E C O N O M I C S   B U L L E T I N

Ramsey pricing in one−way and two−way interconnection between telephone networks

Seonghoon Jeon
Sogang University

Abstract

I derive Ramsey optimal prices in one−way access of long−distance operators and enhanced service providers to local loops. As long−distance services and enhanced services become substitutes due to the advance of Internet telephony, the Ramsey principle requires higher access charges assessed on both services. I also derive Ramsey prices in two−way interconnection between fixed−link and mobile phone networks, which turn out to be formally equivalent to those for the one−way access above. This result suggests that the price of fixed−to−mobile calls should be higher than the price of mobile−to−fixed calls when the substitutability of calls to double subscribers is more prominent than the substitutability of calls of double subscribers, and vice versa.

Citation: Jeon, Seonghoon, (2002) "Ramsey pricing in one−way and two−way interconnection between telephone networks." *Economics Bulletin,* Vol. 12, No. 3 pp. 1−9
Submitted: February 19, 2002. **Accepted:** February 20, 2002.
URL: http://www.economicsbulletin.com/2002/volume12/EB−02L50001A.pdf

1. Introduction

The principle of Ramsey optimal pricing is to meet the regulated operator's break-even requirement with the least possible distortions of prices. It is very relevant to telecommunications pricing since the recovery of large fixed costs in local loop services has been primary concerns. The local loop deficit should be covered by mark-ups of various services involved. The involved services include long-distance services, enhanced services such as Internet services, and mobile phone services as well as local telephone services. Under the market configuration where the fixed-link local service is supplied by a regulated monopolist, and all other services are provided competitively, the Ramsey principle provides the second-best optimal solution for those prices.

The models of one-way access and two-way interconnection are two conceptual frameworks with which we can address relationships among telephone services. In one-way access, one company needs access to the other, but the reverse does not hold. The examples are the access of long-distance carriers and Internet service providers to local loops. Laffont-Tirole(1996) characterized Ramsey prices in the model of one-way supply of access by a vertically integrated monopolist to downstream new entrants in long-distance market. In the paper, I extend their set-up to consider an additional downstream service such as on-line or Internet service providers. An insight obtained additionally is that higher access charges should be assessed on long-distance and enhanced services as two services become more substitutable, for example, due to the advance of Internet telephony.

In two-way interconnection, customers calling each other belong to two different networks, such as fixed-link and mobile phone networks. In this case, each network can be an access-provider to the other network for calls that it terminates on its network. I derive Ramsey prices of four differently directed calls between fixed and mobile networks. Incidentally, the formulas are formally equivalent to those for the above one-way access. The isomorphism is remarkable in that two frameworks have different contexts. Disregarding other factors that affect Ramsey optimal prices, we may say that the price of fixed-to-mobile calls should be higher than the price of mobile-to-fixed calls when the substitutability of calls *to* double subscribers is more prominent than the substitutability of calls *of* double subscribers, and vice versa.

The remainder of the paper is organized as follows. Section 2 derives Ramsey prices for one-way access of long-distance and on-line services to local loops, and discusses the implications. Section 3 does the same work for two-way interconnection between fixed-link and mobile phone networks. Section 4 concludes by making remarks on the relevance of the model to regulatory realities.

2. Ramsey Pricing in One-way Interconnection

We consider four operators: 0 and 1 represent the local and long-distance division, respectively, of an incumbent telephone company, 2 refers to a group of new entrants in the long-distance market, and 3 represents a collection of enhanced service providers.[1]

[1] The incumbent firm is a vertically integrated local monopolist that provides local and long-distance services. The model can be adapted into the case in which the incumbent provides only local services under structural separation or the case in which it fully integrates, and provides enhanced services as well.

Denoting each operator's quantity and price by q_i and p_i, demand and cost functions are simplified as follows:

$$q_0 = q_0(p_0), \qquad q_i = q_i(p_1, p_2, p_3), \quad i = 1,2,3; \tag{1}$$

$$C_0 = c_0(q_0 + q_1 + q_2 + q_3) + k_0, \qquad C_i = c_i q_i, \quad i = 1,2,3. \tag{2}$$

Specification (1) assumes that demand for local call services is independent, while demands for long-distance and on-line services are interrelated. It is implicit in (2) that a unit of local call service is essential in providing long-distance and on-line services. Operator 2 and 3 pay access charges, a_2 and a_3, respectively, to the incumbent company for supplying access to local loop services.[2] Specification (2) assumes that marginal costs, $c_i(i = 0,1,2,3)$, are constant, and that only local call services incur fixed costs k_0.[3] I assume that markets for long-distance and on-line services are competitive, which implies:

$$p_2 = a_2 + c_2, \quad p_3 = a_3 + c_3. \tag{3}$$

The regulator controls the prices that the incumbent charges on final consumers, and other operators in such a way that maximizes social welfare subject to the constraint of zero-profit of the incumbent. I.e. the regulator solves the following optimization program:

$$\begin{aligned} &Max && V_0(q_0) + V(q_1, q_2, q_3) - c_0(q_0 + q_1 + q_2 + q_3) - k_0 - c_1 q_1 - c_2 q_2 - c_3 q_3 \\ &w.r.t. && p_0, p_1, a_2, a_3 \\ &s.t. && p_0 q_0 + p_1 q_1 + a_2 q_2 + a_3 q_3 - c_0(q_0 + q_1 + q_2 + q_3) - k_0 - c_1 q_1 . \end{aligned} \tag{4}$$

In program (4), V_0 and V denote the social values of telephone services; the separability of the former and the interrelationship in the latter reflect the demand structure in (1). This program is an application of the well-known Ramsey principle. Substituting (3) into the constraint of (4), and applying the Lagrangean method, we can reduce it into:

$$\begin{aligned} &Max && V_0(q_0) + V(q_1, q_2, q_3) + l(p_0 q_0 + p_1 q_2 + p_2 q_2 + p_3 q_3) \\ & && -(1 + l)\{c_0(q_0 + q_1 + q_2 + q_3) + k_0 + c_1 q_1 + c_2 q_2 + c_3 q_3\} \\ &w.r.t. && p_0, p_1, p_2, p_3 . \end{aligned} \tag{5}$$

Multiplier l reflects the social premium for the incumbent company's profit, which stems from the fact that the subsidy to make up its losses should be covered by distorting taxes.

Define own- and cross-elasticities of demands as follows:

$$h_i \equiv -\frac{\partial q_i}{\partial p_i}\frac{p_i}{q_i}, \quad h_{ij} \equiv \frac{\partial q_i}{\partial p_j}\frac{p_j}{q_i}, \quad i = 0,1,2,3, \quad j(\neq i) = 0,1,2,3. \tag{6}$$

Then, we can derive the optimal prices according to Ramsey principle as follows:

Proposition 1. Suppose $\frac{\partial V_0}{\partial q_0} = p_0$ and $\frac{\partial V}{\partial q_i} = p_i\,(i = 1,2,3)$. Then,

[2] Currently, enhanced service providers do not pay access charges in most of the countries. This policy is to promote the development of enhanced telecom services. But, at some point, they should, and will, share the large fixed costs of building telecom infrastructure.

[3] I ignore fixed costs in providing other services, which are relatively insignificant.

$$\frac{p_1 - c_0}{p_1} = \frac{l}{1+l}\frac{1}{h_0} \tag{7}$$

$$\frac{p_1 - c_0 - c_1}{p_1} = \frac{l}{1+l}\frac{1}{h_1}\frac{1+\dfrac{p_2 q_2 (h_{13}h_{32} + h_{12}h_3) + p_3 q_3 (h_{12}h_{23} + h_{13}h_2) - p_1 q_1 h_{23}h_{32}}{p_1 q_1 h_2 h_3}}{1-\dfrac{h_{12}h_{23}h_{31} + h_{13}h_{32}h_{21} + h_1 h_{23}h_{32} + h_2 h_{13}h_{31} + h_3 h_{12}h_{21}}{h_1 h_2 h_3}} \tag{8}$$

$$\frac{p_2 - c_0 - c_2}{p_2} = \frac{l}{1+l}\frac{1}{h_2}\frac{1+\dfrac{p_1 q_1 (h_{23}h_{31} + h_{21}h_3) + p_3 q_3 (h_{21}h_{13} + h_{23}h_1) - p_2 q_2 h_{13}h_{31}}{p_2 q_2 h_1 h_3}}{1-\dfrac{h_{12}h_{23}h_{31} + h_{13}h_{32}h_{21} + h_1 h_{23}h_{32} + h_2 h_{13}h_{31} + h_3 h_{12}h_{21}}{h_1 h_2 h_3}} \tag{9}$$

$$\frac{p_3 - c_0 - c_3}{p_3} = \frac{l}{1+l}\frac{1}{h_3}\frac{1+\dfrac{p_1 q_1 (h_{32}h_{21} + h_{31}h_2) + p_2 q_2 (h_{31}h_{12} + h_{32}h_1) - p_3 q_3 h_{12}h_{21}}{p_3 q_3 h_1 h_2}}{1-\dfrac{h_{12}h_{23}h_{31} + h_{13}h_{32}h_{21} + h_1 h_{23}h_{32} + h_2 h_{13}h_{31} + h_3 h_{12}h_{21}}{h_1 h_2 h_3}} \tag{10}$$

[Proof: Equation (7) is the standard inverse-elasticity rule. To derive (8), (9), and (10), obtain the first order conditions with respect to p_1, p_2, p_3. With $\frac{\partial V}{\partial q_i} = p_i$, we can rearrange them into the following simultaneous equations system:

$$\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_1} & \frac{\partial q_3}{\partial p_1} \\ \frac{\partial q_1}{\partial p_2} & \frac{\partial q_2}{\partial p_2} & \frac{\partial q_3}{\partial p_2} \\ \frac{\partial q_1}{\partial p_3} & \frac{\partial q_2}{\partial p_3} & \frac{\partial q_3}{\partial p_3} \end{bmatrix} \begin{bmatrix} p_1 - c_0 - c_1 \\ p_2 - c_0 - c_2 \\ p_3 - c_0 - c_3 \end{bmatrix} = -\frac{l}{1+l}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{11}$$

Then, we can solve p_i ($i = 1, 2, 3$) with Crammer' s rule. Expressing them in terms of elasticities (6), we have (8), (9), and (10). Q.E.D.]

The assumption of $\partial V / \partial q_i = p_i$ is the requirement of prices reflecting marginal social values of services.

These are the extension of what Laffont-Tirole(1996) obtained for the case of only one type of competitive access-users such as long-distance companies. For example, if $h_{i3} = h_{3j} = 0$ $(i(j) = 1,2)$, then equation (9) is reduced to:

$$\frac{\tilde{p}_2 - c_0 - c_2}{\tilde{p}_2} = \frac{l}{1+l}\frac{1}{h_2}\frac{1+\dfrac{p_1 q_1 h_{21}}{p_2 q_2 h_1}}{1-\dfrac{h_{12}h_{21}}{h_1 h_2}} \tag{12}$$

which is what they derived. It says that $\tilde{p}_2$ should be higher than the level according to the simple inverse elasticity rule, $l/(1+l)h_2$, since long-distance services of the incumbent and new entrants are substitutes, i.e., $h_{21} > 0$. That is, the increase in p_2 is more tolerable because the consequent reduction in q_2 is somewhat compensated by the accompanying increase in q_1.

Comparison between (9) and (12) reveals that the existence of another related service makes it necessary to consider many other interrelationships. First of all, the inclusion of h_{23} in the numerator of (9) reflects the effect of change in p_2 on demand for q_3. If $h_{23} > 0$, then p_2 should be even higher than the level according to (12). Moreover, the numerator of (9) includes the terms that show indirect effects, $h_{23}h_{31}$ and $h_{21}h_{13}$. The former shows the indirect effect of change in p_2 on demand for q_1 via the change in demand for q_3, while the latter shows the indirect effect of change in p_2 on demand for q_3 via the change in demand for q_1. When $h_{31} > 0$ and $h_{13} > 0$, these indirect effects reinforce the direct effects due to the relationship of substitutes. The following corollary summarizes the comparison between (9) and (12).

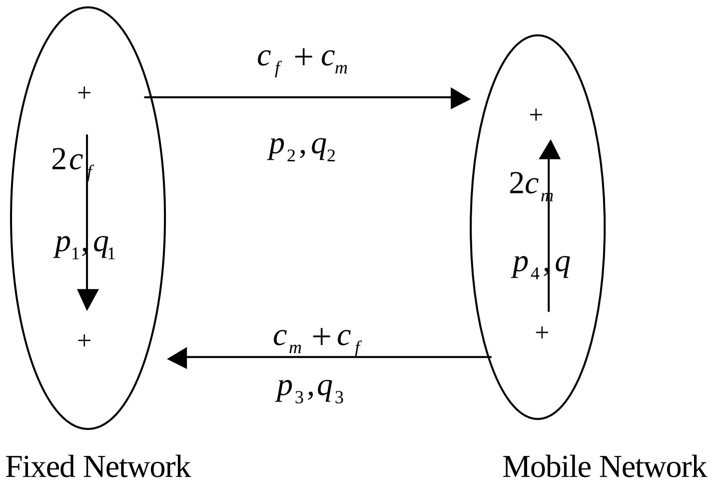
Corollary 1. Suppose $h_{ij} > 0$, $i = 1,2,3$, $j(\neq i) = 1,2,3$. Then, $p_2 > \tilde{p}_2$.

Given condition (3), the practical implication is that when long-distance and enhanced services become substitutes due to the advance of Internet telephony, the Ramsey principle requires higher access charges assessed on both services.

What if long-distance and enhanced services are complementary whatever the reason? Do we expect the reverse result with $p_2 < \tilde{p}_2$ if all of h_{23}, h_{32}, h_{13}, and h_{31} are negative? Formula (9) implies that it is not always the case. We have factors that induce $p_2 < \tilde{p}_2$: i.e., $h_{23} < 0$ and $h_{21}h_{13} < 0$ in the numerator of (9). However, there also exists a countervailing factor, $h_{23}h_{31} > 0$. This indirect effect captures the following chain of reactions: $p_2 \uparrow \Rightarrow q_3^D \downarrow \Rightarrow p_3 \downarrow \Rightarrow q_1^D \uparrow$. A chain of two complementary relationships results in substitute effect. This countervailing substitute effect may not be negligible if the incumbent's revenue of p_1q_1 is important. This exercise shows that the extension of Ramsey formulas to the case of two types of competitive access-users is not trivial; it is useful in understanding indirect effects clearly.

3. Ramsey Pricing in Two-way Interconnection

The following Figure 1 describes the structure of two-way interconnection between fixed and mobile networks. Let (p_1, q_1), (p_2, q_2), (p_3, q_3), and (p_4, q_4) represent prices and quantities for fixed-to-fixed, fixed-to-mobile, mobile-to-fixed, and mobile-to-mobile calls, respectively. Let c_f and c_m denote marginal operating costs of transmitting a call from fixed and mobile networks to switch, respectively.



<Figure 1>

Then, I specify demand and cost functions as follows:

$$q_i = q_i(p_1, p_2, p_3, p_4)\,, \qquad i = 1,2,3,4\,; \tag{13}$$

$$C_f = 2c_f q_1 + c_f(q_2 + q_3) + k_f\,, \qquad C_m = c_m(q_2 + q_3) + 2c_m q_4\,. \tag{14}$$

Demands are interrelated. I assume that only fixed network incurs fixed costs k_f.[4] Let a_f and a_m denote the access charge that one network assesses to the other network for completing a call. In case of off-net calls such as mobile-to-fixed calls q_3, mobile network's perceived marginal costs are $c_m + a_f$, even though the relevant social costs are $c_m + c_f$. I assume that the mobile service market is perfectly competitive, which implies:

$$p_3 = c_m + a_f\,, \qquad p_4 = 2c_m\,, \qquad a_m = c_m \tag{15}$$

The program that the regulator should solve for Ramsey optimal prices is:

$$\begin{aligned} &Max\;\; U(q_1, q_2, q_3, q_4) - 2c_f q_1 - (c_f + c_m)q_2 - (c_m + c_f)q_3 - 2c_m q_4 - k_f \\ &w.r.t.\;\;\; p_1, p_2, a_f \\ &s.t.\;\;\;\;\; p_1 q_1 + p_2 q_2 + a_f q_3 - 2c_f q_1 - (c_f + a_m)q_2 - c_f q_3 - k_f = 0 \end{aligned} \tag{16}$$

U is the social value for four interrelated call services. Plug $a_f = p_3 - c_m$ and $a_m = c_m$ in (15) into (16), and apply the Lagrangean method. Then, we have:

$$\begin{aligned} &Max\;\; U(q_1, q_2, q_3, q_4) + m(p_1 q_1 + p_2 q_2 + p_3 q_3) \\ &\qquad\qquad - (1 + m)\{2c_f q_1 + (c_f + c_m)q_2 + (c_m + c_f)q_3 - k_f\} - 2c_m q_4 \\ &w.r.t.\;\;\; p_1, p_2, p_3 \end{aligned} \tag{17}$$

Langrangean multiplier m reflects the social premium for the fixed-line network operator's revenues.

[4] I ignore fixed costs of mobile operators, which are relatively insignificant. Moreover, mobile operators are more flexible in recovering fixed costs by fixed monthly fees.

The Ramsey optimal solution is:

Proposition 2. Suppose $\frac{\partial U}{\partial q_i} = p_i\,(i = 1,2,3,4)$. Then,

$$\frac{p_1-2c_f}{p_1} = \frac{m}{1+m}\frac{1}{h_1}\frac{1+\frac{p_2q_2(h_{13}h_{32}+h_{12}h_3)+p_3q_3(h_{12}h_{23}+h_{13}h_2)-p_1q_1h_{23}h_{32}}{p_1q_1h_2h_3}}{1-\frac{h_{12}h_{23}h_{31}+h_{13}h_{32}h_{21}+h_1h_{23}h_{32}+h_2h_{13}h_{31}+h_3h_{12}h_{21}}{h_1h_2h_3}} \tag{18}$$

$$\frac{p_2-c_f-c_m}{p_2} = \frac{m}{1+m}\frac{1}{h_2}\frac{1+\frac{p_1q_1(h_{23}h_{31}+h_{21}h_3)+p_3q_3(h_{21}h_{13}+h_{23}h_1)-p_2q_2h_{13}h_{31}}{p_2q_2h_1h_3}}{1-\frac{h_{12}h_{23}h_{31}+h_{13}h_{32}h_{21}+h_1h_{23}h_{32}+h_2h_{13}h_{31}+h_3h_{12}h_{21}}{h_1h_2h_3}} \tag{19}$$

$$\frac{p_3-c_m-c_f}{p_3} = \frac{m}{1+m}\frac{1}{h_3}\frac{1+\frac{p_1q_1(h_{32}h_{21}+h_{31}h_2)+p_2q_2(h_{31}h_{12}+h_{32}h_1)-p_3q_3h_{12}h_{21}}{p_3q_3h_1h_2}}{1-\frac{h_{12}h_{23}h_{31}+h_{13}h_{32}h_{21}+h_1h_{23}h_{32}+h_2h_{13}h_{31}+h_3h_{12}h_{21}}{h_1h_2h_3}} \tag{20}$$

[Proof: Obtain the first-order conditions of program (17) with respect to p_1, p_2, p_3. With $\frac{\partial U}{\partial q_i} = p_i$, we can rearrange them into the following simultaneous equations system:

$$\begin{bmatrix} \frac{\partial q_1}{\partial p_1} & \frac{\partial q_2}{\partial p_1} & \frac{\partial q_3}{\partial p_1} \\ \frac{\partial q_1}{\partial p_2} & \frac{\partial q_2}{\partial p_2} & \frac{\partial q_3}{\partial p_2} \\ \frac{\partial q_1}{\partial p_3} & \frac{\partial q_2}{\partial p_3} & \frac{\partial q_3}{\partial p_3} \end{bmatrix} \begin{bmatrix} p_1-2c_f \\ p_2-c_f-c_m \\ p_3-c_m-c_f \end{bmatrix} = -\frac{m}{1+m}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} - \frac{1}{1+m}(p_4-2c_m)\begin{bmatrix} \frac{\partial q_4}{\partial p_1} \\ \frac{\partial q_4}{\partial p_2} \\ \frac{\partial q_4}{\partial p_3} \end{bmatrix} \tag{21}$$

Simplify equation (21) by substituting $p_4 = 2c_m$ in (15). Then, we can solve p_i ($i = 1$, 2, 3) with Crammer's rule. Expressing them in terms of elasticities (6), we have formulas (18), (19), and (20). Q.E.D.]

The results of Proposition 1 and 2 are formally equivalent. There exists a common factor in the two set-ups; the fixed local loop services are under regulated monopoly, while other services connected to the local loops, such as long-distance and enhanced services in the case of one-way interconnection, and mobile services in the case of two-way interconnection, are competitive. This common feature explains the isomorphism between the two results. However, considering the differences in the underlying contexts of one-way and two-way interconnections, the equivalence is notable at least.

The formulas in (18), (19), and (20) are very complicated. But, demand analyses may be useful in obtaining their implications. Jeon(2000) identified two kinds of substitutability between calls in fixed and mobile interconnection: *substitutability between calls **to** double subscribers* and *substitutability between calls **of** double* *subscribers*. Double subscribers refer to persons who have both fixed-link and mobile phones. Then, the former means that when people place calls to double subscribers, their calls *to* double subscribers' fixed-link phones and their calls *to* double subscribers' mobile phones are somewhat substitutable. On the other hand, the latter means that when double subscribers place calls to other people, fixed phone calls *of* double subscribers to others and mobile phone calls *of* double subscribers to others are somewhat substitutable. The former implies $h_{12} > 0$ and $h_{21} > 0$, while the latter implies $h_{13} > 0$ and $h_{31} > 0$. To obtain the ramifications on optimal prices of substitutability between calls to double subscribers, suppose the following simplification:

$$h_{12} > 0,\ h_{21} > 0, \qquad h_{13} = 0,\ h_{31} = 0, \qquad h_{23} = 0,\ h_{32} = 0.$$ [5]

Then, Proposition 2 implies:

$$\frac{p_2 - c_f - c_m}{p_2} = \frac{m}{1+m}\frac{1}{h_2}\frac{1+\dfrac{p_1 q_1 h_{21}}{p_2 q_2 h_1}}{1-\dfrac{h_{12}h_{21}}{h_1 h_2}}$$

$$\frac{p_3 - c_f - c_m}{p_3} = \frac{m}{1+m}\frac{1}{h_3}$$

We can implement the symmetric exercise to obtain the implication of substitutability between calls of double subscribers. The following corollary summarizes the discussion.

Corollary 2. Suppose $h_{12} > 0$, $h_{21} > 0$, $h_{13} = 0$, $h_{31} = 0$, $h_{23} = 0$, $h_{32} = 0$, and $h_2 = h_3$. Then, we have $p_2 > p_3$. On the other hand, suppose $h_{12} = 0$, $h_{21} = 0$, $h_{13} > 0$, $h_{31} > 0$, $h_{23} = 0$, $h_{32} = 0$, and $h_2 = h_3$. Then, we have $p_2 < p_3$.

This result may be read as follows. Controlling other factors that affect optimal prices, the price of fixed-to-mobile calls should be higher than the price of mobile-to-fixed calls when substitutability of calls to double subscribers is more prominent than substitutability of calls of double subscribers, and vice versa.

4. Conclusion

In deriving Ramsey optimal prices, the paper presumed the market configuration in which the fixed local services are supplied under regulated monopoly, while other services are provided competitively. Even though the current discussion of local competition is very active, most of local service markets are still under monopoly. Moreover, the current deregulatory trend into price caps does not make Ramsey prices irrelevant. In fact, direct implementation of Ramsey prices by regulatory authority is impossible, which is hinted by the complexity of the formulas in this paper. Ramsey pricing is essentially business oriented as noted by Laffont-Tirole(1999, p.63); "the

[5] Jeon(2001) shows that network externalities work to make all kinds of calls complementary. That is, network externalities countervail substitutability. I disregard the effect of network externalities in this discussion.

Ramsey-Boiteux prices are the same as those of an unregulated monopolist, just a notch down. We will therefore say that Ramsey-Boiteux prices are business oriented." So, as liberalization goes on, we may expect that the structure of Ramsey prices will approximate actual prices more closely. Finally, long-distance and mobile service markets may not be perfectly competitive. The Ramsey prices for those services should be adjusted to lower levels in order to offset the effect of monopoly power.

References

Jeon, S (2001) "Telephone Demands in Two-way Interconnected Networks" *Journal of Economic Theory and Econometrics* 7, 1-28.

Laffont, J., and J. Tirole (1996) "Creating Competition Through Interconnection: Theory and Practice," *Journal of Regulatory Economics*, 10:227-256.

Laffont, J., and J. Tirole (1999) *Competition in Telecommunications*, Cambridge, MA: MIT press.