E C O N O M I C S   B U L L E T I N

On the Number of Firms and the Quantity of Innovation

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Abstract

This paper models a dynamic innovation process to examine the relationship between levels of R and D and market structure. In contrast to most of the literature, here R and D increases firms' knowledge stocks, making future R and D less costly. This creates a feedback by which market structures can affect levels of R and D. In general while an increase in the number of firms reduces R and D per firm, industry R and D increases. The model also endogenizes the number of firms using a zero profit condition.

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1 Introduction

This paper develops a model of firm R&D in order to investigate the relationship between R&D levels and market structure.[1] There are two innovations in the model. First, the model treats R&D as an ongoing dynamic process. One of the effects of R&D, whether or not it creates market success, is to create knowledge for the firm doing it. Since knowledge is one of the most important inputs to R&D, there is a natural feedback from today's R&D to tomorrow's R&D productivity—more R&D today implies a higher knowledge stock for the firm tomorrow, which in turn facilitates tomorrow's R&D. This effect is essentially absent from the literature.[2] The second innovation in this model is to endogenize the number of firms doing R&D.

2 A Repeated Patent Race

Consider an industry in which firms are continually doing R&D. Projects are risky, though, so the probability of success of any project is less than one. Firms are Bertrand competitors, so positive rents from R&D only accrue if a firm is the only firm to succeed. A successful innovation creates a temporary monopoly, but the monopoly is destroyed when the next innovation occurs. The model can be seen as a repeated patent race, and as such, provides a dynamic treatment of the issues that Sah and Stiglitz (1987) and others analyze in a static framework.

The industry has n identical, myopic firms. In each period, t, a continuum of research projects, labelled [0,1], are possible. Firm j does m_j projects. Larger amounts of knowledge increase the probability of success, so each of a firm's projects is successful with equal probability $p(k_j)$ where k_j is the knowledge stock of firm j; $p(\cdot)$ is concave

[1] Theoretical literature on market structure and R&D has generated varied results on the nature of this relationship. See for example, Dasgupta and Stiglitz (1980), Dixit (1988) or Loury (1979), Gilbert and Newbery, (1982) or Greenstein and Ramey (1998). Sah and Stiglitz (1987) argue explicitly that there should be no relationship. Empirical evidence is if anything more mixed. Symeonides (1996) summarizes his survey of that literature by emphasizing its inconclusiveness.

[2] Peretto (1996) includes static increasing returns to scale in R&D, and dynamic increasing returns in goods production. Yi (1999) also includes dynamic increasing returns, here feeding back from output to the quantity of R&D. In Joshi and Vonortas (2001), R&D increases the firm's knowledge stock and thus its productivity in goods production. Profits in goods production are used to finance further R&D, further increasing knowledge stocks. In none of these models, though, are there any effects by which R&D feeds back onto its own productivity. R&D productivity is unaffected by knowledge stocks.

so $0 < p' < \infty$, $p'' < 0$; further, $p(0) \geq 0$. The costs of undertaking a project, c, are identical for every project. For a firm, each project undertaken adds to its future knowledge stock, so firm knowledge evolves as $k_{t+1} = f(k_t, m_t)$, and we assume that $f(\cdot)$ is concave in both arguments. Knowledge depreciates, so $f(k, 0) \leq k$; and finally, $f(0, 0) = 0$. Both knowledge, k, and the number of projects, m, are non-negative, and the probability of success, $p \in [0, 1)$.

Firms are Bertrand competitors so the payoff to a firm is

$$P = \begin{cases} \pi, & \text{if at least one of its projects is successful and no other firms' are,} \\ 0, & \text{if any other firms' projects are successful} \end{cases}$$

Assuming that the probability of success for different projects is independent,[3] the probability that at least one of firm j's projects is successful is $q_j(k_j, m_j) = 1 - (1 - p(k_j))^{m_j}$. The probability that no other firm has a successful project is $h_j(\mathbf{k}, \mathbf{m}) = \prod_{i \neq j}(1 - p(k_i))^{m_i}$ where $\mathbf{k}$ and $\mathbf{m}$ are vectors of knowledge endowments of, and numbers of projects undertaken by, firms other than firm j. A firm's R&D costs are linear in the number of projects, with fixed costs C. Thus the expected one-period profit of firm j is $EP = \pi h_j q_j - C - c m_j$. Finally, define total industry R&D to be M_t in period t.

2.1 The Short Run

Consider a population of myopic firms, which maximize their one-period profits with respect to the number of projects undertaken, treating h as fixed by the actions of the other firms. For an arbitrary firm, dropping the j subscript, the problem is written as

$$\max_{\{m\}} EP = \pi h q - cm - C, \tag{1}$$

and the first order condition,

$$\frac{\partial EP}{\partial m} = \pi h \frac{\partial q}{\partial m} - c = 0 \tag{2}$$

can be written as

$$c/\pi = \prod_{i \neq j}(1 - p(k_i))^{m_i} \times [-\ln(1 - p_j)(1 - p_j)^{m_j}]. \tag{3}$$

[3]This assumption is not critical, but greatly simplifies the mathematics.

The number of firms, n, enters the first order condition through the product term, and thus can affect firm levels of R&D.[4] It is the case, though, that it does not affect industry R&D levels This is made explicit by examination of the symmetric equilibrium.

In a symmetric equilibrium, $m_i = m_j$ and $k_i = k_j \forall i, j$, implying that $h(\cdot)$ reduces to $h(k, m_j) = (1 - p(k))^{M - m_j}$, where M is industry R&D. Dropping the j subscript and substituting, the first order condition becomes

$$c/\pi = -\ln(1-p)(1-p)^M. \tag{4}$$

Solving for M,

$$M = \ln\left(\frac{-c}{\pi \ln(1-p)}\right)\left(\frac{1}{\ln(1-p)}\right), \tag{5}$$

which again is not a function of n.[5] M is a function of p, however:

$$\frac{\partial M}{\partial p} = \left[\frac{1}{(\ln(1-p))^2}\frac{1}{1-p}\right] \times \left[1 + \ln\left(\frac{-c}{\pi \ln(1-p)}\right)\right], \tag{6}$$

which is ambiguous in sign. We can see, though, that

$$\frac{\partial M}{\partial p} \gtreqless 0 \text{ as } p \lesseqgtr 1 - \exp\{\frac{-ce}{\pi}\}. \tag{7}$$

so in general if p is small, $\frac{\partial M}{\partial p} > 0$ and if p is large $\frac{\partial M}{\partial p} < 0$. That $\partial M/\partial p$ could be negative seems an odd result, since if the probability of success increases, a firm is more likely to have a successful project. The confounding factor is that the probability of all other firms failing to succeed falls. Since the probability of all other firms failing decreases faster than the probability of success for one firm increases, the expected profit of a marginal project declines.

The first order condition defined above generates the *total* industry R&D, M, as a function of the success probability p, which is determined by the representative firm's knowledge level, k. The evolution of M, then, will be determined by the evolution of p, and so by the evolution of k.

Consider the next period for the representative firm: $k_{t+1} = f(k_t,\ M(p, k_t)/n)$. $\partial k_{t+1}/\partial n = f_2 \times -M/n^2$, which is negative. Thus the more firms there are in an industry, for a given level of knowledge in period t, the lower the knowledge level in

[4] $\frac{\partial^2 EP}{\partial m_j^2} = -\pi h[(\ln(1-p_j)^2(1-p_j)^{m_j}] < 0$, implying that the optimum is a maximum.

[5] Note here that M is positive only if $p > 1 - e^{-c/\pi}$. If $c/\pi = 0.01$, $p > 0.095$ is the necessary condition for $M > 0$.

period $t + 1$. The independence between total industry R&D and number of firms is broken.[6]

3 The Steady State

The analysis can be pursued by an examination of the steady state of the system.

Proposition 1: A steady state, defined by $k = f(k, M(p(k))/n)$ exists.

Proof: Consider an arbitrary m. $f(\cdot)$ is concave in k, and $f(0, m) \geq 0$, thus for an arbitrary m a fixed point, $k^* = f(k^*, m)$ exists. Concavity implies that at the fixed point, $f_1(k, m) < 1$. Now differentiate $f(k, m) - k = 0$, to get $dm/dk = -(f_1 - 1)/f_2 > 0$. Thus in m, k space $k = f(k, m)$ has positive slope everywhere, and since $f(0, 0) = 0$, it passes through the origin. Fixing n, and noting that $m = M/n$, from equation 7 and $\partial p/\partial k > 0$, the optimal $m = m^*(k)$ is concave in k, with negative slope for large k. Therefore, if $m^*(0) > 0$, $m^*(k)$ and $f(\cdot) - k = 0$ intersect at an interior fixed point. Otherwise, since $m^*(\cdot)$ is bounded below by 0, there is a fixed point at $m^* = 0$, $k = 0$; and possibly also at an interior point.

Stability properties of the steady states are easy to describe. In general if one or more interior steady states exist, an interior steady state is stable. If there are two interior steady states, the larger value of k is stable. The stability of the origin depends on the behaviour of $m^*(k)$ for $k > 0$ but small. If $m^*(k) > 0$ for $k = 0$, then the origin is unstable. However if there is a $k_1 > 0$ such that $m^*(k_1) = 0$, then the origin is again stable. The former seems more likely since if $m^*(0) = 0$ then without some exogenous shock, no industry could start.

Finally, notice again that n appears in the expression for k_{t+1} implying that in general the value of the fixed point of k is dependent on the number of firms in the industry, and so the stable number of projects M, is dependent on the number of firms. As n increases, the curve $k = f(k, M/n)$ rotates clockwise around the origin. (See Figure 1.) Whether this increases or decreases the steady state value of M depends on

[6] In the model in Sah and Stiglitz, (1987), which is similar to this model, but static, there is no relation between number of firms and R&D levels. Modelling R&D as an ongoing dynamic process is thus clearly important.

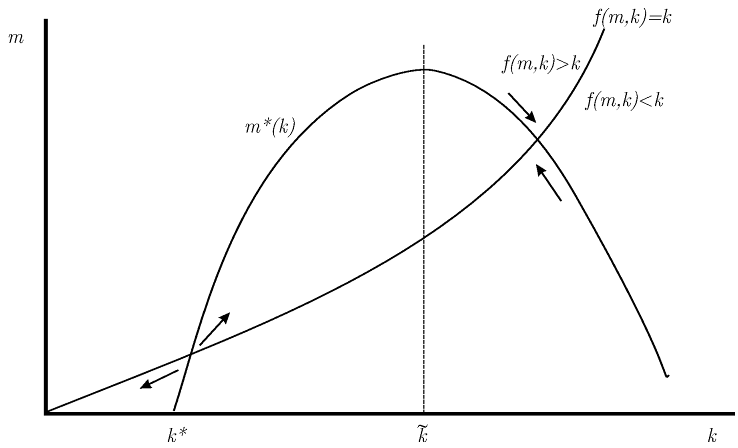

Figure 1: Diagram of Motion for Myopic Firms

functional forms. Define a value $\tilde{k}$ by $p(\tilde{k}) = 1 - \exp\{-ce/\pi\}$. (Shown in Figure 1.) Differentiating $M = mn$, $dM/dn = n \times dm/dn + m$. dM/dn will only be negative if dm/dn is negative, which can only occur if the steady state k is larger than $\tilde{k}$. This will occur if either $k = f(k, m)$ is relatively flat, or if $\tilde{k}$ is small (or both). The first condition holds if f_1 and f_2 are large; that is, if it is easy to accumulate knowledge. The second condition holds when c/π is small, that is, when the costs of undertaking a project are small relative to the potential (though not necessarily expected) profits.

4 An Illustration

This section presents a numerical example of the model, specifying functional forms for the probability of success as a function of knowledge, and for the evolution of knowledge.

Suppose that $k_{t+1} = \delta \ln(m_t + 1)$; and $p_t = \alpha - \beta e^{-k_t}$. Substituting, $p_{t+1} = \alpha - \beta/(m_t + 1)^{\delta}$, which implies that while $m \in [0, \infty)$, $p \in [\alpha - \beta, \alpha)$. In the steady state, $p = \alpha - \beta/(m + 1)^{\delta}$ Substituting into equation 5, and dividing by m, we can write

$$n = \frac{1}{m}\left[\ln\left(\frac{1}{\ln(1-\alpha+\beta(m+1)^{-\delta})}\right)\right]\left[\frac{1}{\ln\left(1-\alpha+\beta(m+1)^{-\delta}\right)}\right]. \tag{8}$$

We can solve implicitly for m, and the resultant relationship is shown in figure 2.

We can see that while firm R&D decreases with the number of firms, industry R&D is not monotonic in the number of firms.

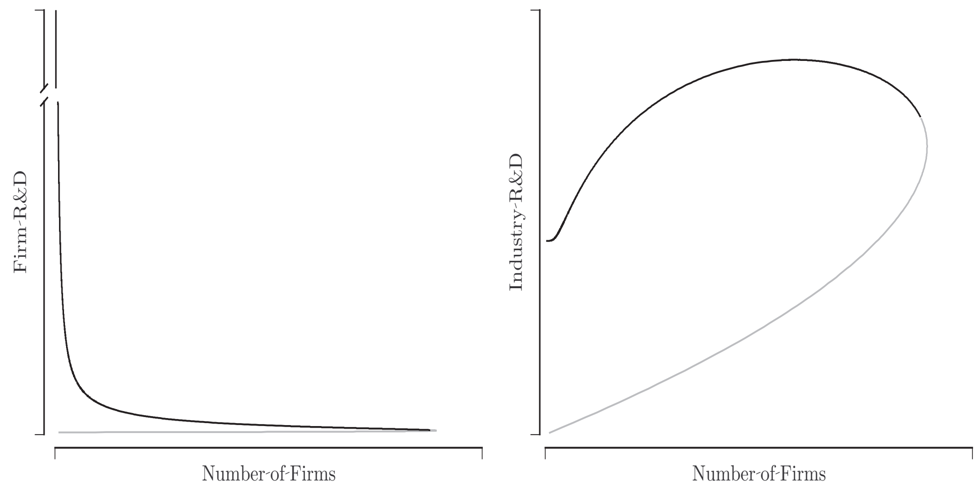

Figure 2: Steady state levels fo firm and industry R&D.

Positive profits in black; negative profits in grey.

We can use the model further to explore the endogeneity issue between industry structure and R&D. A zero profit condition will determine the number of firms. In the figure this is illustrated by plotting positive profit configurations in black, and negative profit configurations in grey. The conclusion to draw here then, is that endogeneity is extremely important. Under the parameters used to draw this figure, the relationship between number of firms and industry R&D is negative at the margin. But changing parameters both moves us along the curve and shifts the curve. The (empirical) analysis becomes more complex.

In the illustration there are five parameters and three variables of interest. We can summarize their effects in Table 1. To interpret, α and β determine the success probabilities; δ measures how easily knowledge is accumulated; π/c is the relative profitability of a uniquely successful project; and C is the fixed cost of R&D.

A crucial point emerges. Considering $\partial M/\partial n$ while ignoring the endogeneity of firms would suggest that there are conditions under which the number of firms and industry R&D move in opposite directions. If, for example, δ is large, we will be on the negatively sloped part of the $M(n)$ curve in Figure 2. Thus, at the margin, we would expect a negative relationship between M and n. But for any changes in parameters, n^* and $M|_{n^*}$ move in the same direction, implying a positive relationship, after adjustments, between number of firms and industry R&D. Empirical research must take into account

	n^*	$M\|_{n^*}$	$\partial M/\partial n\|_{n^*}$
α	+	+	−
β	−	−	−
π/c	+	+	+
δ	+	+	−
C	−	−	+

Table 1: Summary of parameter effects

then, whether the industry has had opportunity to adjust to the shocks, and whether we are observing marginal or infra-marginal changes.

5 Discussion

What we observe in this model is that including a central dynamic aspect of the R&D process has important implications for the relationship between industry structure and levels of R&D. The model is general enough that both positive and negative relationships are possible. Further, imposing a zero profit condition to determine the number of firms permits us to treat R&D levels and industry structure as jointly determined.

The numerical illustration shows that in general more firms always implies less R&D per firm. At the industry level, however, things are more complex and depend on parameter values, and on whether the industry has adjusted to whatever shocks arise to change parameter values. At the margin $\partial M/\partial n$ can be positive or negative, but when responding to parameter shocks n and M show a positive relationship.

This model is very general and quite robust. It could be extended quite naturally to include forward-looking firms who understand that one reason for doing R&D today is to increase R&D productivity tomorrow. This would change the particulars of the optimization problem, but would still permit the endogenization of the number of firms, and the joint determination of R&D levels. A second extension is to use the model for welfare analysis. There are externalities in the R&D process and in general this creates a divergence between private and social incentives. This model could be used to ask whether policy is useful in this situation.

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