E C O N O M I C S B U L L E T I N

Prior Restrictions on Bargaining Contract Curves

Jan K. Brueckner
University of Illinois at Urbana−Champaign

Abstract

It is well−known that the efficient−bargain model imposes no general restrictions on the slope of the contract curve. As a result, both upward− and downward−sloping curves are consistent with the theory. Less is known, however, about the effect on the contract curve of changes in the demand and supply variables that underlie employer and union indifference maps and help determine curve's position. To aid empirical researchers, this paper analyzes the effects of demand and supply variables on the position of the contract curve and states the minimal prior restrictions that can be placed on these effects.

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1. Introduction

McDonald and Solow's (1981) efficient-bargain model has attracted considerable attention in the collective bargaining literature. Under this model, bargaining outcomes lie along a contract curve comprised of tangencies between union and employer indifference curves in wage-employment space. The efficient-bargain model has become the principal alternative to the standard monopoly-union model of the bargaining process, under which the union picks its preferred point on the employer's demand curve for labor.[1]

It is well-known that the efficient-bargain model imposes no general restrictions on the slope of the contract curve. As a result, both upward- and downward-sloping curves are consistent with the theory. Less is known, however, about the effect on the contract curve of changes in the demand and supply variables that underlie employer and union indifference maps and help determine the curve's position. Examples of such variables are the alternative wage (a supply variable) and community population in a public sector model (a demand variable). To aid empirical researchers, the present paper analyzes the effects of demand and supply variables on the position of the contract curve and states the minimal prior restrictions that can be placed on these effects. Brueckner and O'Brien (1989) were able to reject the efficient-bargain model for several public employee samples by appealing to these restrictions.

2. Analysis

To begin the analysis, it is assumed that union preferences over the wage w and employment L are represented by the utility function $V(w, L, \beta)$, where β is a supply variable. This function is strictly quasi-concave in its first two arguments, and an increase in β is assumed to flatten indifference curves in $L-w$ space (w is on the vertical axis). The marginal rate of substitution V_L/V_w is thus a decreasing function of β. An example of such a utility function is given by McDonald and Solow (1981). Letting $z(w)$ (a strictly concave function) give the utility of working at wage w, expected utility for a union member is $z(w)L/N + z(w_a)(1 - L/N)$, where w_a is the wage in alternative (non-union) employment and L/N is the probability of union employment (N is union membership). The alternative wage plays the role of β under this formulation, and it is easily seen that the marginal rate of substitution between w and L is decreasing in w_a.

It is convenient to invert the equation $V(w, L, \beta) = v$ defining an indifference curve so that it reads $w = h(L, \beta, v)$ (v is some constant utility level). The function h satisfies $h_L = -V_L/V_w < 0, h_{LL} > 0$ (by quasi-concavity), and $h_v = 1/V_w > 0$ (the effect of β is discussed below).

Without specifying the details of the institutional setting, the employer's objective function is written $U(w, L, \alpha)$, where α is a demand variable. As an example, suppose that the employer's output is produced with labor alone according to the strictly concave function $f(L)$ and that utility is measured by profit. Then U becomes $pf(L) - wL$, where p (which plays the role of the demand variable) is output price.

[1]Contributions to the literature on efficient bargains include Brown and Ashenfelter (1986), Eberts and Stone (1986), and MaCurdy and Pencavel (1986).

The labor demand curve used in the monopoly-union bargaining model is found by choosing L to maximize U for fixed w, solving $U_L(w, L, \alpha) = 0$. Under the assumption $U_{LL} < 0$, the second-order condition for this problem is satisfied, and it follows that U_L is positive to the left and negative to the right of the labor demand curve. The employer's indifference curves are defined by $U(w, L, \alpha) = u$ for some constant u, and the slope of an indifference curve equals $-U_L/U_w$. Given the behavior of U_L and the fact that U_w is negative, indifference curves are upward-sloping to the left and downward-sloping to the right of the labor demand curve. An increase in the demand variable is assumed to increase the indifference curve slope ($-U_L/U_w$ is an increasing function of α), so that a larger α makes the indifference curves steeper to the left and flatter to the right of the demand curve. The higher α also moves the demand curve to the right. As in the case of the union indifference curves, it is convenient to invert the equation $U(w, L, \alpha) = u$ so that the employer's indifference curves can be written $w = g(L, \alpha, u)$, where $g_L = -U_L/U_w$ and $g_u = 1/U_w < 0$ (the effect of α will be discussed below).

Before discussing the contract curve, further analysis of the functions h and g is needed. First, it is necessary to express the assumption that V_L/V_w is decreasing in β in terms of the function h. The assumption means that the absolute slope expression $-h_L$ is decreasing in β provided that the utility level v adjusts to hold w constant as β changes. The adjustment in v restricts the focus to an indifference curve passing through a particular (L, w) point, which by assumption becomes flatter. Thus, it must be the case that $-(h_{L\beta} + h_{Lv}[dv/d\beta]) < 0$, where $dv/d\beta$ is the utility change required to keep w constant as β increases. By differentiation of $w = h(L, \beta, v)$, $dv/d\beta$ equals $-h_\beta/h_v$, so that the required condition is

$$h_{L\beta} - h_{Lv}h_\beta/h_v > 0. \tag{1}$$

By exactly analogous reasoning, g must satisfy

$$g_{L\alpha} - g_{Lu}g_\alpha/g_u > 0 \tag{2}$$

for U_L/U_w to be decreasing in α.

Normality of w (alternatively L) in union preferences requires that the union indifference curves become steeper (flatter) moving vertically (horizontally) in the (L, w) plane. Because employer utility rises moving toward the origin, the opposite behavior is required of the employer indifference curves, with normality of w (alternatively L) requiring the curves to become flatter (steeper) moving vertically (horizontally). By calculations similar to those above, these requirements yield, respectively, the following normality conditions:[2]

$$h_{Lv} < 0 \tag{3}$$

$$h_{LL} - h_{Lv}h_L/h_v > 0 \tag{4}$$

$$g_{Lu} < 0 \tag{5}$$

$$g_{LL} - g_{Lu}g_L/g_u < 0. \tag{6}$$

[2]It can be shown that in the examples considered above, the union indifference map exhibits normality for both w and L, while L is inferior in the employer indifference map (a horizontal movement, however, has an ambiguous effect on the slope of the employer's curves).

With the appropriate background in place, the analysis of the contract curve can now begin. The contract curve is defined by following equation system:

$$w = g(L, \alpha, u) \tag{7}$$

$$w = h(L, \beta, v) \tag{8}$$

$$g_L(L, \alpha, u) = h_L(L, \beta, v) \tag{9}$$

Equations (7) and (8) indicate that employer and union indifference curves intersect, and (9) says that the intersection involves a tangency. For this tangency to represent a Pareto-efficient bargaining outcome, it must be the case that $g_{LL} < h_{LL}$, so that the employer's indifference curve is less convex than the union's.[3]

The equation of the contract curve is found by treating L in the system (7) – (9) as fixed and solving for the remaining variables. The resulting solution is written[4]

$$w = w(L, \alpha, \beta), \tag{10}$$

and the goal of the analysis is to sign the partial derivatives this function. Totally differentiating the system (7) – (9), the slope of the contract curve is given by

$$\begin{aligned} \partial w/\partial L &= g_L + \frac{g_{LL} - h_{LL}}{h_{Lv}/h_v - g_{Lu}/g_u} \\ &= \frac{[g_{LL} - g_{Lu}g_L/g_u] - [h_{LL} - h_{Lv}h_L/h_v]}{h_{Lv}/h_v - g_{Lu}/g_u}. \end{aligned} \tag{11}$$

The sign of (11) depends on whether L and w are normal goods in employer and union preferences. If w is normal in both cases, then the numerator of (11) is negative (recall (4) and (6)). If L is normal in both cases, then the denominator is also negative given that $h_v > 0$ and $g_u < 0$ (recall (3) and (5)). Normality of both goods therefore implies that (11) is positive and that the contract curve is upward sloping. If, on the other hand, w is inferior and L is normal for both the employer and the union, then (4) and (6) are reversed. The numerator of (11) then becomes positive, and the contract curve is downward sloping. Similarly, if w is normal but L is inferior, then (3) and (5) are reversed, the denominator of (11) becomes positive, and the contract curve is again downward sloping. Finally, if L (alternatively w) is neither normal nor inferior for both the employer and union, implying that equality holds in (3) and (5) (in (4) and (6)), then the contract curve is vertical (horizontal). In all admissible circumstances other than those just listed, the sign of (11) is indeterminate and the contract curve could slope up or down.[5]

Even though an explicit proof has not appeared in the literature, the above results are generally known. The effects of the demand and supply variables α and β on the position of the contract curve are, however, less well understood. The following analysis develops the

[3] Satisfaction of this condition is not guaranteed and must be assumed.

[4] Alternatively, the equation could give L in terms of w. The choice of dependent variable is immaterial.

[5] When the utility functions take the forms considered in the above examples, the contract curve turns out to be upward sloping. This is shown explicitly by McDonald and Solow (1981).

minimal prior restrictions that can be placed on these effects. Total differentiation of (7) – (9) shows that

$$\partial w/\partial\alpha = \frac{g_{L\alpha} - g_{Lu}g_\alpha/g_u}{h_{Lv}/h_v - g_{Lu}/g_u} \tag{12}$$

$$\partial w/\partial\beta = -\frac{h_{L\beta} - h_{Lv}h_\beta/h_v}{h_{Lv}/h_v - g_{Lu}/g_u} \tag{13}$$

Given that the numerators of (12) and (13) are positive by (1) and (2), the signs of $\partial w/\partial\alpha$ and $\partial w/\partial\beta$ depend on the sign of the common denominator. Although the latter expression has a determinate sign when L is normal or inferior in both indifference maps, the goal is to make a statement that is independent of preferences.[6] The following result can be established:

> **Proposition**. *When the contract curve is flat or upward sloping, an increase in the demand variable shifts the curve down ($\partial w/\partial\alpha < 0$) and an increase in the supply variable shifts it up ($\partial w/\partial\beta > 0$). When the contract curve is downward sloping, demand and supply shifts may have either sign but must be in opposite directions.*

To prove this result, note first that since $\partial w/\partial\alpha$ has the sign of $h_{Lv}/h_v - g_{Lu}/g_u$ while $\partial w/\partial\beta$ has minus this sign, demand and supply shifts are always in opposite directions regardless of the slope of the contract curve. The shifts can be signed, however, in the flat and upward-sloping cases by noting that (12) and the first line of (11) together imply

$$\partial w/\partial L - g_L = \frac{g_{LL} - h_{LL}}{g_{L\alpha} - g_{Lu}g_\alpha/g_u}\partial w/\partial\alpha. \tag{14}$$

Since the denominator of (14) is positive and $g_{LL} - h_{LL} < 0$ must hold for the tangency point to be optimal, it follows that

$$\text{sign}[\partial w/\partial\alpha] = -\text{sign}[\partial w/\partial L - g_L]. \tag{15}$$

To interpret (15), note that the right-hand side depends on the difference between the slope of the contract curve and the (negative) slope of the indifference curves at the tangency point.[7] While this difference can have either sign when the contract curve is downward sloping, the difference must be positive when the curve is flat or upward sloping. Therefore, in the flat and upward-sloping cases, (15) implies that $\partial w/\partial\alpha$ is negative and hence that $\partial w/\partial\beta$ is positive, as claimed. When the contract curve is downward sloping, this result is preserved provided that the curve is flatter than the indifference curves, in which case $\partial w/\partial L > g_L$. The reverse impacts occur when the contract curve is steeper than the indifference curves (in this case $\partial w/\partial\alpha$ and $\partial w/\partial\beta$ are respectively positive and negative). However, since these

[6] Using a specialized model, Gyourko and Tracy (1988) proved that the contract curve shifts down in response to a demand increase when L is normal in both the union and employer indifference maps. This result, which can be seen in (12), is never used in their paper since the analysis is devoted to estimating reduced-form wage equations.

[7] Recall the $g_L = h_L$ at the tangency.

slope relationships are unobservable, all that can be said in the downward-sloping case is that demand and supply shifts must be in opposite directions.[8]

The proposition is illustrated in Figures 1 and 2, which show the effect of an increase in the demand variable (recall that this flattens the employer's indifference curves). Figure 1 illustrates the downward shift of an upward-sloping contract curve, case A in Figure 2 shows the same outcome in the case of relatively flat downward-sloping curve, and case B shows the upward shift of a steep downward-sloping curve. Note that cases A and B are based on different union indifference maps.[9]

Using the prior restrictions from the proposition, Brueckner and O'Brien (1989) rejected the efficient bargain model for three national cross-section samples of fire, police, and sanitation workers. The estimated contract curve for each sample was downward sloping, so that only the weak opposite-sign restriction applies. However, in each case, the coefficients on community population (the most important demand variable) and the manufacturing wage (a key supply variable) were both positive and significant, in violation of the opposite-sign restriction.

3. Conclusion

This paper has analyzed the effects of supply and demand variables on the position of the contract curve and developed prior restrictions on the directions of these effects. Estimated contract curves must conform to these restrictions to be consistent with the underlying model. It is hoped that the restrictions will prove useful to researchers engaged in empirical implementation of the efficient-bargain model.

4. References

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[8] Note that an implication of this discussion is that the slope of the contract curve is numerically larger (smaller) than the slope of indifference curves as $h_{Lv}/h_v - g_{Lu}/g_u$ is negative (positive). Since an upward-sloping contract curve thus implies that $h_{Lv}/h_v - g_{Lu}/g_u$ is negative, the case where the curve is upward sloping as a result of both the numerator and denominator of (11) being positive is ruled out. This case can also be ruled out from first principles since positivity of both terms in (11) can be shown to imply $g_{LL} > h_{LL}$, in violation of the condition for the optimality of the tangency point.

[9] Figure 1 applies when the utility functions have the forms in the above examples. Explicit calculations in McDonald and Solow (1981) confirm that an increase in $w_a(p)$ shifts the contract curve up (down).

McDonald, I.M. and R.M. Solow (1981) "Wage Bargaining and Employment" *American Economic Review* **71,** 896-908.

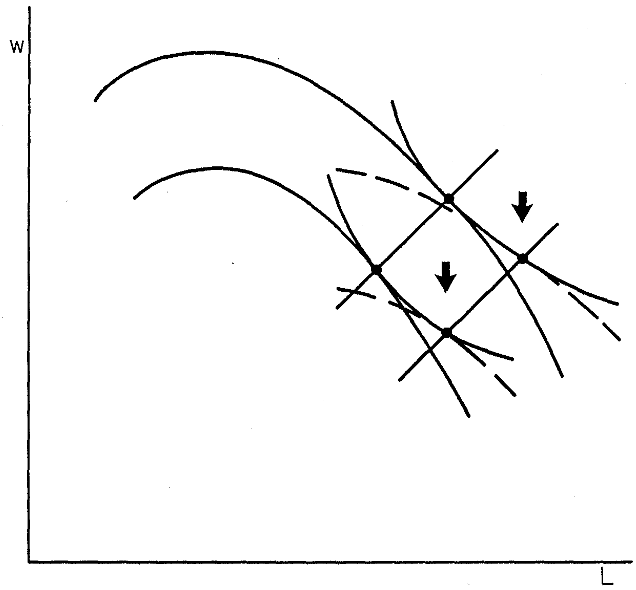

Fig. 1

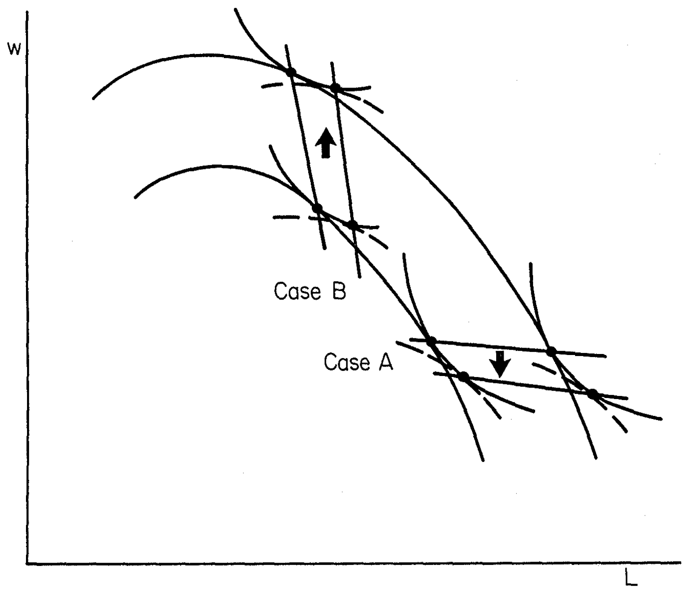

Fig. 2