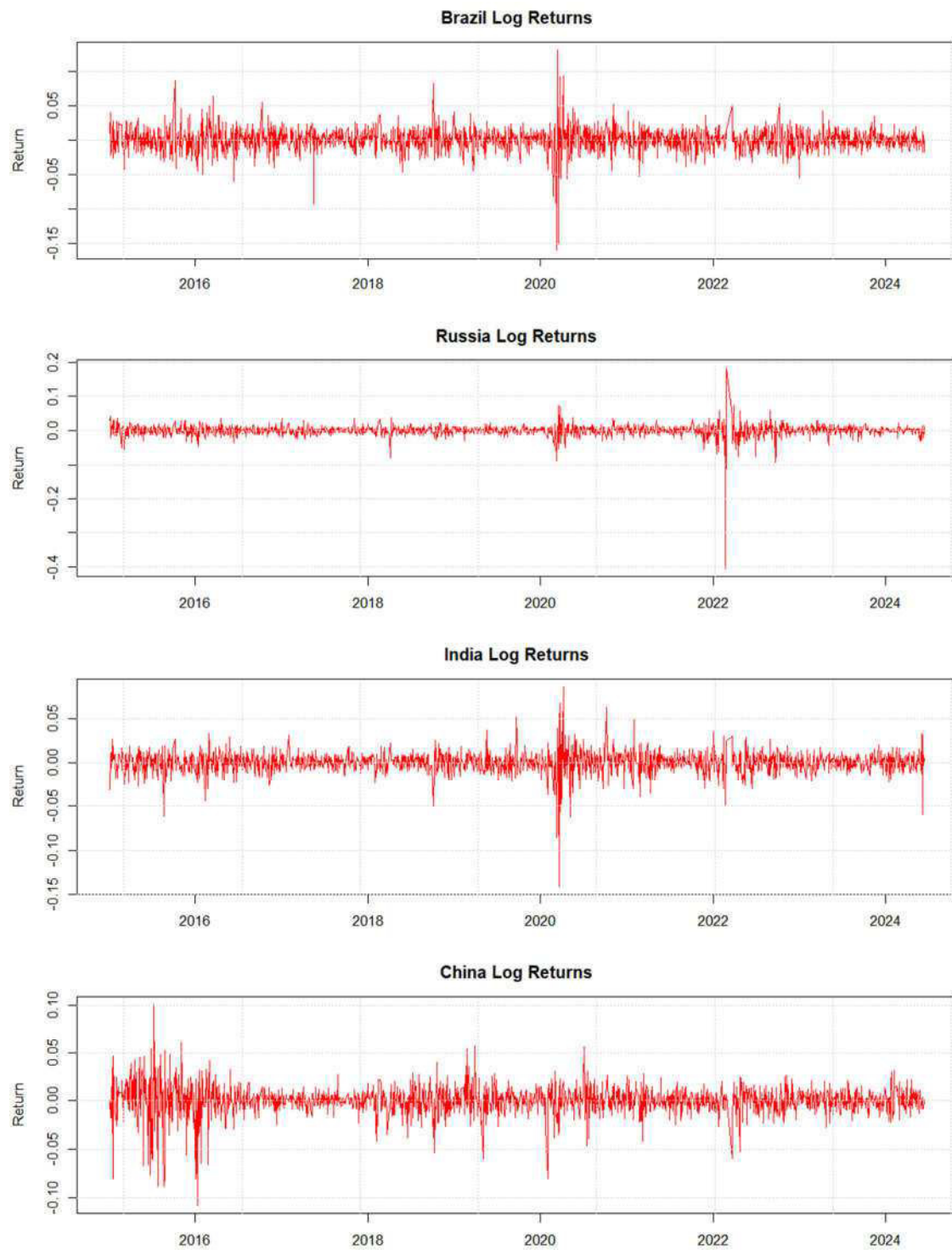


Figure 3: BRIC Return Indices

Table 2: Bai-Perron Breakpoints Identification for Return and Volatility Series

| Break Date | SupF     | P-value  | Asset  | Series     |
|------------|----------|----------|--------|------------|
| NA         | 3.423925 | 0.471491 | Brazil | Return     |
| NA         | 1.329265 | 0.946096 | Russia | Return     |
| NA         | 3.055712 | 0.542987 | India  | Return     |
| NA         | 2.347114 | 0.701207 | China  | Return     |
| Break Date | SupF     | P-value  | Asset  | Series     |
| 19/05/2016 | 148.7621 | 0.0000   | Brazil | Volatility |
| 19/02/2020 | 148.7621 | 0.0000   | Brazil | Volatility |
| 14/03/2016 | 234.7336 | 0.0000   | Russia | Volatility |
| 02/02/2018 | 234.7336 | 0.0000   | Russia | Volatility |
| 16/01/2019 | 234.7336 | 0.0000   | Russia | Volatility |
| 19/02/2020 | 234.7336 | 0.0000   | Russia | Volatility |
| 02/06/2016 | 325.5358 | 0.0000   | India  | Volatility |
| 11/09/2018 | 325.5358 | 0.0000   | India  | Volatility |
| 19/02/2020 | 325.5358 | 0.0000   | India  | Volatility |
| 08/04/2016 | 1509.971 | 0.0000   | China  | Volatility |
| 21/02/2017 | 1509.971 | 0.0000   | China  | Volatility |
| 06/02/2018 | 1509.971 | 0.0000   | China  | Volatility |
| 30/05/2019 | 1509.971 | 0.0000   | China  | Volatility |

Note: \*NA denotes no breakpoint detected.

Table 2 reports the multiple-breakpoint analysis for the BRIC return and volatility series. No breakpoint is detected in the stationary return series for all BRIC countries over the period 2015 to 2020. The test fails to reject the null hypothesis of structural stability for the return series with no permanent significant baseline shift over the sample period. In other words, these series possess a constant unconditional mean. On the other hand, the multiple breakpoint test reveals strong evidence of significant structural changes in the BRIC equity market volatilities, where all SupF statistics are highly significant with p-values less than 0.05. Brazil, Russia and India all indicate volatility regime shifts during February 2020, with the exception of China. Brazil experienced another volatility regime shift in 2016. Russia exhibited four volatility breakpoints between 2016 and 2020, indicating heightened sensitivity to geopolitical and economic events. India recorded an additional two structural breaks, around 2016 and 2018, which imply changing market uncertainty and financial instability. China showed the strongest structural instability (the highest SupF statistic) among the BRIC markets with persistent volatility regime changes. Overall, the results imply that the BRIC markets experienced substantial shifts in volatility behavior during the studied period, providing important insights into corporate structural breaks for volatility and risk management analysis.

Table 3: GARCH with Dummy Variables

| Parameter          | Estimate   | Standard Error | t-statistic | p-value |
|--------------------|------------|----------------|-------------|---------|
| [Brazil]. $\omega$ | 1.5057E-05 | 0.0000         | 24.3944     | 0.0000  |
| [Brazil]. $\alpha$ | 1.1224E-01 | 0.0029         | 39.1004     | 0.0000  |
| [Brazil]. $\beta$  | 8.3670E-01 | 0.0065         | 129.2117    | 0.0000  |
| [Brazil]. Dummy1   | 4.5557E-12 | 0.0000         | 0.0002      | 0.9999  |
| [Brazil]. Dummy2   | 1.4537E-08 | 0.0000         | 0.1696      | 0.8654  |
| [Russia]. $\omega$ | 5.2248E-06 | 0.0000         | 2.6629      | 0.0077  |
| [Russia]. $\alpha$ | 8.9122E-02 | 0.0019         | 47.5718     | 0.0000  |
| [Russia]. $\beta$  | 8.7258E-01 | 0.0143         | 61.2303     | 0.0000  |
| [Russia]. Dummy1   | 4.9548E-09 | 0.0000         | 0.0125      | 0.9900  |
| [Russia]. Dummy2   | 2.6106E-08 | 0.0000         | 0.0595      | 0.9525  |
| [Russia]. Dummy3   | 1.1559E-14 | 0.0000         | 0.0000      | 1.0000  |
| [Russia]. Dummy4   | 2.4847E-08 | 0.0000         | 0.0252      | 0.9799  |
| [India]. $\omega$  | 3.9818E-06 | 0.0000         | 2.0533      | 0.0400  |
| [India]. $\alpha$  | 9.6144E-02 | 0.0013         | 76.4271     | 0.0000  |
| [India]. $\beta$   | 8.6916E-01 | 0.0092         | 94.1592     | 0.0000  |
| [India]. Dummy1    | 1.2075E-15 | 0.0000         | 0.0000      | 1.0000  |
| [India]. Dummy2    | 8.5924E-07 | 0.0000         | 1.0072      | 0.3138  |
| [India]. Dummy3    | 3.0869E-08 | 0.0000         | 0.0109      | 0.9913  |
| [China]. $\omega$  | 1.4370E-06 | 0.0000         | 0.7736      | 0.4391  |
| [China]. $\alpha$  | 8.6569E-02 | 0.0022         | 39.6121     | 0.0000  |
| [China]. $\beta$   | 9.1081E-01 | 0.0033         | 277.0999    | 0.0000  |
| [China]. Dummy1    | 2.9905E-15 | 0.0000         | 0.0000      | 1.0000  |
| [China]. Dummy2    | 3.1986E-08 | 0.0000         | 0.1820      | 0.8556  |
| [China]. Dummy3    | 4.3052E-07 | 0.0000         | 0.8587      | 0.3905  |
| [China]. Dummy4    | 4.2395E-08 | 0.0000         | 0.2123      | 0.8318  |

Table 3 reports the results for the breakpoint dummy variables incorporated into the GARCH model for the BRIC stock markets. The estimated dummy coefficients are found to be extremely small compared to the ARCH ( $\alpha$ ) and GARCH ( $\beta$ ) coefficients across all BRIC markets. For instance, Dummy1 and Dummy2 for the Brazilian market are 4.5557E-12 and 1.4537E-08. These impacts are considered quite small relative to the  $\alpha$  and  $\beta$  coefficients, which have values 1.1224E-01 and 8.3670E-01. In addition, it is found that all dummy coefficients are statistically insignificant at the 5% level for all BRIC countries. Figure 4 illustrates the pairwise correlations among the BRIC markets, where the plots do not reveal any obvious or persistent structural breaks. Overall, the correlations display temporary fluctuations associated with periods of market stress, reverting to their long-run levels shortly thereafter. In general, all BRIC stock markets exhibit

strong volatility clustering and persistence, indicating that the structural breaks do not provide additional explanatory power beyond the time-varying volatility process.

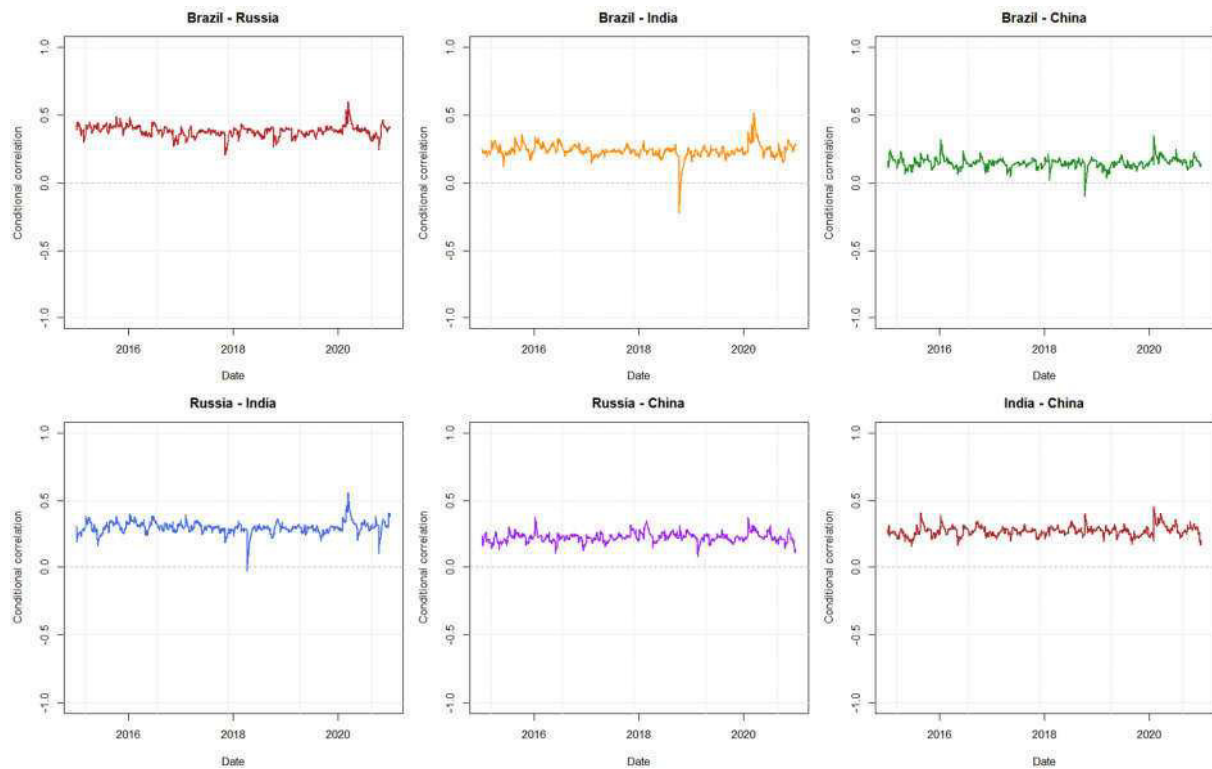


Figure 4: Pairwise Correlations Among BRIC

Table 4: ADCC-GARCH Estimation Results

| Parameter          | Estimate | Standard Error | t-statistic | p-value |
|--------------------|----------|----------------|-------------|---------|
| [Brazil]. $\omega$ | 0.0000   | 0.0000         | 9.5706      | 0.0000  |
| [Brazil]. $\alpha$ | 0.0818   | 0.0086         | 9.4715      | 0.0000  |
| [Brazil]. $\beta$  | 0.8732   | 0.0106         | 82.2567     | 0.0000  |
| [Brazil]. $\nu$    | 5.7764   | 0.8652         | 6.6761      | 0.0000  |
| [Russia]. $\omega$ | 0.0000   | 0.0000         | 3.4751      | 0.0005  |
| [Russia]. $\alpha$ | 0.0922   | 0.0123         | 7.5002      | 0.0000  |
| [Russia]. $\beta$  | 0.8591   | 0.0159         | 54.0955     | 0.0000  |
| [Russia]. $\nu$    | 6.0194   | 0.9020         | 6.6732      | 0.0000  |
| [India]. $\omega$  | 0.0000   | 0.0000         | 0.8276      | 0.4079  |
| [India]. $\alpha$  | 0.0666   | 0.0278         | 2.3932      | 0.0167  |
| [India]. $\beta$   | 0.9006   | 0.0114         | 78.8560     | 0.0000  |
| [India]. $\nu$     | 4.9843   | 0.6731         | 7.4054      | 0.0000  |
| [China]. $\omega$  | 0.0000   | 0.0000         | 0.2606      | 0.7944  |
| [China]. $\alpha$  | 0.0670   | 0.0506         | 1.3228      | 0.1859  |
| [China]. $\beta$   | 0.9320   | 0.0395         | 23.5931     | 0.0000  |
| [China]. $\nu$     | 3.7247   | 0.8642         | 4.3101      | 0.0000  |
| [Joint] $a$        | 0.0153   | 0.0084         | 1.8173      | 0.0692  |
| [Joint] $b$        | 0.8811   | 0.0557         | 15.8101     | 0.0000  |
| [Joint] $g$        | 0.0000   | 0.0102         | 0.0000      | 1.0000  |
| [Joint] $mv$       | 5.8598   | 0.3205         | 18.2807     | 0.0000  |

Table 4 provides the estimation results without the inclusion of breakpoints in the volatility series. Overall, all BRIC stock markets exhibit strong volatility persistence (ranging from 0.8591 to 0.9320) and time-varying dependence. For the transitory volatility coefficients,  $\alpha$ s are all significant for Brazil, Russia, and India, suggesting that recent market shocks have an immediate impact on current volatility, with the exception of the Chinese stock market, which implies weaker short-term shock sensitivity. For the individual shape parameter,  $\nu$ , all return series show the presence of heavy tails and support the use of a Student-t distribution. For the joint estimation, the dynamic persistence parameter,  $b$ , is 0.8811, suggesting that the correlation among BRIC markets is strongly persistent over time. The short-run correlation shock parameter,  $a$ , is marginally significant at the 10% level, implying that temporary shocks have a relatively limited immediate impact on conditional correlations. The asymmetry parameter,  $g$ , is insignificant, suggesting that negative shocks do not generate additional asymmetric impact in the dynamic correlations. Overall, the results suggest that the BRIC equity markets exhibit persistent volatility and stable long-run correlation dynamics with heavy-tailed risk characteristics.

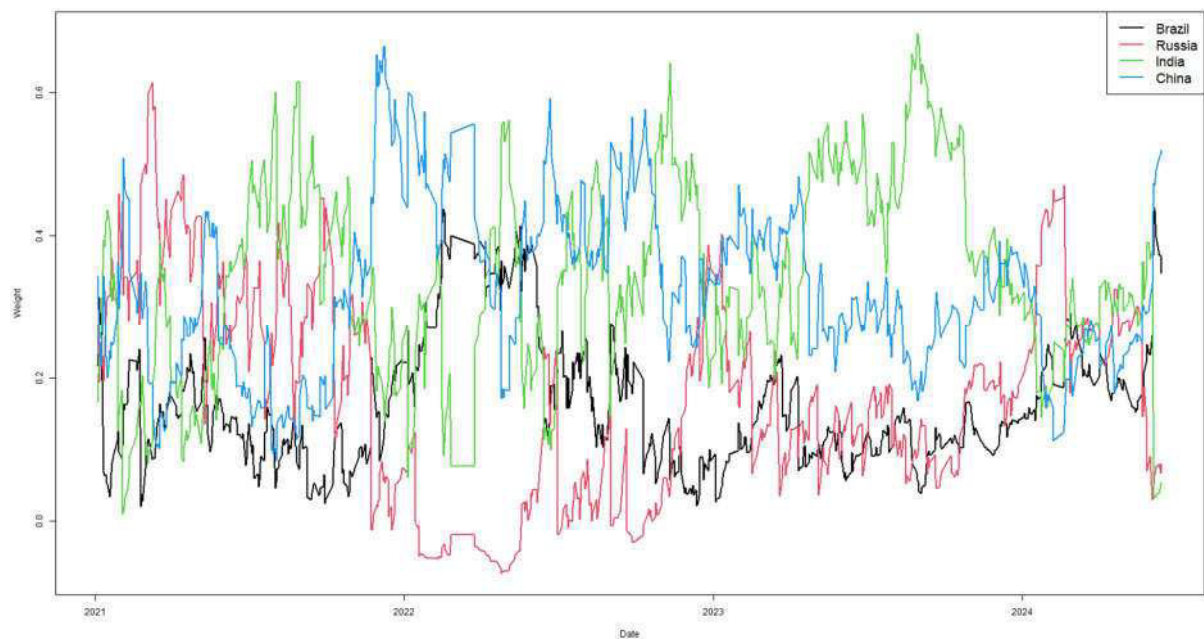


Figure 5: Optimal Weight Allocation for BRIC Portfolio

Figure 5 provides a quick view of the portfolio weight allocation in the BRIC from 2021 to 2025. Russia and India dominated the optimal weights during the 2021-2022 period. In the subsequent period after 2022, the portfolio shifted towards China and India as the primary drivers. Brazil remained a baseline residual over the studied period, and Russia was largely phased out, especially after the Russia-Ukraine conflict started in early 2022 with extreme downward volatility.

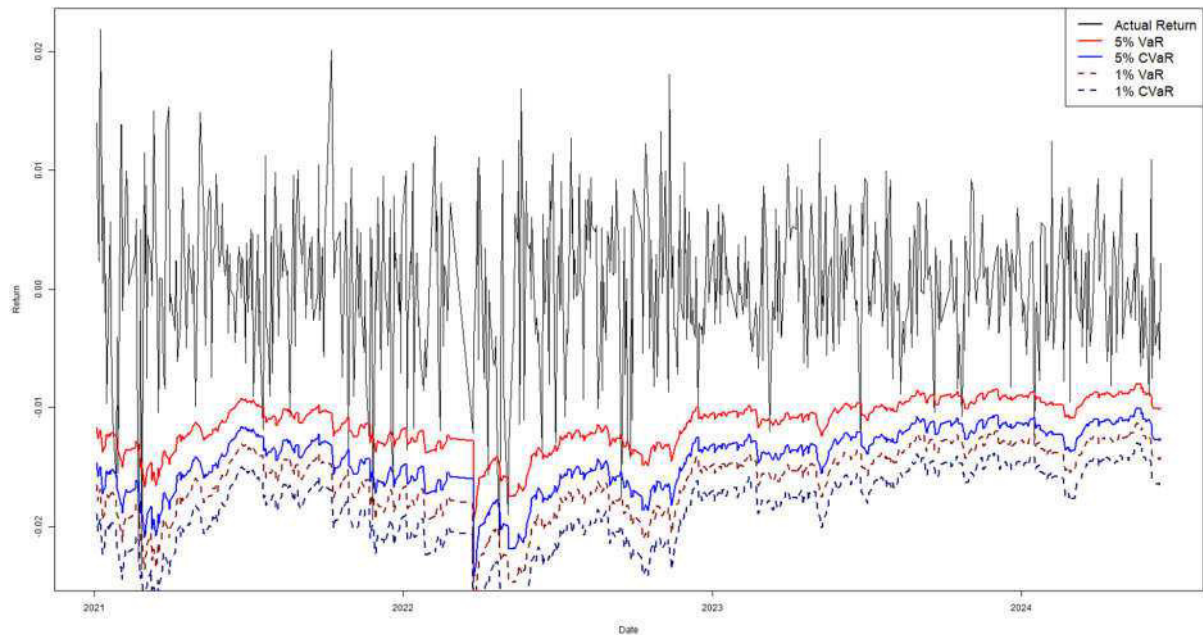


Figure 6: The 5% VaR and 5% ES Daily Forecasts

Figure 6 illustrates the dynamic VaR and ES forecasts from the year 2021 to 2025. The 5% and 1% VaR series represent the minimum expected loss threshold at the 95% and 99% confidence levels, whereas the 5% and 1% ES consistently run above (or plot deeper than) the VaR line, capturing the average tail loss. For the period from 2021 to mid-2022, the plots exhibit strong volatility clustering with both the VaR and ES plots dynamically deepening to adapt to the highly volatile downside risks. In the period after 2022, both VaR and ES show a steady upward trend, indicating a reduction in downside risk.

Table 5: Backtesting Results

| Optimal-weight |            |          |                     |                |              |                |
|----------------|------------|----------|---------------------|----------------|--------------|----------------|
| 750-day        | Violations | Expected | Kupiec LR Statistic | <i>p-value</i> | AZ Statistic | <i>p-value</i> |
| 95% 1-Day      | 26         | 36.5     | 3.518962            | 0.06067        | -0.06795     | 0.94582        |
| 99% 1-Day      | 6          | 7.3      | 0.248959            | 0.61781        | -0.09471     | 0.92454        |
| 1000-day       | Violations | Expected | Kupiec LR Statistic | <i>p-value</i> | AZ Statistic | <i>p-value</i> |
| 95% 1-Day      | 27         | 36.5     | 2.849875            | 0.09138        | -0.03570     | 0.97152        |
| 99% 1-Day      | 5          | 7.3      | 0.822948            | 0.36432        | -0.00820     | 0.99346        |
| Equal-weight   |            |          |                     |                |              |                |
| 750-day        | Violations | Expected | Kupiec LR Statistic | <i>p-value</i> | AZ Statistic | <i>p-value</i> |
| 95% 1-Day      | 25         | 36.5     | 4.26783             | 0.03884        | 0.72885      | 0.46608        |
| 99% 1-Day      | 8          | 7.3      | 0.06575             | 0.79762        | 0.64777      | 0.51712        |
| 1000-day       | Violations | Expected | Kupiec LR Statistic | <i>p-value</i> | AZ Statistic | <i>p-value</i> |
| 95% 1-Day      | 31         | 36.5     | 0.91735             | 0.33817        | 0.64604      | 0.51825        |
| 99% 1-Day      | 9          | 7.3      | 0.37230             | 0.54175        | 0.72942      | 0.46574        |

Table 5 provides the summary of the backtest results for the VaR and ES forecasts of the ADCC-GARCH model using 750-day and 1,000-day rolling estimation windows under both the optimal-weighting and equal-weighting schemes. For optimal-weight portfolio, overall, the model provides satisfactory forecasting performance at both the 95% and 99% confidence levels. For the 750-day window length, the portfolio encountered fewer violation counts (26) than statistically expected ( $np=36.5$ ), which suggests that the VaR model is slightly conservative where it tends to safely over-provide for risk rather than underestimating downside exposure. The LR statistic is 3.518962 and the *p-value* is 0.06067, which is larger than the 5% significance level. Meanwhile, the 99% VaR has a LR statistic of 0.248959 and a *p-value* 0.617809, indicating that the observed number of VaR violations is statistically consistent with the expected frequency. In short, we fail to reject the null hypothesis for the 5% VaR, although it is close to the borderline of being overly conservative. For the ES model, the AZ statistic for the 95% level is -0.06795 with a *p-value* 0.945824, and the AZ for the 99% level is -0.09471 with a *p-value* of 0.924547, suggesting that the difference between observed and expected outcomes is entirely within normal, random statistical bounds. As a result, the backtesting results illustrate that the VaR and ES models are statistically accurate and reliable on a daily horizon at the 99% and 95% confidence levels.

Similar results were observed for the 1,000-day rolling window. The Kupiec test again fails to reject the null hypothesis for both the 95% VaR and 99% VaR, confirming that the forecasted violation rates remain consistent with the expected values. For the AZ tests, both the 95% and 99% confidence levels providing strong evidence that the ES forecasts are well calibrated.

Comparing the two window lengths, the 1,000-day rolling window exhibits slightly smaller AZ test statistics together with larger *p-values*, indicating marginally better calibration for the 95% and 99% ES forecasts. For the VaR forecasts, expanding the window size improved the *p-value*

for the 95% VaR from 0.06067 (750-day) to 0.09138 (1,000-day). Overall, both rolling window lengths produce statistically adequate risk forecasts, although the 1,000-day window provides slightly superior out-of-sample performance due to its greater parameter stability while maintaining accurate coverage of extreme losses.

When compared to the equal-weight portfolio, overall, the optimal-weight portfolio demonstrates superior and more consistent risk forecasting performance, particularly in terms of ES calibration and the statistical stability of VaR forecasts. For the 750-day rolling window, the equal-weight portfolio fails the Kupiec test at the 95% confidence level ( $LR=4.26783$ ,  $p\text{-value} = 0.03884$ ), implying that the fixed allocation produces an inaccurate estimate of downside risk. Under the 1,000-day rolling window, the optimal-weight portfolio produces smaller Kupiec test statistics and substantially lower AZ statistics than the equal-weight portfolio. This indicates that the dynamically optimized portfolio provides a more accurate representation of extreme downside risk. Overall, the empirical findings suggest that dynamically adjusting portfolio weights according to the evolving conditional covariance structure enables more effective risk management than maintaining a fixed, equal-weight allocation, particularly during periods of changing market volatility and dependence.

## **4. Conclusion**

This study provides a risk management framework designed to enhance portfolio data analysis amid heightened global uncertainty driven by pandemics, systemic financial crises, and escalating geopolitical events. Our empirical analysis reveals that although the BRIC return series remained structurally stable between 2015 and 2020, multiple breakpoints are found in the volatility series of the BRIC stock markets, especially during a global pandemic event such as COVID-19. All BRIC markets demonstrated multiple volatility regime shifts; however, the inclusion of breakpoint dummy variables in the rolling estimation of the GARCH was not statistically significant. This implies that the rolling GARCH framework is sufficient to capture regime changes and volatility adjustments over time. Under the standard ADCC-GARCH estimation, strong volatility persistence, heavy-tails, and stable long-run correlations were found among the BRIC markets. The portfolio allocation indicates that China and India became the key drivers after 2022, especially during the Russia-Ukraine conflict. The dynamic VaR and ES forecasts captured periods of heightened downside risk, followed by market stabilization after 2022. The backtest results ensure that both the VaR and ES models are statistically reliable and slightly conservative, showing that the ADCC-GARCH framework is capable of quantifying tail risk and enhancing portfolio risk management in the BRIC stock markets.

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