E C O N O M I C S   B U L L E T I N

Skill gaps: existence and efficiency

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Abstract

Given the interdependences between human capital accumulation and technological change, skill gaps may arise in equilibrium. However, they are not necessarily inefficient, and in this paper we present a model in which the simple absence of such a skill gap can be inefficient.

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1. Introduction

Technological change cannot expand without sufficiently qualified workers and, at the same time, workers' productivity is not independent of their assimilated level of technology. Empirical analysis provides evidence on the influence of human capital and innovation on growth, as well as on the interdependence of these two phenomena. Nevertheless, the results depend on the set of countries considered, with the influence of these phenomena varying across economies, depending on some structural features. In this regard, see Mankiw, Romer and Weil (1992), Benhabib and Spiegel (1994), Nonneman and Vanhoudt (1996) and Murthy and Chien (1997), among others. The identification of the distinctive characteristics of the economies that determine the influence of each of these two activities on the rate at which the economy grows continues to be an open question.

One specific point is of particular interest, namely the question of skill gaps, understood as skill levels of the labour force that are below the frontier of the technological knowledge. In our view, there is merit in proposing models in which this problem can be analysed, and in this paper we present a model of endogenous technological change with human capital accumulation inspired in Lucas (1988) and Romer (1990), in which we introduce human capital obsolescence due to the expansion of knowledge in line with Zeng (1997).

In this model we determine the conditions in which such skill gaps appear and discuss the efficiency of this deficit of workers' skills with respect to the technological frontier. As main conclusions, we find that the skill gaps arise in the market equilibrium when innovation runs too fast and that they are not necessarily inefficient. In fact, for the case we consider the gap is never inefficient, and the simple absence of such a gap appears as inefficient.

2. The model

The economy is composed by individuals that offer inelastically one unit of time in the labour market each period, with the population normalised to one. The utility function $U = \int_0^\infty \ln(c_t) e^{-\rho t} dt$ captures the welfare provided by the stream of consumption c over time, with ρ being the intertemporal discount rate. We consider four productive sectors: the R&D sector, that give rise to innovations; the intermediate goods sector, that produces different varieties of capital goods; the final good sector; and the education sector.

Human capital accumulation. Let N be the range of the variety of intermediate goods, which measures the technical level reached by the society, and let $\tilde{N}$ be the level of the knowledge assimilated by workers (obviously, $\tilde{N} \leq N$). We assume that the new techniques are incorporated by firms even when workers have not incorporated the new knowledge. Therefore, we allow for a gap between the production techniques and the labour quality. Such a gap gives rise to losses in workers' productivity.

In order to increase their knowledge, workers devote a fraction u_E of their time to education, which takes place according to $\dot{\tilde{N}} = \delta_E u_E \tilde{N}^{1-\nu} N^{\nu}$, where δ_E is the productivity parameter and $\nu \in [0,1]$. Moreover, we assume that workers' human capital increases with the acquired knowledge with elasticity ε, $H = \tilde{N}^{\varepsilon}$, which implies that human capital accumulation follows:

$$\dot{H} = \delta_H \, u_E H^{\frac{\varepsilon-\nu}{\varepsilon}} N^{\nu}, \tag{1}$$

with $\delta_H = \varepsilon\delta_E$. As usual in human capital models, we assume $\delta_H > \rho$.

With productive processes changing over time, workers' human capital becomes a poor indicator of their productivity. As an alternative indicator, and in line with Zeng (1997), we define the effective human capital as $H_E = H/N^{\varepsilon}$, that is to say, the existing human capital related to its maximum value attainable given the available technology. With this formulation, the parameter ε can be interpreted as the degree of obsolescence that the expansion of knowledge induces on human capital.

R&D. Following Romer (1990) and introducing the effective human capital, the technology of the R&D sector is given by:

$$\dot{N} = \delta_N u_N \left(H / N^{\varepsilon}\right) N = \delta_N u_N H N^{1-\varepsilon}, \tag{2}$$

where δ_N is a productivity parameter and u_N is the fraction of time devoted to innovation. This equation shares with Jones (1995) a greater flexibility with respect to the effects of the stock of knowledge on its own expansion than is the case in Romer (1990), where the influence is considered as linear.

Intermediate and final goods. The technology of the final good sector is:

$$Y = (u_Y H / N^{\varepsilon})^{\alpha} \int_0^N x_i^{1-\alpha} di, \tag{3}$$

where u_Y is the fraction of time devoted to final good firms ($u_Y + u_N + u_E = 1$) and x_i is the amount of the i-th variety of capital. The symmetry in the use of each variety implies $x = x_i$. Each unit of every intermediate good is produced linearly from η units of final good, in such a way that physical capital can be defined as $K = \eta N x$. Taking this into account, the final good technology becomes:

$$Y = C + \dot{K} = (u_Y H / N^{\varepsilon})^{\alpha} N x^{1-\alpha} = \eta^{\alpha-1} (u_Y H)^{\alpha} N^{\alpha(1-\varepsilon)} K^{1-\alpha}. \tag{4}$$

For the sake of clarity, we will concentrate on the case $\alpha \geq \varepsilon + \nu$, corresponding to a direct influence of H_E on final output which is no lower than the sum of the degree of obsolescence the knowledge generates on human capital and its influence on education[1].

[1] In the reverse case, the same possibilities would arise but it is more complex to delimit the circumstances that lead to each of them.

3. Long-run effective human capital and the possibility of skill gaps

The steady state is characterised by a constant distribution of workers' labour supply among the different sectors and constant growth rates for output, consumption and the different stocks, which implies $g = g_N = g_H / \varepsilon$ (g_z denotes the rate of growth of variable *z*, whereas *g* stands for that of output, which equals that of consumption and physical capital).

Consumers maximise welfare subject to their budget constraint and the technology of human capital accumulation. Let *w* be the wage received for each unit of human capital hired by either final good firms or R&D firms, whereas *r* denotes the interest rate and *f* the individuals' financial wealth (in the form of bonds issued by firms). Then, consumers face the following problem:

$$\max_{\{c_t, u_{Et}\}} \int_0^{\infty} \ln(c_t) e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{f} = w(1 - u_E)H + rf - c,$$

$$\dot{H} = \delta_H u_E H^{\frac{\varepsilon - \nu}{\varepsilon}} N^{\nu},$$

$$H \le N^{\varepsilon}.$$

This problem can be summarised by the following Hamiltonian:

$$\mathrm{H}^{\mathrm{c}} = \ln c + q_f\left[w(1 - u_E)H + rf - c\right] + q_H \delta_H u_E H^{\frac{\varepsilon - \nu}{\varepsilon}} N^{\nu} + \lambda\left(N^{\varepsilon} - H\right),$$

where q_f and q_H are the shadow prices of financial wealth and human capital, respectively, and λ is the Lagrange multiplier associated to the constraint on human capital. The first order conditions of maximum are given by:

$$c^{-1} = q_f, \tag{5}$$

$$q_f w = q_H \delta_H H^{-\nu/\varepsilon} N^{\nu}, \tag{6}$$

$$\dot{q}_f = (\rho - r) q_f, \tag{7}$$

$$\dot{q}_H = \rho q_H - q_f w(1 - u_E) - q_H \varepsilon^{-1}(\varepsilon - \nu)\delta_H u_E H^{-\nu/\varepsilon} N^{\nu} + \lambda, \tag{8}$$

$$H \le N^{\varepsilon},\ \lambda \ge 0,\ \lambda\left(N^{\varepsilon} - H\right) = 0.$$

Let us consider the case in which the constraint on human capital does not bind, so that $\lambda = 0$ and $H_E < 1$. Conditions (5) and (7) imply that consumption grows according to $g = r - \rho$. Carrying (6) to (8) we deduce the dynamics of the shadow price of human capital. Then, taking time derivatives in (6) and making use of the previous results we obtain

$$g = \frac{\delta_H (1 - \nu u_E / \varepsilon) H_E^{-\nu/\varepsilon} - \rho}{\varepsilon}. \tag{9}$$

From this expression, together with the technology of education given in (1), we deduce that (with $H_E < 1$) the effective human capital is related with the growth rate in the following way:

$$H_E^{\nu/\varepsilon} = \frac{\delta_H}{(\varepsilon+\nu)g+\rho}. \quad (10)$$

On the other hand, from (4), the inverse demand functions for labour and for any variety of capital on the part of competitive firms in the final good sector are given by:

$$w_Y = \alpha\eta^{\alpha-1}(u_Y H)^{\alpha-1} N^{\alpha(1-\varepsilon)} K^{1-\alpha}, \quad (11)$$

$$p = (1-\alpha)(u_Y H)^{\alpha} N^{-\alpha\varepsilon} x^{-\alpha}, \quad (12)$$

where w_Y is the wage paid by the final good sector and p is the rental price of each unit of every variety of intermediate goods.

Given the inverse demand function (12), the intermediate goods firms maximise their profits by setting a rental price $p = (1-\alpha)^{-1} r\eta$, from which the monopolistic profit is given by $\pi = \alpha p x$. The competition to purchase the patents leads to a patent price P_N equal to the discounted future flow of monopolistic profits. With the profit being constant in steady state, this implies $P_N = \pi / r$.

The R&D firms pay to the human capital the competitive wage, $w_N = \delta_N P_N N^{1-\varepsilon}$, which after the substitution of the previous results can be written as $w_N = \delta_N \alpha(1-\alpha)(u_Y H)^{a} N^{1-\varepsilon-\alpha\varepsilon} x^{1-\alpha} / r$. This wage equals the final good sector wage in (11) in any equilibrium with innovation, what implies that $H_E = [(1-\alpha)\delta_N u_Y]^{-1}(g+\rho)$. From this expression, together with equation (10) and the R&D technology given in (2), we obtain, after some algebra, the following implicit equation that solves the steady state value of H_E:

$$G(H_E) = (2-\alpha)\delta_H H_E^{-\nu/\varepsilon} - (2-\alpha-\varepsilon-\nu)\rho - (1-\alpha)\nu\delta_N H_E - (1-\alpha)\varepsilon\delta_N \delta_H^{-1}\rho H_E^{(\varepsilon+\nu)/\varepsilon} = 0. \quad (13)$$

It can be shown that $G' < 0$, $\partial H_E / \partial\delta_H > 0$, $\partial H_E / \partial\delta_N < 0$ and $\partial H_E / \partial\rho < 0$; that is to say, G is a decreasing function and the effective human capital is increasing in the productivity of education but decreasing in that of innovation and in the discount rate. For H_E to be strictly less than one, the condition $G(1) < 0$ must hold, which is equivalent to

$$\delta_N > \delta_N^d = \frac{(2-\alpha)(\delta_H - \rho) + (\varepsilon+\nu)\rho}{(1-\alpha)\nu\delta_H + (1-\alpha)\varepsilon\rho}\delta_H. \quad (14)$$

When this is not the case ($\delta_N \le \delta_N^d$), the equilibrium is a corner solution in which $H_E = 1$. This means that, if the productivity of innovation δ_N is high enough relative to that of education, δ_H, then the expansion of knowledge will take place at a rate which is too fast to be assimilated by the workers, giving rise to a skill gap ($H_E < 1$). By contrast, when $\delta_N \le \delta_N^d$, the new knowledge is totally assimilated.

The question we can now pose is whether or not the skill gap implied by the case in which $H_E < 1$ is reflecting an inefficient allocation of resources. In order to answer the question, in the next section we solve the centralised solution and compare it with the market equilibrium.

4. Skill gaps and efficiency

A benevolent central planner faces the problem:

$$\max_{\{c_t, u_{Yt}, u_{Et}\}} \int_0^{\infty} \ln(c_t) e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{K} = \eta^{\alpha-1}(u_Y H)^{\alpha} N^{\alpha(1-\varepsilon)} K^{1-\alpha} - c,$$

$$\dot{N} = \delta_N (1 - u_Y - u_E) H N^{1-\varepsilon}$$

$$\dot{H} = \delta_H u_E H^{\frac{\varepsilon-\nu}{\varepsilon}} N^{\nu},$$

$$H \le N^{\varepsilon},$$

which can be summarised by the following Hamiltonian:

$$\mathrm{H}^{\mathrm{p}} = \ln c + \theta_K \left[\eta^{\alpha-1}(u_Y H)^{\alpha} N^{\alpha(1-\varepsilon)} K^{1-\alpha} - c\right] + \theta_N \delta_N (1 - u_Y - u_E) H N^{1-\varepsilon} +$$
$$+ \theta_H \delta_H u_E H^{\frac{\varepsilon-\nu}{\varepsilon}} N^{\nu} + \lambda\left(N^{\varepsilon} - H\right),$$

where θ_K, θ_N and θ_H are the shadow prices of physical capital, knowledge and human capital, respectively, and λ is again the Lagrange multiplier associated to the constraint on human capital. The optimal solution verifies the following first order conditions:

$$c^{-1} = \theta_K, \tag{15}$$

$$\theta_K \alpha Y / u_Y = \theta_N \delta_N H N^{1-\varepsilon} = \theta_H \delta_H H^{\frac{\varepsilon-\nu}{\varepsilon}} N^{\nu}, \tag{16}$$

$$\dot{\theta}_K = [\rho - (1-\alpha) Y / K]\theta_K, \tag{17}$$

$$\begin{aligned} \dot{\theta}_N = {} & \rho\theta_N - \theta_K \alpha (1-\varepsilon) Y / N - \theta_N (1-\varepsilon)\delta_N (1 - u_Y - u_E) H N^{-\varepsilon} - \\ & - \theta_H \nu \delta_H u_E H^{\frac{\varepsilon-\nu}{\varepsilon}} N^{\nu-1} - \lambda \varepsilon N^{\varepsilon-1}, \end{aligned} \tag{18}$$

$$\dot{\theta}_H = \rho\theta_H - \theta_K \alpha Y / H - \theta_N \delta_N (1 - u_Y - u_E) N^{1-\varepsilon} - \theta_H \varepsilon^{-1}(\varepsilon - \nu)\delta_H u_E H^{-\nu/\varepsilon} N^{\nu} + \lambda, \tag{19}$$

$$H \le N^{\varepsilon},\ \lambda \ge 0,\ \lambda\left(N^{\varepsilon} - H\right) = 0.$$

Let us assume first, as in the decentralised problem, that $H_E < 1$, so that $\lambda = 0$. Taking into account the relationships between the different growth rates in steady state, the two former equations imply that the stocks K, N and H increase over time at the same rate at which their respective shadow prices decrease. Making use of this result, and substituting (16) in (18) we have:

$$H_E = \frac{g+\rho}{\delta_N\left[1-\varepsilon+(\varepsilon+\nu-1)u_E\right]}, \tag{20}$$

whereas carrying (16) to (19) we obtain again equation (9). Thus, the expression (10), which results from (9) and the technology of education, also holds in the centralised solution. Then, the system of equations given by (10) and (20) allows us to obtain the optimal long-run value of H_E as the solution to the following (again implicit) equation:

$$G^p(H_E) = \delta_H H_E^{-\nu/\varepsilon} - (1-\varepsilon-\nu)\rho - \nu\delta_N H_E - (1-\nu-\varepsilon)\varepsilon\delta_N\delta_H^{-1}\rho H_E^{(\varepsilon+\nu)/\varepsilon} = 0, \tag{21}$$

with the same properties as G in the market equilibrium. The condition that now ensures that $H_E < 1$ is $G^p(1) < 0$ or, which is the same,

$$\delta_N > \delta_N^p = \frac{\delta_H - \rho + (\varepsilon+\nu)\rho}{\nu\delta_H + (1-\varepsilon-\nu)\varepsilon\rho}\delta_H. \tag{22}$$

Therefore, when this condition holds, the skill gaps are efficient. By contrast, when $\delta_N \leq \delta_N^p$, the optimal allocation involves an effective human capital equal to one and thus the existence of a skill gap is inefficient.

By comparing the critical values of the productivity of innovation in (14) and (22), it is easy to show that, under our assumptions about the parameters, δ_N^d is always above δ_N^p. As a consequence, whenever $\delta_N > \delta_N^d$, both the centralised and the decentralised solutions involve a skill gap; in other words, the existence of such a skill gap in the market allocation is not in itself an indicator of inefficiency. Whether the size of the gap is above or below the optimal is a separate question.

The most surprising result arises when $\delta_N^d > \delta_N > \delta_N^p$. Contrary to what could be expected, in this case the centralised allocation involves a skill gap, but the market equilibrium does not. As a result, the absence of the skill gap appears as an indicator of inefficiency.

Finally, when $\delta_N^p > \delta_N$ the workers assimilate all the knowledge in the market long-run equilibrium, with this absence of a skill gap also being a characteristic of the optimal allocation.

Why could it be optimal to maintain the knowledge assimilated by workers below the technological frontier, when the market allocation leads to fully updated workers? This seems a paradox, since the presence of skill gaps entails a social cost in that the lack of knowledge on the part of workers limits the exploitation of the productivity gains derived from the technical progress. On the other hand, the full updating of the individuals' knowledge is also costly, because it requires the corresponding investment in education. Only in the circumstances where the productivity of education is high enough (relative to that of innovation, as we have seen) can it be efficient to devote sufficient time to education so as to assimilate all the new knowledge.

Moreover, given that innovation involves important external effects (both positive and negative), and that the intermediate goods market is not competitive, the market allocation may involve too many or too few incentives for innovation. This last situation arises when $\varepsilon \le \alpha$ and, as a result of the slow innovation, the investment in education by workers might be sufficiently large so as to acquire all the new knowledge, although this feature is not shared by the optimal allocation.

If we take the efficiency analysis a stage further, a new question arises: when the skill gap is a feature of the market equilibrium and its existence is not inefficient (i.e., when $\delta_N > \delta_N^d$), what can we say about whether the efficiency requires workers' human capital to be closer to, or further away from, the knowledge frontier? Let us compare equations (13) and (21). Under our assumptions about the parameters, we have that

$$G(H_E) - G^p(H_E) = (1-\alpha)(\delta_H H_E^{-\nu/\varepsilon} - \rho) + \alpha\nu\delta_N H_E - (\nu + \varepsilon - \alpha)\varepsilon\delta_N\delta_H^{-1}\rho H_E^{(\varepsilon+\nu)/\varepsilon} > 0,$$

provided that $H_E < 1$. As a result, the effective human capital in the market allocation is above the optimal one: $H_E > H_E^p$. This means that the skill gap in the market equilibrium is lower than its efficient level, a situation which we might refer to as "over-education". Given that the inverse relationship between the growth rate and the effective human capital in (10) also holds in the optimal solution, this implies that the market growth rate is below the optimal. This is due to a low investment in the sectors of human capital and knowledge accumulation, which drive the growth in output. In fact, it can be shown that the fractions of time devoted by the market allocation to education and innovation are both lower than the efficient. The low investment in R&D reduces the obsolescence of human capital and, despite the lower rate of human capital accumulation, allows human capital to be permanently closer to the technology frontier than the optimum in the long-run equilibrium[2].

5. Conclusions

In this paper we have illustrated the way in which skill gaps may appear in an economy with endogenous technological change. The key point is that some investment in human capital accumulation must accompany the expansion of knowledge in order to exploit the potential productivity gains. As we have shown, such skill gaps appear as a consequence of too rapid rate of innovation, induced by a productivity in the R&D sector which is too high with respect to that of education.

Nevertheless, the existence of a skill gap cannot be considered as an indicator of inefficiency. On the contrary, our model includes a case in which the inefficiency proceeds precisely from the absence of such a gap. The cause for this seemingly paradoxical result is a rate of innovation that is below the optimal, which implies a lower obsolescence of human capital and allows for a full acquisition of knowledge through the investment in education. This is in clear contrast with the higher optimal rate of innovation, which makes such an investment too costly and gives rise to a skill gap.

[2] Relaxing the assumption $\alpha \ge \varepsilon + \nu$, the opposite result may arise, with a skill gap in the market allocation higher than the optimal ("under-education").

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