E C O N O M I C S B U L L E T I N

Labor productivity and dynamic efficiency

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Abstract

This note exhibits sufficient conditions concerning the skills of old workers ruling out overaccumulation stationnary equilibria in an OLG model with productive capital. Using a Cobb−Douglas economy, we show that such conditions seem to be largely fullfilled in the industrialized countries.

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1 Introduction

In an overlapping generation model, Diamond (1965) showed that an economy can reach a steady state with capital over-accumulation. Such an economy is said to be dynamically inefficient since a Pareto-improvement can be achieved by allowing the current generation to devour a portion of the capital stock and leaving the consumption of all future generations intact.

This result, further improved by Galor and Ryder (1991), has motivated a large body of research. Indeed, several authors have elaborated on the nature of that inefficiency. Weil (1989) examined the role of finite horizons. Tirole (1985) showed that rational bubbles on assets bringing a real rent appear in dynamically inefficient situations. Thibault (2000) focuses on altruistic preferences, while Abel *et al.* (1989) and Dechert and Yamamoto (1992) extend Diamond's result to the case of uncertainty.

We shed light on a crucial assumption of the standard OLG model: agents do not work during their second period of life. This strong hypothesis has been first noted by Blanchard (1985), when comparing his perpetual youth model to Diamond's one. The present note follows the idea originally contained (but not fully developed) in Blanchard (1985).[1] Indeed, by contrast to Diamond (1965), we consider a very simple OLG model in which agents work during their second period of life. We assume age-specific labor productivity which is related to the age-earnings profile studied in labor economics. Contrary to the Blanchard model, such a framework is tractable and allows to encompass most of the Diamond models.

As the wage income of the second period increases, the intuition suggests that savings decrease. Therefore, the more productive the old workers, the lower the capital stock, and the higher the interest factor. The paper builds on this intuition and presents general technologies and preferences under which it is correct. In particular, we exhibit sufficient conditions ruling out dynamic inefficiency if the labor productivity of old agents is sufficiently strong. Finally, using a Cobb-Douglas economy we show that such conditions seem to be largely fullfilled in the industrialized countries.

2 The model

Consider a perfectly competitive economy which extends over infinite discrete time. Individuals are identical within as well as across generations and live for two periods. In each period t, N_t individuals are born and the population grows at rate $n > -1$. Agents work throughout their life. Young (old) are endowed with one $(1+\gamma)$ unit(s) of labor. Hence, $\gamma \geq -1$ is the age-specific labor productivity component. The

[1] Recently, Decreuse (2001) provides an altruistic interpretation to the Blanchard (1985) perpetual youth model.

period t wage per efficient unit of labor is w_t. Young born at time t earn w_t, consume c_t and save s_t. When old they consume, d_{t+1}, both their labor earnings $(1+\gamma)\,w_{t+1}$ and the proceeds of their savings $R_{t+1}s_t$. Agents thus face the following budget constraints:

$$w_t = c_t + s_t \qquad \text{and} \qquad (1+\gamma)w_{t+1} + R_{t+1}s_t = d_{t+1} \tag{1}$$

An agent born at t maximizes his life cycle utility $U(c_t, d_{t+1})$ subject to (1).

Assumption 1 *$U(c,d)$ is twice continuously differentiable, monotonically increasing, strictly quasi-concave, over the interior of the consumption set $\mathbb{R}^2_+$, and satisfies: $\lim_{x\to 0} U_c(x,d) = \lim_{x\to 0} U_d(c,x) = +\infty$, $\lim_{x\to+\infty} U_d(c,x) = 0$ and $\lim_{x\to+\infty} U_c(c,x) > 0$. Moreover, both periods'consumption goods c and d are normal.*

The output produced at time t using two inputs, capital K_t and efficient labor $L_t = N_t + N_{t-1}\,(1+\gamma)$, is governed by a production function with constant returns to scale $F(K_t, L_t) = L_t f(k_t)$, where $k_t \equiv K_t/L_t$.

Assumption 2 *f is twice continuously differentiable, positive, strictly increasing, strictly concave and satisfies $\lim_{k\to 0} f(k) = 0$, $\lim_{k\to 0} f'(k) = +\infty$, and $\lim_{k\to+\infty} f'(k) = 0$.*

Each factor is paid its marginal product. Assuming that capital fully depreciates after one period, we obtain:

$$w_t = w(k_t) = f(k_t) - k_t f'(k_t) \qquad \text{and} \qquad R_t = R(k_t) = f'(k_t) \tag{2}$$

The capital stock of period $t+1$ is financed by the savings of period t:

$$(1+n)k_{t+1} = s_t \tag{3}$$

3 The results

The saving function s is given by:

$$s_t = \arg\max_s U(w_t - s, R_{t+1}s + (1+\gamma)w_{t+1}) \equiv s(w_t, (1+\gamma)w_{t+1}, R_{t+1}) \tag{4}$$

Under Assumption 1, $s(.,.,.)$ is increasing with respect to the first argument and decreasing with respect to the second argument. Intuitively, the higher the wage income of the second period, the lower the savings. Hence, an increase of the labor productivity of old agents tends to lower savings. If the substitution effect dominates the income effect, s is increasing with respect to the interest factor.

Definition 1 *A steady state is a stationary capital/labor ratio* $\widehat{k}$ *which satisfies:*
$$\widehat{k} = \phi(\gamma, \widehat{k})/(1+n), \text{ where } \phi(\gamma, k) = s(w(k), (1+\gamma)w(k), R(k))$$

In order to evaluate the efficiency properties of stationary equilibria, we extend Galor and Ryder's (1989) existence results to the case where there are old workers.

Proposition 1 *There is a steady state with a positive capital stock if* $k_0 > 0$ *and:*

(i) $s_3(w_1, w_2, R) \geq 0$ *for all* $(w_1, w_2, R) \geq 0$.

(ii) $\lim_{k \to 0} \dfrac{kf''(k)s_1}{f''(k)[s_3 - (1+\gamma)ks_2] - (1+n)} > 1$.

Moreover, the steady state is unique and globally stable if:

(iii) the unique function Λ *such that* $k_{t+1} = \Lambda(k_t)$ *is strictly concave.*

Proof.[2] According to (2), (3) and (4) we obtain $\Pi(k_t, k_{t+1}) = s(w(k_t), (1+\gamma)w(k_{t+1}), R(k_{t+1})) - (1+n)k_{t+1} = 0$. According to (i), $\Pi_2(.,.) < 0$ and using the implicit functions theorem, there exists a single valued function Λ such that $k_{t+1} = \Lambda(k_t)$. We have $\Lambda(k_t) = k_t f''(k_t)s_1/\Pi_2(k_t, k_{t+1}) > 0$. From obvious economics arguments,[3] $\Lambda(0) = 0$. Condition (ii) guarantees that the function Λ is steeper than the 45° line at the origin, and since $\lim_{k \to +\infty} f'(k) = 0$ implies that $\lim_{k \to +\infty} \Lambda(k)/k = 0$, there exists at least a $\tilde{k} > 0$ such that $\Lambda(\tilde{k}) = \tilde{k}$. Hence $\tilde{k}$ is a steady state. Moreover, the strict concavity of Λ implies the uniqueness and global stability of $\tilde{k}$. ■

Consider the resource constraint of the whole economy: $x_t = f(k_t) - (1+n)k_{t+1}$ where $x_t = (N_t c_t + N_{t-1}d_t)/(N_t + N_{t-1}(1+\gamma))$ is the period t consumption per efficient unit of labor. The stationary consumption flow is maximized whenever $R = 1+n$, i.e. at the golden rule. As usual the economy can be Pareto-improved when R is greater than $1+n$ by simply making the young reduce their savings.

Definition 2 *A steady state* $\widehat{k}$ *is dynamically efficient if and only if* $R(\widehat{k}) \geq 1+n$.

We first extend the theoretical result provided by Galor and Ryder (1991) (Corollary 1) in the case of $\gamma = -1$. Indeed, if the golden rule level of capital, $k^* = f'^{-1}(1+n)$, is greater than the upper bound of the attainable per-capita capital, $\breve{k}$ (i.e. $f(\breve{k}) = (1+n)\breve{k}$), steady state equilibria are dynamically efficient[4]. We can refine this result. Let k^D be the highest Diamond equilibrium (obviously $k^D < \breve{k}$). Then, steady state equilibria[5] are dynamically efficient if $k^D \leq k^*$.

[2] It is a simple extension of the results provided by Galor and Ryder (1989) in case of $\gamma = -1$.

[3] Homogenous preferences prevent agents from getting into debt. Hence $0 \leq s_t \leq w_t \leq f(k_t)$. If $k_t = 0$, since $0 \leq (1+n)k_{t+1} \leq f(k_t)$ and $\lim_{k \to 0} f(k) = 0$ we have $k_{t+1} = 0$ i.e. $\Lambda(0) = 0$.

[4] Under Assumption 2 there exists an upper bound to attainable capital, $\breve{k}$. Since $s_t < w_t < f(k_t)$ we have $\phi(\gamma, k) < f(k)$. Hence all the steady states lie in $[0, \breve{k})$.

[5] As $\phi_\gamma(\gamma, k) < 0$, if the model has a steady state, the Diamond model has a steady state.

We now study the impact of the age-earnings profile on the dynamic efficiency of steady states. This analyze, of course, requires a classical framework in which the steady state is a single-valued function of γ for all $\gamma \in [-1, +\infty)$.

Proposition 2 *Assume (i), (ii), (iii) of proposition 1 are satisfied for all* $\gamma \geq -1$. *Then: (a) the economy experiences a unique steady state* $\widehat{k}$.
(b) there exists a unique $\widehat{\gamma} \geq -1$ *such that* $\hat{k}$ *is dynamically efficient if and only if* $\gamma \geq \hat{\gamma}$.

Proof. Assertion (a) follows directly from proposition 1.
(b) Let k be given. According to (1) and Assumption 1, for all $\gamma \geq -1$, $\phi(\gamma, k) \in (-\frac{(1+\gamma)w(k)}{R(k)}, w(k))$. Hence, $\phi(-1, k) \in\ (0, w(k))$. Moreover, $\phi_\gamma(\gamma, k) = w(k)s_2 < 0$. Let $\psi(s, \gamma) = -U_c(w(k) - s, R(k)s + (1+\gamma)w(k)) + R(k)U_d(w(k) - s, R(k)s + (1+\gamma)w(k))$. For all γ, $\phi(\gamma, k)$ solves $\psi(\phi(\gamma, k), \gamma) = 0$. Assume that $\phi(\gamma, k) \geq 0$ for any γ. Then, from Assumption 1, $\lim_{\gamma \to +\infty} \psi(\phi(\gamma, k), \gamma) < 0$, which is impossible. Therefore, since $\phi_\gamma(\gamma, k) < 0$ we have $\lim_{\gamma \to +\infty} \phi(\gamma, k) < 0$. As, for any k, $\lim_{\gamma \to +\infty} \phi(\gamma, k) < 0$ we have $\lim_{\gamma \to +\infty} \hat{k} \leq 0$. Since $\hat{k} > 0$, $\lim_{\gamma \to +\infty} \hat{k} = 0$. Hence $\lim_{\gamma \to +\infty} f'(\hat{k}) = +\infty$. Thus, there exists a unique $\widehat{\gamma} \geq -1$ above which $R(\widehat{k}) \geq 1 + n$. ■

As the productivity of old workers increases, the savings motive dies down, and the stationary interest factor rises. Hence, for any given population growth rate, the steady state is (dynamically) efficient if old workers are sufficiently productive. More precisely, if the Diamond equilibrium k^D is efficient then the equilibrium is always efficient and $\hat{\gamma} = -1$. Conversely, if k^D is inefficient, then $\hat{\gamma} > -1$.

4 A Cobb-Douglas illustration

Consider a Cobb-Douglas economy described by:

$U(c, d) = \beta \ln c + (1-\beta) \ln d,\ f(k) = Ak^\alpha,$ where $A > 0,\ \alpha \in (0,1)$ and $\beta \in (0,1)$

The long-run intensive capital stock and interest factor are given by:

$\hat{k} = \left[\frac{(1-\beta)(1-\alpha)A}{1+n+(1+\gamma)\left(1+\beta\frac{1-\alpha}{\alpha}\right)}\right]^{\frac{1}{1-\alpha}}$ and $R(\hat{k}) = \frac{\alpha}{1-\alpha}\frac{1}{1-\beta}\left[1+n+(1+\gamma)\left(1+\beta\frac{1-\alpha}{\alpha}\right)\right]$

Therefore, the unique steady-state is dynamically efficient if and only if:

$$1 + \gamma \geq \frac{(1-\beta)(1-\alpha) - \alpha}{\alpha + \beta(1-\alpha)}(1+n)$$

In the Diamond case, the condition of dynamic efficiency does not depend on n. Hence, the weight of the first period consumption in the utility function and the elasticity of the production function w.r.t. the capital stock must be sufficiently high. The more general framework needs that γ is sufficiently high.

Consider an economy endowed with the following characteristics: (i) the lifetime horizon at birth is 70 years, divided in two periods of 35 years, (ii) the growth rate of the population is 1% per year, which means 35% per period, (iii) $\alpha = .35$, (iv) $\beta = \dfrac{1}{1 + (0.975)^{35}}$ (implying a 2.5% psychological discount rate on an annual basis). Then the condition of dynamic efficiency is, γ greater or equal to $\hat{\gamma} = -.83$.

This condition is largely fullfilled in the industrialized countries where the time path of wage earnings strictly increases. This conclusion is in line with the empirical evidence established by Abel *et al* (1989) and remains valid if we authorize the agents to be retired during part of their second period of life. Indeed, individuals spend at least their 15 first years at school.

References

Abel, A.B., M.G. Mankiw, L.H. Summers, and R.J. Zeckhauser (1989) “ Assessing dynamic efficiency: theory and evidence ” *Review of Economic Studies* **56**, 1-20.

Blanchard, O.J. (1985) “ Debts, deficits and finite horizons ” *Journal of Political Economy* **93**, 223-247.

Dechert, D., and K. Yamamoto (1992) “ Asset valuation and production in an overlapping-generations model with production shocks ” *Review of Economic Studies* **59**, 389-405.

Decreuse, B. (2001) “ Dying of selfishness ” *Economic Theory* **17**, 481-488.

Diamond, P.A. (1965) “ National debt in a neoclassical growth model ” *American Economic Review* **55**, 1126-1150.

Galor, O., and H.E. Ryder (1989) “ Existence, uniqueness, and stability of equilibrium in an overlapping-generations model with productive capital ” *Journal of Economic Theory* **49**, 360-375.

Galor, O., and H.E. Ryder (1991) “ Dynamic efficiency of steady state equilibria in an overlapping-generations model with productive capital ” *Economics Letters* **35**, 385-390.

Thibault, E. (2000) “ Existence of equilibrium in an OLG model with production and altruistic preferences ” *Economic Theory* **15**, 709-715.

Tirole, J. (1985) “ Asset bubbles and overlapping generations ” *Econometrica* **53**, 1071-1100.

Weil, P. (1989) “ Overlapping families of infinitely-lived agents ” *Journal of Public Economics* **38**, 183-198.