E C O N O M I C S B U L L E T I N

On the strategic choice of spatial price policy: the role of the pricing game rules.

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Abstract

The strategic choice of spatial price policy under duopoly crucially depends on the rules of price competition. We show that under simultaneous price competition and under leader−follower price competition (with the discriminatory firm being the leader), the pricing policy game is not, as stated by Thisse and Vives (1988), a Prisoner's Dilemma.

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1. Introduction

Thisse and Vives (1988) (TV) analyze the strategic choice of spatial price policy in a duopoly market with homogeneous product and inelastic demand. They assume that if both firms choose the same price policy, firms compete in prices simultaneously. But given that if firms choose different price policies there is no pure strategy equilibrium in the pricing game under simultaneous competition, they assume that the mill-pricing firm becomes a price leader and the discriminatory firm is a follower. [1] As price discrimination is a dominant strategy but firms would increase profits under f.o.b. pricing the problem is like a Prisoner's Dilemma.

In this paper, we extend the analysis of TV by considering other types of competition when firms choose different price policies: leader-follower price competition (with the discriminatory firm being the leader) and simultaneous price competition (allowing mixed strategies). We conclude that the tendency for firms to price discriminate found by TV does not in general hold and that equilibrium price policies crucially depend on the rules of price competition. In the leader-follower game equilibrium price policies depend on the consumer's reservation value and in the most relevant case there are two asymmetric equilibria in which one firm price discriminates and the other firm prices uniformly. Under simultaneous price competition, we find a mixed strategies equilibrium when firms choose different price policies and show that the price policy game has two equilibria: both firms choose price discrimination or f.o.b. pricing.

2. The model

Two firms, 1 and 2, produce a homogeneous good in a spatial market $[0,1]$ and are located at the left and at the right endpoints of the market respectively. The (constant) marginal cost is identical for both firms and normalized to zero. Consumers are distributed uniformly along the interval $[0,1]$. Each consumer has a reservation value, R, for the good, and buys one unit from the firm with the lowest delivered price. When both firms have the same delivered price at a location the consumer chooses the supplier with the lowest transportation cost.[2] The transportation cost is $t(d) = td$, where d is the distance from the location of the consumer to the producer. We will assume that $R > t$.[3] The delivered price at a location x must cover the transport cost.[4] The timing of the game is as follows. Stage 1: Firms choose the price policy simultaneously. Stage 2: Firms decide the price level simultaneously if both firms choose the same policy. When firms choose different price policies, we consider two kinds of competition: simultaneous and leader-follower competition where the discriminatory firm is the leader.

3. The choice of price policy

We solve the game by backward induction to obtain the subgame perfect equilibria. In the *second stage*, there are several cases depending on the outcome of the previous stage:

3.1. *Both firms price according to f.o.b.*

Firms will select mill prices simultaneously. The demand for each firm is given by:

[1] Other works that also consider this assumption are De Fraja and Norman (1993) and Eber (1997).

[2] See Lederer and Hurter (1986) for a justification of this assumption.

[3] This assumption guarantees that the whole market will be served regardless of the firms' pricing policies.

[4] See Lederer and Hurter (1986) and Thisse and Vives (1988).

$$D_i(p_i, p_j) = \begin{cases} 1 & \text{if } p_i \le p_j - t \\ 1/2 + (p_j - p_i)/2t & \text{if } p_j + t > p_i > p_j - t \\ 0 & \text{if } p_i \ge p_j + t \end{cases} \quad i, j = 1,\ 2, j \ne i.$$

The profit functions are $\Pi_i\ (p_i, p_j\ ) = p_i D_i\ (p_i, p_j)$, $i\ , j\ = 1,\ 2,\ j \neq\ i$. These profit functions are quasi-concave, ensuring the existence of a price equilibrium. The equilibrium mill prices are given by $p_1{}^U = p_2{}^U = t$. The equilibrium profits are $\Pi_1{}^{UU} = \Pi_2{}^{UU} = t/2$.

3.2. *Both firms use delivered pricing*

Denote as $p_1(x)$ and $p_2(x)$ the delivered prices of firm 1 and firm 2 at a location x, $0 \le x \le 1$. At a given location x, competition is *à la* Bertrand: with cost asymmetries if $x \neq 1/2$ and with the same cost if $x = 1/2$. Thus, in equilibrium the delivered price at x will equal the transportation cost of the firm located further from x: $p_1\ (x) = p_2(x) = \max\{tx,\ t(1 - x)\}$ for all $x \in [0,1]$. [5] Firms' profits are $\Pi_1^{DD} = \int_0^{1/2}\{ t(1-x) - tx\}dx = t/4$ and $\Pi_2^{DD} = \int_{1/2}^{1}\{ tx - t(1-x)\}dx = t/4$.

3.3. *One firm is committed to f.o.b. and the other uses delivered pricing*

As noticed by TV there is no equilibrium in pure strategies when firms choose different pricing policy, under simultaneous price competition. They assume that the mill-pricing firm becomes a price leader and the discriminatory firm is a follower. They conclude that price discrimination is a dominant strategy and that the pricing policy game is like a Prisoner's Dilemma. We next show that their result crucially depends on the rules of price competition.

4. The discriminatory firm is the price-leader

Note that the fob pricing firm's optimal response to the delivered price policy $p_2(x)$ is a mill price $p_1{}^*$ such that $p_1^* = \arg\max_{p_1} p_1 \int_Z dz$ where $Z = \{x \in [0,1] : p_1 + tx < p_2(x)\}$ is the firm 1's market area. The following Lemma characterizes the equilibrium prices.

Lemma 1. The backward induction solution is given by a pricing policy : $p_2{}^*(x) = [\overline{\Pi}^*(R)/x] + tx$, for the discriminatory firm and a mill price $p_1{}^* = p_H(\overline{\Pi}^*(R)) = [R + \sqrt{R^2 - 4t\overline{\Pi}^*(R)}]/2$, for the fob-pricing firm, where $\overline{\Pi}^*$ is the firm 1's profit.

Figure 1 illustrates the equilibrium pricing policies. The discriminatory firm's policy maintains firm 1 indifferent between prices $p_1 \in [\ p_L,\ p_H]$ with a profit of $\overline{\Pi}$ for firm 1.[6] [7] Given firm 2's pricing policy, $p_2(x)$, and firm 1's mill price, p_1, the market boundary $\tilde{x}(p_1)$ is determined by $p_1 + t\tilde{x}(p_1) = p_2(\tilde{x}(p_1))$, which yields $\tilde{x}(p_1) = [\ p_2(\tilde{x}(p_1)) - p_1]/t$. To maintain firm 1 indifferent between

[5] See Lederer and Hurter (1986) for a formal proof.

[6] See Prescott and Visscher (1977) for a similar idea in a model of sequential choice of location.

[7] The price p_L is the highest price that allows firm 1 to capture all the market and to obtain a profit $\overline{\Pi}$ (note that $p_L = \overline{\Pi}$) and p_H is the highest price that allows firm 1 to obtain $\overline{\Pi}$ given the rival's price policy and R.

$p_1 \in [p_L, p_H)$ it must be satisfied that $\Pi_1(p_1) = \overline{\Pi}$. Therefore the optimal pricing policy for the discriminatory firm is $p_2(x) = (\overline{\Pi}/x) + tx$. Note that given the price policy, $p_2(x) = (\overline{\Pi}/x) + tx$, the mill-pricing firm is indifferent between prices $[p_L, p_H)$, and the discriminatory firm maximizes profits when the mill-pricing firm charges the highest price p_H.[8] The price p_H, which depends on the consumer reservation value, is given by $p_H(\overline{\Pi}) = [R + \sqrt{R^2 - 4t\overline{\Pi}}]/2$ and firm 1's market share is $x_H(\overline{\Pi}) = [R - \sqrt{R^2 - 4t\overline{\Pi}}]/2t$.

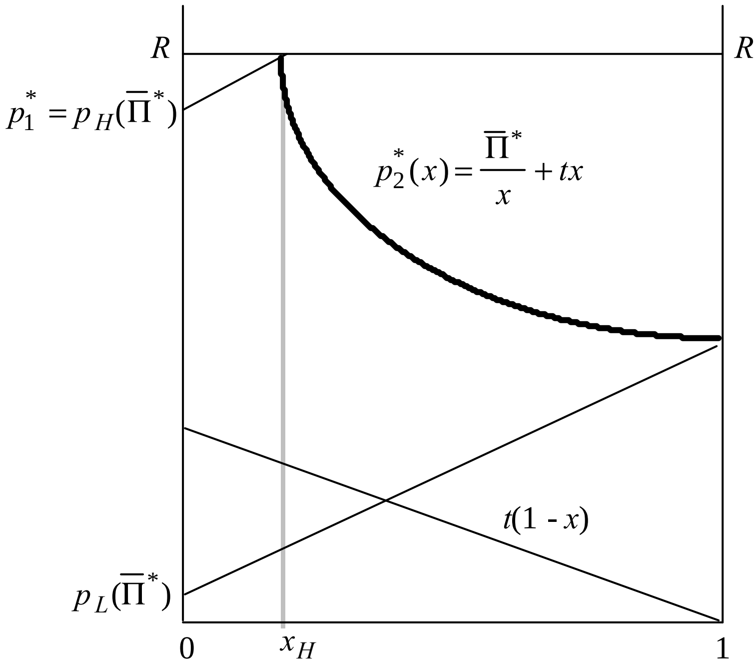

Figure 1. Equilibrium pricing policies when the mill-pricing firm is the follower.

Thus, the discriminatory firm's profit, maintaining firm 1 with a profit $\overline{\Pi}$, is:

$$\Pi_2(\overline{\Pi}) = \int_{[R - \sqrt{R^2 - 4t\overline{\Pi}}]/2t}^{1} \{(\overline{\Pi}/x) + tx - t(1-x)\}dx = -\overline{\Pi}\ln x_H(\overline{\Pi}) + tx_H(\overline{\Pi})[1 - x_H(\overline{\Pi})]$$

The first order condition of the profit maximization problem is given by:

$$\Pi_2'(\overline{\Pi}) = -\ln x_H(\overline{\Pi}) - \overline{\Pi}[x_H'(\overline{\Pi})/x_H(\overline{\Pi})] + tx_H'(\overline{\Pi})[1 - 2x_H(\overline{\Pi})] = 0 \quad (1)$$

If $\Gamma = R - \sqrt{R^2 - 4t\overline{\Pi}^*}$, condition (1) can be rewritten as:

$$-\ln[\Gamma/2t] - [2t\overline{\Pi}^* /(\Gamma \cdot \sqrt{R^2 - 4t\overline{\Pi}^*})] + [t/\sqrt{R^2 - 4t\overline{\Pi}^*}][(t - \Gamma)/t] = 0 \quad (2)$$

[8] The optimal price policy for the discriminatory firm holds the mill pricing firm indifferent between prices $[p_L, p_H]$. In order for firm 1 to choose p_H its profits at prices $[p_L, p_H)$ must be ε-below $\overline{\Pi}$. We have to change slightly the discriminatory pricing policy: $p_2(x) = \{(\overline{\Pi}/x) + tx - \delta$ for $x_H < x \le 1$; R for $x \le x_H\}$, with $\delta > 0$, $\delta \to 0$.

Therefore, the backward induction solution is given by $p_2^*(x) = [\overline{\Pi}^*(R)/x] + tx$ and $p_1^* = p_H(\overline{\Pi}^*(R)) = [R + \sqrt{R^2 - 4t\overline{\Pi}^*(R)}]/2$, and the equilibrium profits are $\Pi_1^* = \Pi_1^{UD} = \overline{\Pi}^*(R)$ and $\Pi_2^* = \Pi_2^{UD} = \int_{[R - \sqrt{R^2 - 4t\overline{\Pi}^*(R)}]/2t}^{1} \{[\overline{\Pi}^*(R)/x] + tx - t(1-x)\}dx$. Given that it is not possible to obtain an explicit expression for $\overline{\Pi}^*$ from condition (2), we consider a numerical approximation. The equations of linear regression are $\overline{\Pi}^* = -0.15064t + 0.36548R$ (r^2= 0.9998, where r^2 is the determination coefficient) and $\Pi_2^* = 0.098288t + 0.36777R$ ($r^2 \cong 1$). When firm 2 is the mill pricing and firm 1 the discriminatory firm we obtain the symmetric results, $\Pi_1^{UD} = \Pi_2^{DU}$ and $\Pi_1^{DU} = \Pi_2^{UD}$. Table 1 summarizes the possible outcomes of the second stage. The following proposition states the main result of this subsection.

		Firm 2 U	Firm 2 D
Firm 1	U	$\frac{t}{2}, \frac{t}{2}$	Π_1^{UD}, Π_2^{UD}
Firm 1	D	Π_1^{DU}, Π_2^{DU}	$\frac{t}{4}, \frac{t}{4}$

Table 1. Summary of firms' profits.

Proposition 1. *If the discriminatory firm is the leader when firms choose different pricing policies then (i) if $t < R \leq \overline{R}$ the pricing policy game has two Nash equilibria in pure strategies: either both firms price uniformly or both firms price discriminate. (ii) If $\overline{R} < R < \overline{\overline{R}}$ spatial price discrimination is a dominant strategy and the pricing policy game is a Prisoner's Dilemma. (iii) If $R > \overline{\overline{R}}$, the pricing policy game has two Nash asymmetric equilibria in which one firm prices according to f.o.b. and the other price discriminates. This case is the most relevant given that $\overline{R} = 1.0923t$ and $\overline{\overline{R}} = 1.0962t$.*

Proof. Given Table 1, we have three possibilities:
(i) When $t < R \leq \overline{R}$ the equilibrium profits are such that $\Pi_1^{UU} \geq \Pi_1^{DU} > \Pi_1^{DD} > \Pi_1^{UD}$ and $\Pi_2^{UU} \geq \Pi_2^{UD} > \Pi_2^{DD} > \Pi_2^{DU}$. Therefore, there are two Nash equilibria: (U, U) and (D, D).
(ii) When $\overline{R} < R < \overline{\overline{R}}$ the equilibrium profits are such that $\Pi_1^{DU} > \Pi_1^{UU} > \Pi_1^{DD} > \Pi_1^{UD}$ and $\Pi_2^{UD} > \Pi_2^{UU} > \Pi_2^{DD} > \Pi_2^{DU}$. Therefore, (D, D)* is the dominant strategy Nash equilibrium.
(i) When $R > \overline{\overline{R}}$, the equilibrium profits are such that $\Pi_1^{DU} > \Pi_1^{UU}$, $\Pi_1^{UD} > \Pi_1^{DD}$ and $\Pi_2^{UD} > \Pi_2^{UU}$, $\Pi_2^{DU} > \Pi_2^{DD}$. Thus, there are two Nash equilibria: (U, D) and (D, U). If $R = \overline{\overline{R}}$ then (D, D) would be also a Nash equilibrium. Q.E.D.

5. Simultaneous price competition

The analysis above provides us with an intuition as to the equilibrium outcome when mixed strategies are allowed in the simultaneous pricing game. We have obtained that the equilibrium price policy of the discriminatory firm is such that the mill-pricing firm is indifferent between prices in the interval $[p_L(\bar{\Pi}), p_H(\bar{\Pi})]$. In order to find an equilibrium in the simultaneous game, we would only need to prove that there exists a distribution function for the mill pricing firm with support in an interval $[p_l(\tilde{\Pi}), p_h(\tilde{\Pi})]$ such that the best response of the discriminatory firm to that mixed strategy is $p_2(x, \tilde{\Pi}) = (\tilde{\Pi}/x) + tx$. Lemma 2, 3 and 4 give us some properties that the equilibrium must satisfy.

Lemma 2. In the market area of the mill-pricing firm the full price of the mill-pricing firm, is lower than or equal to the transportation cost from the discriminatory firm.[9]
Proof. If this condition is not satisfied the discriminatory firm might undercut the full price of the mill-pricing firm in order to capture a greater market area. Q.E.D.

Lemma 3. The value of $\tilde{\Pi}$ is $t/8$.
Proof. $\tilde{\Pi}$ cannot be less than $t/8$ since firm 1 can always ensure this profit by charging $p_1 = t/2$ given that the discriminatory firm must cover transportation costs. Thus $\tilde{\Pi} \geq t/8$. On the other hand, Lemma 2 implies that $p_2(x,\tilde{\Pi}) = (\tilde{\Pi}/x) + tx$ cannot be always above $t(1-x)$ (however, the intersection between $p_1 + tx$ and $p_2(x)$ would be over $t(1-x)$ which contradicts Lemma 2). The values of $\tilde{\Pi}$ that satisfy this condition are $\tilde{\Pi} \leq t/8$, which completes the proof. Q.E.D.

Lemma 4. The support of the mixed strategy for the mill-pricing firm is the interval $[t/8, t/2]$.
Proof. The lower extreme of the support must satisfy $p_l \geq t/8$ given that $p_1 = t/8$ is the highest price that allows firm 1 to capture the whole market and obtain a profit of $\tilde{\Pi} = t/8$. Lemma 2 implies that the intersection between $p_1 + tx$ and $p_2(x)$ cannot be over $t(1-x)$. This condition is satisfied if $p_h \leq t/2$. Finally if we do not consider the complete interval the discriminatory firm could change its strategy to obtain more profits. Q.E.D.

Lemma 5. (i) The distribution function for the mill firm $F_1^*(p_1) = 1 - k[e^{-t/(t-2p_1)}]/(t-2p_1)$ where $k = (3/4)te^{(4/3)}$ with support $[t/8, t/2]$ and
(ii) the pricing policy $p_2^*(x) = \begin{cases} t(1-x) & \text{for } x \in [0,1/4) \\ (t/8x) + tx & \text{for } x \in [1/4,1] \end{cases}$ for the discriminatory firm.
constitute a mixed Nash equilibrium with an associated profit $\tilde{\Pi} = t/8$ for the mill pricing firm.
Proof. See Appendix.

As noticed by Gabszewicz and Thisse (1992) "price discrimination operates, in some sense, with respect to mill pricing as mixed strategies operate with respect to pure strategies by enlarging the space of strategies". Therefore, it results natural that in equilibrium the mill-pricing firm follows a mixed strategy while the discriminatory firm uses a pure strategy.

The following proposition states the main result of this section.

[9] That is, $p_h(\tilde{\Pi}) + tx \leq t(1-x)$ for $x \in [0, x_h]$, where x_h denotes the marginal consumer at p_h.

Proposition 2. *Under simultaneous price competition the price policy game has two Nash equilibria in pure strategies: both firms price uniformly (fob) or both firms price discriminate.*

Proof. Given Lemma 5, the expected profit of the discriminatory firm is given by $\Pi_2^{UD} = \Pi_2^e(t/8) = \int_{t/8}^{t/2}\left\{\int_{t/8p_1}^{1}[(t/8x)+tx-t(1-x)]dx\right\}dF_1^*(p_1)$. Assume that the mill-pricing firm follows a pure strategy $p_1 = t/2$ (and the discriminatory firm the pricing policy $p_2^*(x) = (t/8x) + tx$) then the secure profit for firm 2 would be $\Pi_2 = \int_{1/8}^{1/2}[(t/8x)+tx-t(1-x)]dx = [(3+2\ln 4)/16]t$. Thus, $\Pi_2^{UD} = \Pi_2^e(t/8) < [(3+2\ln 4)/16]t < t/2 = \Pi_2^{UU}$ and from Table 1 we conclude that the pricing policy game has two Nash equilibria in pure strategies: both firms price uniformly or both firms price discriminate. Q.E.D.

6. Concluding remarks

We have shown that the general tendency for firms to price discriminate found by TV crucially depends on the rules of price competition. In particular, spatial price discrimination is a dominant strategy only when the mill-pricing firm is the leader and the discriminatory firm the follower. When the leader-follower roles are reversed, equilibrium price policies depend on the consumer's reservation value and in the most relevant case there are two asymmetric equilibria in which one firm price discriminates and the other firm price uniformly. Under simultaneous price competition in all subgames, we find a mixed strategies equilibrium when firms choose different pricing policies and we demonstrate that the pricing policy game has two perfect Nash equilibria: price discrimination and f.o.b. pricing. Note that the fob-fob equilibrium Pareto dominates the discriminatory equilibrium.

Appendix

Proof of Lemma 5

We shall obtain a distribution function for the mill-pricing firm with support [$t/8,t/2$] such that the best response of the discriminatory firm to that mixed strategy is $p_2^*(x)$. We solve the profit maximization problem of the discriminatory firm at a generic point of the market. The discriminatory firm sells the product to the consumer located at x if its delivered price, $p_2(x)$, is lower than or equal to the full price of the mill pricing firm, $p_1 + tx$. Thus, the probability of this event is $P(p_1 + tx \geq p_2(x)) = P(p_1 \geq p_2(x) - tx) = 1 - F_1(p_2(x) - tx)$, where $F_1(p_2(x) - tx)$ is the distribution function of the mill pricing firm evaluated at $p_2(x) - tx$. So the expected profit of the discriminatory firm at x is $\Pi_2^e(x) = [p_2(x) - t(1-x)][1 - F_1(p_2(x) - tx)]$. The first order condition of the maximization problem is

$$\partial\Pi_2^e(x)/\partial p_2(x) = [1 - F_1(p_2(x) - tx)] - [p_2(x) - t(1-x)]f_1(p_2(x) - tx) = 0 \qquad \text{(A1)}$$

where $f_1(p_2(x) - tx)$ is the density function. (A1) can be rewritten as

$$[1 - F_1(p_2(x) - tx)] = [p_2(x) - t(1-x)]f_1(p_2(x) - tx) \qquad \text{(A2)}$$

We want to obtain the density function $f_1(.)$ such that $p_2^*(x) = (t/8x) + tx$ is a solution for this maximization problem. By substituting this value in (A2) we get

$$[1-F_1(t/8x)]=[(t/8x)+2tx-t]f_1(t/8x) \quad \text{(A3)}$$

(A3) can be expressed as

$$[1-F_1(z)]=\{[z^2+(t^2/4)-tz]/z\}f_1(z) \quad \text{(A4)}$$

where $(t/8x)=z$. Given that $f_1(z)=F_1'(z)$, then (A4) is a variable coefficient first order linear differential equation. It is straightforward to check from the solution of this differential equation that the equilibrium distribution function for the mill firm is given by
$F_1^*(p_1)=1-k[e^{-t/(t-2p_1)}]/(t-2p_1)$ where $k=(3/4)te^{(4/3)}$. Q.E.D.

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