E C O N O M I C S   B U L L E T I N

Does Altruism Mitigate Free−riding and Welfare Loss?

Akram Temimi
University of Alabama

Abstract

A warm−glow motivation for charitable giving has recently been explored as a possible solution to the problem of inefficient private provision of public goods. However, the introduction of warm−glow affects both the efficient level of public good provision as well as the equilibrium level. Hence it is not clear whether warm−glow mitigates or exacerbates inefficiency. We revisit Andreoni's (1989) model of impure altruism and formally analyze this question. Cornes and Sandler's (1986) index of easy riding and a version of Debreu's (1951) coefficient of resource utilization are used as measures of free−riding and welfare loss.

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1. Introduction

The impure public good model is gaining widespread acceptance as a powerful framework for analyzing charitable giving. Impure public good models are characterized by the presence of a commodity which jointly generates both private and public benefits (Cornes and Sandler (1984, 1986), Andreoni (1989, 1990), among others). That is, a person who makes voluntary contributions to the provision of a public good may benefit both from the act of contribution *per se* and from the total supply of the public good. In such models, individuals may not be solely motivated by pure altruism where they care only about the total amount of charitable giving. Individuals may also be motivated by the "warm-glow" of having contributed. The combination of both motivations is referred to as impure altruism.

Cornes and Sandler (1994) indicate that the privatization of benefits may "attenuate" free-riding motives. This great hope for warm-glow is also suggested by Olson (1965), Sugden (1984, 1986), Palfrey and Rosenthal (1988), and Andreoni (1989, 1990), and is motivated by the idea that the propensity to contribute to public goods is greater when agents also care about their donations *per se*. More specifically, the existing literature shows that the equilibrium level of charitable contributions in most of these impure public good models is higher than in pure public good models.

The importance of higher overall contributions, however, is only important to the extent that it makes people better off. Cornes and Sandler (2000) point out that "policies that can increase public good supply and improve everyone's well-being have desirable normative properties, and, as such, are more interesting than policies that just augment public good provision." Indeed since the introduction of warm-glow to preferences affects both the equilibrium level <u>and</u> the efficient level of public goods provision, it is not clear *a priori* whether warm-glow mitigates or exacerbates inefficiency and free-riding even if it leads to an increase in equilibrium contributions. This point has not been addressed in the literature, which has focused exclusively on the equilibrium level of provision.

In this paper we use Andreoni's (1989) index of altruism and analyze the effect of impure altruism on the efficiency of private provision of public goods. We use two alternative measures of efficiency. First we consider Cornes and Sandler's (1984) proposed index of free-riding, derived by pairing an equilibrium level of public goods with the efficient level arising from a given income distribution. Second, we consider a measure of welfare loss based on the coefficient of resource utilization introduced in the classic work of Debreu (1951). Using these two measures, we show via an example that the degree of altruism has an ambiguous effect on free-riding and welfare loss. These results challenge our expectations that "warm-glow" may solve the problem of inefficient private provision of public goods.

2. An Impure Public Good Model

Following Andreoni (1989, 1990), we consider an economy with n identical consumers who have preferences over a private good, a public good, and the individual's contribution to the public good. That is, the twice-differentiable, strictly increasing, strictly quasi-concave utility function of the representative agent is given by

$$U_i = U(x_i, G, g_i), \tag{1}$$

where x_i is the level of the private good, G is the aggregate level of the public good, and g_i is the agent's contribution to the public good. The price of the private good is normalized to be one, and the technology for transforming the private good into the public good is linear. Hence we can express the aggregate public good level as the sum of the individual contributions,

$$G = \sum_j g_j = G_{-i} + g_i, \tag{2}$$

and the representative agent's budget constraint is given by

$$I = x_i + g_i \tag{3}$$

where I is the individual's endowment of the private good. The representative agent chooses x_i, G, and g_i to maximize (1) subject to the constraints given by (2) and (3). Substituting in the budget constraint and assuming that each individual treats G_{-i} as fixed, the maximization problem can be rewritten as

$$\max_G \; U(I + G_{-i} - G,\; G,\; G - G_{-i}), \tag{4}$$

The first-order conditions for this problem implicitly define the demand for the public good as a function of the exogenous components of (4),

$$G = f_i(I + G_{-i},\; G_{-i}). \tag{5}$$

We can then directly express the representative agent's demand for contributions as

$$g_i = f_i(I + G_{-i},\; G_{-i}) - G_{-i}. \tag{6}$$

Note that the first argument of f_i comes directly from the public good argument of the utility function and the second argument comes directly from the private contributions argument of the utility function.

We can therefore consider the marginal propensity to donate for altruistic reasons and the marginal propensity to donate for egoistic reasons as

$$f_{ia} = \frac{\partial f_i}{\partial (I_i + G_{-i})} \quad \text{and} \quad f_{ie} = \frac{\partial f_i}{\partial G_{-i}},$$

respectively. Andreoni then defines consumer i's altruism index as

$$\alpha_i = \frac{f_{ia}}{f_{ia} + f_{ie}}.$$

That is, an agent's index of altruism is the proportion of his total propensity to contribute to the public good explained by altruistic motives. In particular, if $\alpha_j > \alpha_i$ then agent j can be considered to be more altruistic than agent i because his propensity to contribute to public goods is driven less by the "warm-glow" from contributions than from the benefits of consuming the public good.

Recall that in the impure altruism model presented here, all agents are identical. In this case, we can say that the economy's altruism index is simply the altruism index of the representative consumer.

3. Equilibrium and Optimality

A standard result in the impure public good literature is that the Nash equilibrium level of contributions will be suboptimal. In this paper, we analyze how an economy's altruism index relates to its "degree of inefficiency". To do this, we consider measures of free-riding and welfare loss. Cornes and Sandler (1986) suggest measuring the degree of free-riding by the ratio $\lambda = \frac{G_{EQ}}{G_{OPT}}$ where G_{EQ} is the equilibrium level of the public good and G_{OPT} is the point on the Pareto frontier at which each individual's share of total contributions is the same as his equilibrium contribution shares. Since we are considering symmetric equilibria, the equilibrium and optimal shares of contributions are simply $1/n$. So G_{OPT} is the point on the Pareto frontier which arises from an equal distribution of initial wealth.

Arguably, however, the importance of free-riding by itself is only important to the extent that it is evidence of welfare loss. Cornes and Sandler again point out that "equilibrium may be very close to the optimum, yet be associated with a large welfare loss. Conversely, a substantial shortfall of [equilibrium] below [optimality] may turn out not to matter much in welfare terms."[1] For this reason, we also compare equilibrium contributions to optimal contribution levels by considering a special case of Debreu's (1951) coefficient of resource utilization in which "the inefficiency is now described by the number of dollars representing the value of the physical resources which could be thrown away without preventing the achievement of the prescribed levels of satisfaction." Equivalently, we consider the degree to which we could scale back initial wealth and not prevent the achievement of the equilibrium level of utility. That is, if g^I_{EQ} and g^I_{OPT} are the equilibrium and optimal levels of the representative agent's contributions as a function of initial wealth, and $g^{\gamma I}_{OPT}$ is similarly defined, then we index welfare loss by the γ which satisfies

$$U(I - g^I_{EQ}, ng^I_{EQ}, g^I_{EQ}) = U(\gamma I - g^{\gamma I}_{OPT}, ng^{\gamma I}_{OPT}, g^{\gamma I}_{OPT}).$$

If $\gamma = 1$ then there is no welfare loss at equilibrium. If $\gamma = 0$, then welfare loss is maximized. As γ increases, welfare loss decreases.

4. The Effect of Altruism on Free-riding and Welfare Loss

Consider an economy of $n \geq 2$ consumers, each with the following utility function

$$U(x_i, G, g_i) = a \ln x_i + b \ln G + c \ln g_i. \tag{7}$$

[1] Although the free-riding index doesn't given us direct information about the size of welfare losses, one might expect that a larger λ is associated with a smaller welfare losses. In fact, this is not always the case and is a partial justification for considering free-riding *and* welfare loss.

This specification is ubiquitous in the voluntary contribution literature. See Feldstein and Clotfelter (1976), Abrams and Schmitz (1984), Shiff (1985), Lucas and Stark (1985), Cox (1987), and Andreoni (1988, 1990).

4.1 The Equilibrium Level of Contributions

The representative consumer maximizes utility given by (7) subject to the constraints given by (2) and (3). The first-order conditions for utility maximization are

$$\frac{\partial U/\partial G}{\partial U/\partial x_i - \partial U/\partial g_i} = \frac{b/G}{a/x_i - c/g_i} = 1,$$

which equates the marginal rate of substitution between the public good and private goods to the marginal rate of transformation between the public good and private goods. Substituting in the constraints given by (2) and (3), the first order conditions can be re-written in terms of G and G_{-i}:

$$b(I + G_{-i} - G)(G - G_{-i}) - aG(G - G_{-i}) + cG(I + G_{-i} - G) = 0. \tag{8}$$

The G that satisfies (8) gives the individual's demand for the public good

$$G = f_i(I + G_{-i},\ G_{-i})$$

as described by (5). In the symmetric equilibrium we know that $G - G_{-i} = G/n$, so we can find the Nash equilibrium level of the public good directly from (8):

$$G_{EQ} = \frac{(\frac{b}{n} + c)nI}{a + \frac{b}{n} + c}. \tag{9}$$

4.2 The Efficient Level of Contributions

The efficient level of public goods is determined by a modified Samuelson condition, the sum of the marginal rates of substitution (between the public good and net private good) is equal to one. Since we have identical agents, this becomes

$$n\frac{b/G}{a/x_i - c/g_i} = 1.$$

Using the fact that we are concerned only with the symmetric optimum, substitute $x_i = I - G/n$ and $g_i = G/n$ into the modified Samuelson condition to find that the optimal level of public good is

$$G_{OPT} = \frac{(b + c)nI}{a + b + c}. \tag{10}$$

4.3 The Altruism Index

Using (8) and then substituting in the equilibrium level of contributions given by (9), the agent's altruism index in equilibrium is given by

$$\alpha = \frac{f_{ia}}{f_{ia} + f_{ie}} = \frac{(\frac{b}{n} + c)^2}{(\frac{b}{n})^2 + 2c\frac{b}{n} + c(a + c)}. \tag{11}$$

Let α be the altruism index of the representative agent in an economy, E, comprised of n consumers with utility function $U(x_i, G, g_i) = a \ln x_i + b \ln G + c \ln g_i$ and initial endowment I. Similarly, let $\hat{\alpha}$ be the altruism index of the representative agent in an economy, $\hat{E}$, comprised of n consumers with utility function $U(x_i, G, g_i) = \hat{a} \ln x_i + \hat{b} \ln G + \hat{c} \ln g_i$ and initial endowment I. Then, using Andreoni's language, economy E has a higher index of altruism than economy $\hat{E}$ if $\alpha > \hat{\alpha}$.

Recall that our goal is to determine the relationship between an economy's index of altruism and the degrees of free-riding and welfare loss. To do this, we first analyze how the intensity of preference over G and g_i is related to the altruism index.

Proposition 1. *In economies E and $\hat{E}$, if the preference for the private good is the same ($a = \hat{a}$), and the preference for warm glow is the same ($c = \hat{c}$), then economy E has a higher index of altruism than economy $\hat{E}$ if the preference for the public good is higher in economy E ($b > \hat{b}$).*

This result follows immediately from the comparative static results on α:

$$\frac{d\alpha}{db} = \frac{2ac(\frac{b}{n} + c)n}{[(\frac{b}{n})^2 + 2c\frac{b}{n} + c(a+c)]^2} > 0. \tag{12}$$

Proposition 2. *In economies E and $\hat{E}$, if the preference for the private good is the same ($a = \hat{a}$), and the preference for the public good is the same ($b = \hat{b}$), then economy E has a higher index of altruism than economy $\hat{E}$ if the preference for warm glow is sufficiently high and is higher in economy E ($c > \hat{c} > \frac{b}{n}$).*

This result follows immediately from the comparative static results on α:

$$\frac{d\alpha}{dc} = -\frac{a(\frac{b}{n} + c)(\frac{b}{n} - c)n^2}{[(\frac{b}{n})^2 + 2c\frac{b}{n} + c(a+c)]^2}. \tag{13}$$

Propositions 1 and 2 indicate that, all other things equal, an economy with a larger b or a larger c (provided $c > \frac{b}{n}$) has a larger altruism index.

4.4 Free-riding and Welfare Loss

Now we find the degree of free-riding, λ, and welfare loss, γ, for this economy. It follows from (9) and (10) that the degree of free-riding is

$$\lambda = \frac{G_{EQ}}{G_{OPT}} = \frac{(\frac{b}{n} + c)(a+b+c)}{(a + \frac{b}{n} + c)(b+c)}. \tag{14}$$

Recall that the degree of welfare loss in this model is given by the γ satisfying

$$U(I - g^I_{EQ}, ng^I_{EQ}, g^I_{EQ}) = U(\gamma I - g^{\gamma I}_{OPT}, ng^{\gamma I}_{OPT}, g^{\gamma I}_{OPT}).$$

Substituting in the utility function, γ satisfies

$$\left(I - \frac{G^I_{EQ}}{n}\right)^a \left(G^I_{EQ}\right)^b \left(\frac{G^I_{EQ}}{n}\right)^c = \left(\gamma I - \frac{G^{\gamma I}_{OPT}}{n}\right)^a \left(G^{\gamma I}_{OPT}\right)^b \left(\frac{G^{\gamma I}_{OPT}}{n}\right)^c.$$

Then substituting in G_{EQ} and G_{OPT} from (9) and (10), the degree of welfare loss is

$$\gamma = \lambda^{\left(\frac{b+c}{a+b}\right)} \left(\frac{a+b+c}{a+\frac{b}{n}+c} \right)^{\left(\frac{a}{a+b}\right)}. \tag{15}$$

4.5 Results

We now present our main results. Recall from proposition 1 and 2 that a higher index of altruism can be induced by either a greater preference for the public good or by warmer glow. Proposition 1 implies that, all other things equal between two economies, the economy with the greater intensity of preference for the public good has a higher index of altruism. Now we determine whether such an economy necessarily has more free-riding and greater welfare loss.

Proposition 3. *In economies E and $\hat{E}$ where $a = \hat{a}$ and $c = \hat{c}$, if E has a higher index of altruism than $\hat{E}$ (i.e., if $b > \hat{b}$), then E may have more or less free-riding and welfare loss than $\hat{E}$.*

The comparative statics results to support Proposition 3 are:

$$\frac{d\lambda}{db} = -\frac{a(n-1)(\frac{ac}{n} - (\frac{b}{n})^2 + \frac{c^2}{n})}{(b+c)^2(a+\frac{b}{n}+c))^2} \gtrless 0$$

and

$$\frac{d\gamma}{db} = \frac{\gamma \left(a\frac{b}{n}(a+b)\frac{(n-1)}{n} - (\frac{b}{n}+c)(a+\frac{b}{n}+c)\ln(\frac{b+c}{\frac{b}{n}+c})^a \lambda^c) \right)}{(a+b)^2(\frac{b}{n}+c)(a+\frac{b}{n}+c)} \gtrless 0.$$

Note that $\frac{d\lambda}{db}$ and $\frac{d\gamma}{db}$ may not have the same sign. So, for example, the degree of free-riding may be larger while the welfare loss is smaller.

Now recall from Proposition 2 that, all other things equal between two economies, the economy with the greater warm-glow has a higher index of altruism (provided the warm-glow is strong enough). Now we determine whether an economy being more altruistic in *this manner* implies that is has less free-riding and smaller welfare loss.

Proposition 4. *In economies E and $\hat{E}$ where $a = \hat{a}$ and $b = \hat{b}$, if E has a higher index of altruism than $\hat{E}$ (i.e., if $c > \hat{c} > \frac{b}{n}$), then E will have less free-riding but may have bigger or smaller welfare loss than $\hat{E}$.*

The comparative statics results to support Proposition 4 are:

$$\frac{d\lambda}{dc} = \frac{a\frac{b}{n}(n-1)(a+b(1+\frac{1}{n})+2c)}{(b+c)^2(a+\frac{b}{n}+c)^2} > 0$$

and

$$\frac{d\gamma}{dc} = \frac{\gamma \left(a\frac{b}{n}(n-1) + (\frac{b}{n}+c)(a+\frac{b}{n}+c)\ln\lambda \right)}{(a+b)(\frac{b}{n}+c)(a+\frac{b}{n}+c)} \gtrless 0.$$

Propositions 3 and 4 combine to answer the title question of this paper — does altruism mitigate free-riding and welfare loss? Our analysis shows that the answer is ambiguous in general. Even in this highly stylized model, economies that are more altruistic may or may not have more free-riding and greater welfare loss. We also learn that the answer depends on *how* an economy is more altruistic. The one unambiguous effect derives from Proposition 4: if one economy is more altruistic than another due to having a larger warm-glow effect, then the more altruistic economy will have a smaller degree of free-riding. This result perhaps confirms the expectation that the "privatization" of benefits may solve the problem of inefficient private provision of public goods. However, although the more altruistic economy (due to more warm-glow) has less free-riding, it may have greater welfare loss.

5. Concluding Remarks

The literature on private provision of public goods indicates that in the presence of warm-glow, equilibrium contributions are higher than in pure public good models. The importance of higher overall contributions, however, is only important to the extent that it makes people better off. This point has been completely ignored in the literature. The focus has been exclusively on the equilibrium level of contributions. Our analysis shows via an example that a higher index of altruism may be associated with more or less free-riding and welfare loss.

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