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Taxation and international oligopoly

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Abstract

The combined use of specific and ad valorem taxation as a policy response to the welfare losses caused by international oligopoly is explored. With Nash competition between countries, taxation is inferior to quantity control. In contrast, when countries cooperate production control and taxation lead to identical outcomes. If a single country regulates the oligopoly, taxation can strictly dominate production control.

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1 Introduction

This note analyzes the use of *ad valorem* and specific taxation to control international oligopoly. The outcome with taxation is contrasted to direct government control of production under cooperation between countries, Nash competition and single-government intervention.

It is a common perception that direct production control is the optimal second-best policy. With a single tax instrument this is correct: taxation is inferior to production control both with collusion between governments and when governments act independently. Using the two tax instruments, this remains the case with Nash competition. In contrast, if governments collude, then both production control and taxation achieve the same outcome. Furthermore, when a single country regulates the duopoly, taxation may achieve an outcome *strictly better* than that of direct production control.

2 The Framework

Consider a two-country world in which a traded good is produced by a duopolistic industry. Each country plays host to one of the firms. The preferences of the consumers in each country are represented by the aggregate social welfare function

$$U = U\left(X_j\right) - \ell_j,\ U' > 0,\ U'' < 0,\ j = 1, 2, \tag{1}$$

where X_j is consumption of the duopoly's output in country j and ℓ_j is labor supply. The wage rate in both countries is fixed at unity. With price q_j for the consumption good, the budget constraint for the consumer in country j is $q_j X_j = \ell_j + \pi_j$, where π_j is profit income. Maximizing (1) subject to the budget constraint generates an inverse demand function $q_j = \psi\left(X_j\right)$, with $\psi\left(X_j\right) \equiv U'\left(X_j\right)$. The indirect utility function can be derived as

$$U = U\left(\psi^{-1}\left(q_j\right)\right) - q_j \psi^{-1}\left(q_j\right) + \pi_j \equiv V\left(q_j\right) + \pi_j. \tag{2}$$

The firms produce with costs determined by the cost function

$$C = C\left(x_i^1 + x_i^2\right), C\left(0\right) \geq 0, C' > 0, C'' \geq 0, i = 1, 2, \tag{3}$$

where x_i^j is the supply of firm i to country j. Firm 1 is located in country 1. With specific tax t_j and *ad valorem* tax τ_j, the profit level of firm i is

$$\pi^i = \sum_{j=1}^{2} \left[\left[1 - \tau_j\right] \psi\left(x_1^j + x_2^j\right) - t_j\right] x_i^j - C\left(x_i^1 + x_i^2\right). \tag{4}$$

When firms choose output levels, they act as Cournot-Nash competitors. Lump-sum taxes cannot be used, so each firm must at least break-even.

The following two assumptions are imposed:

Assumption 1. The profit of firm i is a strictly concave function of $\left(x_i^1, x_i^2\right)$ and, when $\tau_j = t_j = 0$, $j = 1, 2$, there exists $\widetilde{x} \equiv \left(\widetilde{x}_1^1, \widetilde{x}_1^2, \widetilde{x}_2^1, \widetilde{x}_2^2\right)$ such that $\pi^i\left(\widetilde{x}\right) > 0$, $i = 1, 2$.

Assumption 2. The inverse demand function satisfies $\lim_{X_j \to \infty} \psi\left(X_j\right) = 0$ and $\lim_{X_j \to 0} \psi\left(X_j\right) \leq K < \infty$.

3 Cooperative Control

With cooperative control, the governments choose their policies to maximize the sum of country welfare levels.

3.1 Quantity Control

Define $\overline{x}$ by $\psi\left(\overline{x}\right) = C'\left(\overline{x}\right)$; $\overline{x}$ is the output consistent with marginal cost pricing.

Proposition 1 *(i) If $\pi^i\left(\overline{x}\right) < 0$, the optimal outcome under government control of production occurs at the highest output level consistent with zero profit for the firms. (ii) If $\pi^i\left(\overline{x}\right) \geq 0$ the optimal outcome has marginal cost pricing.*

Proof. The situation is symmetric for the two countries, so the maximization of joint welfare is equivalent to the maximization of individual welfare. Hence the solution $\left\{\widehat{x}_1^1, \widehat{x}_1^2, \widehat{x}_2^1, \widehat{x}_2^2\right\}$ to the optimization

$$\max \sum_{j=1}^{2} \left[U\left(x_1^j + x_2^j\right) - C\left(x_j^1 + x_j^2\right)\right] \text{ s.t. } \pi^i \geq 0, \; i = 1, 2, \tag{5}$$

where $C\left(x_j^1 + x_j^2\right)$ is labor demand in country j, is the same as the solution $\left\{\widehat{x}_1, \widehat{x}_2\right\}$ to the simpler optimization

$$\max U\left(x_j\right) - C\left(x_j\right) \text{ s.t. } \pi^i \geq 0, \; i = 1, 2, \tag{6}$$

in the sense that $\widehat{x}_j = \widehat{x}_i^1 + \widehat{x}_i^2 = \widehat{x}_1^j + \widehat{x}_2^j$. The Kuhn-Tucker conditions for the optimization (6) are

$$U' - C' + \lambda\left[\psi + \psi' x_j - C'\right] = 0, \tag{7}$$

and

$$\psi x_j - C \geq 0, \lambda \geq 0, \lambda\left[\psi x_j - C\right] = 0. \tag{8}$$

Since $U' \equiv \psi$, there are two potential solutions. The first has $\psi > C'$ and $\psi x_j - C = 0$. In this case, $\psi + \psi' x_j - C' < 0$, so the optimum is given by the greater of the two solutions to $\pi^i\left(x_j\right) = 0$. The second potential solution involves $\psi = C'$ so marginal cost pricing ensues with $\psi x_j - C \geq 0$. ■

Denote the optimal output in the marginal cost pricing outcome, which is also the consumption level in both countries, by x^* and the optimal output level when $\pi^i\left(\overline{x}\right) < 0$ by x^{**}. Note that x^* is first-best and (in the absence of lump-sum transfers) x^{**} is second-best optimal.

3.2 Taxation

Let $q_j \equiv q_j(t_1, t_2, \tau_1, \tau_2)$ be the equilibrium price in country j as a function of the tax rates determined via optimization by the firms. The cooperative tax optimum is the solution to

$$\max_{\{t_1,t_2,\tau_1,\tau_2\}} \sum_{j=1}^{2} V(q_j) + \pi_j, \tag{9}$$

subject to

$$\pi^i = \sum_{j=1}^{2} [[1-\tau_j]\psi - t_j] x_i^j - C \geq 0,\ i = 1,2,\ \ [\tau_j q_j + t_j]\left[x_1^j + x_2^j\right] = 0,\ \ j = 1,2. \tag{10}$$

The first constraint in (10) is profitability for the firms and the second government budget balance.

Theorem 2 *(i) If $\pi^i(\overline{x}) \geq 0$, an ad valorem tax $\tau = 1$ and a specific tax $t = -C'$ lead to marginal cost pricing and attain the first-best outcome x^*. (ii) If $\pi^i(\overline{x}) < 0$, the optimal combination of ad valorem and specific taxes achieves the second-best outcome x^{**}.*

Proof. The proof is a straightforward adaptation of Proposition 1 in Myles (1996). ■

The explanation for this result is that the *ad valorem* tax reduces the perceived market power of the firms and the specific tax acts as a fixed price-per-unit. These ensure that the firms expand output and the welfare loss due to the imperfect competition is eliminated. Hence coordination by countries can control international oligopolies by the use of domestic tax instruments alone; trade policies are unnecessary.

4 Nash Equilibrium

The next two results show that quantity control is strictly superior to taxation with Nash competition between governments.

Given $U'\left(x_1^j + x_2^j\right) = q_j$, $j = 1,2$ it follows that

$$\ell_j = U'\left(x_1^j + x_2^j\right)\left[x_1^j + x_2^j\right] - \pi^j. \tag{11}$$

With quantity control, utility in country j can then be written as

$$U = U\left(x_1^j + x_2^j\right) + \left[x_j^{j'}\psi\left(x_1^{j'} + x_2^{j'}\right) - x_{j'}^j\psi\left(x_1^j + x_2^j\right)\right] - C\left(x_j^1 + x_j^2\right), j \neq j'. \tag{12}$$

The central term in (12) represents the trade surplus (or deficit) of country j in its trade with country j'.

The optimization problem for country j is to maximize (12) through choice of x_j^j and $x_j^{j'}$, subject to the conditions

$$x_j^j \psi \left(x_1^j + x_2^j \right) + x_j^{j'} \psi \left(x_1^{j'} + x_2^{j'} \right) - C \left(x_j^1 + x_j^2 \right) \geq 0, \; x_j^{j'} \geq 0. \tag{13}$$

Theorem 3 *The Nash equilibrium in quantity setting is identical to the cooperative outcome.*

Proof. The symmetric equilibrium between countries is described by

$$\psi - x^o \psi' - C' + \lambda \left[\psi + x^h \psi' - C' \right] + \mu^h = 0, \tag{14}$$

$$\psi + x^o \psi' - C' + \lambda \left[\psi + x^o \psi' - C' \right] + \mu^o = 0, \tag{15}$$

where $x^h \equiv x_j^j, x^o \equiv x_j^{j'}, j \neq j'$ and μ^h, μ^o are the multipliers on the corresponding non-negativity constraints. Conditions (14) and (15) have two solutions: (i) $\lambda = 0, x^o = 0, \psi = C'$; and (ii) $\left[x^h + x^o \right] \psi - C = 0, x^o > 0, x^h > 0$. These correspond to the allocations x^* and x^{**} respectively. ■

It is interesting to note that this theorem shows that even though choice is non-cooperative, the first-best is still sustained. There is no need for explicit cooperation between the governments to coordinate outputs.

With taxation, imposing budget balance gives the objective function in country j

$$U \left(x_1^j + x_2^j \right) - \psi \left(x_1^j + x_2^j \right) x_{j'}^j + \left[\left[1 - \tau_{j'} \right] \psi \left(x_1^{j'} + x_2^{j'} \right) - t_{j'} \right] x_j^{j'} - C \left(x_j^1 + x_j^2 \right). \tag{16}$$

Theorem 4 *With Nash competition, taxation is inferior to production control.*

Proof. Assume marginal cost pricing is the solution under production control so the profit constraint is not binding. Using symmetry and budget balance the first-order condition arising from the optimization of (16) is then

$$\frac{\partial x^h}{\partial \tau} \left[\psi - 2x^h \psi' - C' \right] + \frac{\partial x^o}{\partial \tau} \left[\psi + 2 \left[1 - \tau \right] x^h \psi' - C' \right] = 0. \tag{17}$$

Since $\frac{\partial x^h}{\partial \tau}$ has the opposite sign to $\frac{\partial x^o}{\partial \tau}$ (using the comparative statics of profit maximization), $\psi = C'$ can never be a solution to (17). Since marginal cost pricing is first-best, the tax outcome must be inferior. ■

This conclusion is very much in the flavour of a standard strategic trade policy result. Even though there are two tax instruments available, Nash equilibrium in taxation is still inefficient.

5 Single-Country Control

This section determines the outcome when country 2 chooses not to intervene in the market. In this case the only policy intervention arises from the actions of country 1.

5.1 Quantity Control

Assume country 1 acts alone in controlling the output of the firm located within its borders and that it holds Nash conjectures about the behavior of firm 2. Country 1 therefore seeks to maximize (12) and firm 2 has objective

$$\max_{\{x_2^1, x_2^2\}} x_2^1 \psi\left(x_1^1 + x_2^1\right) + x_2^2 \psi\left(x_1^2 + x_2^2\right) - C\left(x_2^1 + x_2^2\right). \tag{18}$$

The necessary conditions are given by

$$\psi\left(x_1^1 + x_2^1\right) - x_2^1 \psi'\left(x_1^1 + x_2^1\right) - C'\left(x_1^1 + x_1^2\right) = 0, \tag{19}$$

where the substitution $U'\left(x_1^1 + x_2^1\right) \equiv \psi'\left(x_1^1 + x_2^1\right)$ has been used,

$$\psi\left(x_1^2 + x_2^2\right) + x_1^2 \psi'\left(x_1^2 + x_2^2\right) - C'\left(x_1^1 + x_1^2\right) = 0, \tag{20}$$

and

$$\psi\left(x_1^j + x_2^j\right) + x_2^j \psi'\left(x_1^j + x_2^j\right) - C'\left(x_2^1 + x_2^2\right) = 0, \; j = 1, 2. \tag{21}$$

The equilibrium is the simultaneous solution to (19)-(21) and is denoted $\widehat{x} \equiv \left(\widehat{x}_1^1, \widehat{x}_1^2, \widehat{x}_2^1, \widehat{x}_2^2\right)$. Assumption 3 is imposed.

Assumption 3. The profit of firm 1 is positive at the equilibrium $\widehat{x}$, that is $\pi^1\left(\widehat{x}\right) > 0$.

Proposition 5 *(i) If $C'' = 0$ then $\widehat{x}_2^1 = 0$, $\widehat{x}_1^2 = \widehat{x}_2^2$, the good is sold at marginal cost in country 1, and country 1 runs a trade surplus. (ii) If $C'' > 0$ then $\widehat{x}_1^2 < \widehat{x}_2^2, \widehat{x}_1^1 > \widehat{x}_2^1$, total production is greater in country 1, and consumption is greater in country 1.*

Proof. (i) Denote the common value of constant marginal cost by c. it follows from (19) and (21) that

$$\psi\left(\widehat{x}_1^1 + \widehat{x}_2^1\right) - \widehat{x}_2^1 \psi'\left(\widehat{x}_1^1 + \widehat{x}_2^1\right) = c = \psi\left(\widehat{x}_1^1 + \widehat{x}_2^1\right) + \widehat{x}_2^1 \psi'\left(\widehat{x}_1^1 + \widehat{x}_2^1\right), \tag{22}$$

so $\widehat{x}_2^1 = 0$ and $\psi\left(\widehat{x}_1^1 + \widehat{x}_2^1\right) = c$. Hence from (20) and (21), $\widehat{x}_1^2 = \widehat{x}_2^2$.

(ii) From (19) and (21) it follows that $C'\left(\widehat{x}_1^1 + \widehat{x}_1^2\right) > C'\left(\widehat{x}_2^1 + \widehat{x}_2^2\right)$ so $\widehat{x}_1^1 + \widehat{x}_1^2 > \widehat{x}_2^1 + \widehat{x}_2^2$. The relative values of marginal costs imply from (20) and (21) that $\widehat{x}_1^2 \psi'\left(\widehat{x}_1^2 + \widehat{x}_2^2\right) > \widehat{x}_2^2 \psi'\left(\widehat{x}_1^2 + \widehat{x}_2^2\right)$ so $\widehat{x}_1^2 < \widehat{x}_2^2$. Combining these inequalities, it follows that $\widehat{x}_1^1 > \widehat{x}_2^1$. Finally, equating (19) and (20) via marginal cost gives $\psi\left(\widehat{x}_1^1 + \widehat{x}_2^1\right) = \psi\left(\widehat{x}_1^2 + \widehat{x}_2^2\right) + \widehat{x}_1^2 \psi'\left(\widehat{x}_1^2 + \widehat{x}_2^2\right) + \widehat{x}_2^1 \psi'\left(\widehat{x}_1^1 + \widehat{x}_2^1\right)$ or $\psi\left(\widehat{x}_1^1 + \widehat{x}_2^1\right) < \psi\left(\widehat{x}_1^2 + \widehat{x}_2^2\right)$ which implies $\widehat{x}_1^1 + \widehat{x}_2^1 > \widehat{x}_1^2 + \widehat{x}_2^2$. ■

5.2 Taxation

In the presence of taxation, firm i maximizes profit

$$\pi^i = [1 - \tau] \psi\left(x_1^1 + x_2^1\right) x_i^1 + \psi\left(x_1^2 + x_2^2\right) x_i^2 - t x_i^1 - C\left(x_i^1 + x_i^2\right), \quad i = 1, 2. \tag{23}$$

The optimal choice of output in country 1 is determined by

$$[1-\tau]\,\psi\left(x_1^1+x_2^1\right)+[1-\tau]\,\psi'\left(x_1^1+x_2^1\right)x_i^1-t-C'\left(x_i^1+x_i^2\right)=0,\quad i=1,2, \tag{24}$$

which can be simplified using government budget balance to give

$$\psi\left(x_1^1+x_2^1\right)+[1-\tau]\,\psi'\left(x_1^1+x_2^1\right)x_i^1-C'\left(x_i^1+x_i^2\right)=0,\quad i=1,2. \tag{25}$$

The optimal choice in country 2 follows from

$$\psi\left(x_1^2+x_2^2\right)+\psi'\left(x_1^2+x_2^2\right)x_i^2-C'\left(x_i^1+x_i^2\right)=0,\quad i=1,2. \tag{26}$$

The solution of (25) and (26) is denoted by $x(\tau)\equiv\left(x_1^1(\tau),x_1^2(\tau),x_2^1(\tau),x_2^2(\tau)\right)$.

Since an increase in τ reduces perceived market power, output will increase as τ increases, reaching a feasible maximum at $\tau=1$ with the specific tax, t, equal to the negative of marginal cost. The optimal solution is therefore $\tau=1$, with output $\widetilde{x}\equiv x(1)$, and $t=-C\left(x_i^1(1)+x_i^2(1)\right)$. The question now is whether this solution provides a greater welfare level than the direct control of production.

5.2.1 Case 1: $C''=0$

Theorem 6 *With single country regulation, direct control of production and the use of optimal ad valorem and specific taxes lead to the same level of welfare when marginal cost is constant.*

Proof. When $C''=0$, (26) shows that $\widehat{x}_1^2=\widehat{x}_2^2=\widetilde{x}_1^2=\widetilde{x}_2^2$ so both equilibria result in marginal cost pricing in country 1. Since $\widehat{x}_2^1=0$ this implies $\widehat{x}_1^1=\widetilde{x}_1^1+\widetilde{x}_2^1$. Under quantity control, country 1 is supplied entirely by firm 1. Output, and labor use are therefore higher for firm 1 than with taxation. Exports of country 1 are the same in both cases, so the trade surplus of country 1 must be greater with quantity control. Welfare differences between the two are determined by the sum of the labor supply and trade balance effects. The value of imports into country 1 with taxation is $\widetilde{x}_2^1c$. The labor usage with quantity control is $\widehat{x}_1^1c$ and with taxation $\widetilde{x}_1^1c$. However, since $\widehat{x}_1^1=\widetilde{x}_1^1+\widetilde{x}_2^1$, these effects exactly cancel. ■

5.2.2 Case2: $C''>0$

In this case the same factors are at work: the labor use of country 1 is higher with production control but this is offset by a greater trade surplus. The nature of costs then suggests that the production-control equilibrium is likely to provide a lower level of welfare since the higher level of production in country 1 must be generated by a technology with increasing marginal cost. To show that this reasoning can be substantiated, the equilibria are now explicitly calculated for a parameterized example.

Example. Assume that the utility function is quadratic with

$$U=a\left[x_1^j+x_2^j\right]-\frac{b}{2}\left[x_1^j+x_2^j\right]^2-\ell_j,\quad j=1,2, \tag{27}$$

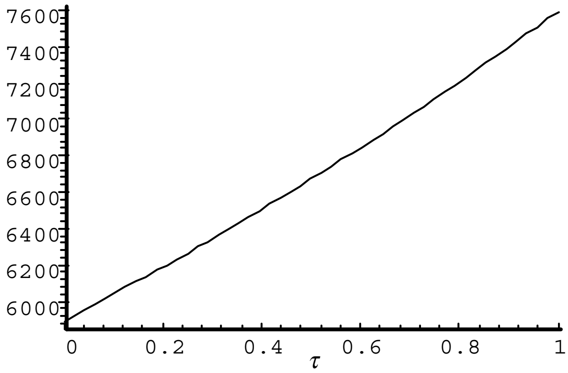

Figure 1: Welfare as a Function of τ

and that the cost function is also quadratic

$$C = F + d\left[x_i^1 + x_i^2\right] + e\left[x_i^1 + x_i^2\right]^2, \quad i = 1, 2. \tag{28}$$

With parameter values $a = 200$, $b = 1$, $d = 10$, $e = 1$, the equilibrium in the quantity control case is given by $\widehat{x}_1^1 = 57$, $\widehat{x}_1^2 = 9.5$, $\widehat{x}_2^1 = 14.25$, $\widehat{x}_2^2 = 38$ with $q_1 = 128.75$ and marginal cost for firm 1 is 143. As already noted, the trade-balance effect drives price below marginal cost. The welfare level is $6238.5-F$. The equilibrium with taxation is $x_1^1(\tau) = x_2^1(\tau) = \frac{570}{21-5\tau}$ and $x_1^2(\tau) = x_2^2(\tau) = 190\frac{\tau-3}{5\tau-21}$. Figure 1 plots the welfare level of country 1 as a function of τ.

Figure 1 shows the level of welfare attained by production control can be exceeded by the use of taxation, a consequence of taxation reducing the use of labor in country 1 relative to production control. For $F = 0$, this occurs for *ad valorem* tax rates in excess of 0.22.

Theorem 7 *The use of taxation can generate a higher level of welfare than direct production control in single country regulation.*

6 Conclusions

When countries cooperate, the paper showed that the combined use of specific and *ad valorem* taxation is at least as effective as direct production control. The use of the taxes succeeds in eliminating the welfare losses in the international economy due to the existence of imperfect competition. Furthermore, no other policy intervention (such as tariffs *etc.*) are necessary to control the oligopoly. It is interesting to note how the structure of these taxes (with the optimal specific tax being negative) differs substantially from the policy of cigarette taxation in the European Union (where a positive specific tax is employed).

With competition between countries, the Nash equilibrium in quantity-setting is first-best so no coordination is required. In contrast, competition in tax-setting leads to an inferior outcome. If a single country acts in isolation, it can do at least as well by employing taxation than it can through controlling the production of its “home“ firm. An example demonstrated that taxation can sometimes be strictly preferable.

7 References

Myles, G.D. (1996) ”Imperfect competition and the optimal combination of *ad valorem* and specific taxation”, *International Tax and Public Finance*, **3**, 29 - 44.