E C O N O M I C S   B U L L E T I N

J curve for abatement with transboundary pollution

Begona Casino
Department of Economic Analysis, University of Valencia

Abstract

In this note an alternative framework to Selden and Song's (1995) work is proposed in order to clarify some of their results and verify whether the relationship between pollution abatement and national income can be drawn as a J curve when there is more than one country involved. As Selden and Song do, pollution is considered as a flow externality. Their model is extended to a two country case which requires the use of differential games. The optimisation problem is decomposed into two stages, the pollution abatement and the accumulation problem. A J curve for pollution abatement is replicated and a static comparative analysis confirms that the smaller the rate of discount and/or the less polluting a technology is, the higher the stock of capital, current expenditure and pollution abatement will be at the steady state.

I would like to thank David Ulph and Roger Salmons for originally suggesting the problem addressed in this note. This work has circulated as a part of a working paper of the "Instituto Valenciano de Investigaciones Economicas" WP−EC 99−20. My gratitude also extends to an anonymous reviewer for her/his comments. Usual caveats apply. Finantial support from the "Caja de Ahorros del Mediterraneo−British Council" is gratefully acknowledged.
Citation: Casino, Begona, (2002) "J curve for abatement with transboundary pollution." *Economics Bulletin,* Vol. 17, No. 2 pp. 1−9
Submitted: July 11, 2002. **Accepted:** September 11, 2002.
URL: http://www.economicsbulletin.com/2002/volume17/EB−02Q20002A.pdf

1 Introduction

Most of the theories on economic growth implicitly assume that if some wastes are produced by the economic process they can be disposed of at no cost to the society. Nevertheless, this assumption is not satisfied in the real world. Thus, any theory of optimal economic growth should take into account these spillover effects. The earliest work dealing with growth and pollution is Forster's (1973) model.

Empirical evidence seems to support the fact that environmental quality declines in the first stages of economic growth and it improves once economies reach a certain degree of development. This behavior might be explained by factors such as changes in the composition of production and consumption. That is, low income levels are usually related to a small industrial sector and, consequently, with low levels of pollution, while emissions of pollutants increase as industrial activities develop. Nevertheless, this trend may change once economies reach a high level of income and specialize in service sectors. Selden and Song (1995) derive this behavior of emissions making a simple change to Forster's model.

This note focuses on Selden and Song's paper and extends the analysis to a two-country case in order to verify whether the relationship between pollution abatement and national income can be drawn as a J curve when pollution is a transboundary problem. Moreover, the two-country model provides an appropriate setting to study the sources of asymmetries that could explain the observed differences among abatement efforts across countries. The following section states the model. In Section 3 a J curve for abatement is derived and Section 4 concludes this note.

2 The Model

The economic process produces goods and services that provide utility to consumers and wastes that generate certain disutility. Thus, social welfare depends positively on consumption and negatively on the damage caused by pollution.

Consider two countries, each one denoted by $i = 1, 2$, and assume that the flow of gross benefits provided by consumption is an increasing and concave function, $B_i\left(c_i(t)\right)$, and that marginal benefits tend to infinity as consumption approaches zero. Suppose also that pollution, $P_i(t)$, is more harmful as it increases. Thus the damage function, $D_i(P_i(t))$, will have the following properties: $D_i^{'}(P_i(t)) > 0, D_i^{''}(P_i(t)) > 0, \forall P_i(t) \geq 0$, with $D_i(0) = 0$.

Additionally, consider a production function that depends exclusively on capital and labor. Assuming, for simplicity, a time invariant production function and a stationary population, the production function can be defined in terms of capital per capita. Suppose then that production is an increasing and concave function of capital per capita, $F_i(k_i(t))$, with $F_i(0) = 0$.

Since pollution is regarded as a negative externality of production, gross emissions are positively related to the stock of capital, $g_i(t) = G_i(k_i(t))$. For simplicity, we will make the assumption that emissions per unit of output are constant, $g_i(t) = G_i(k_i(t)) = \theta_i F_i(k_i(t))$, with $\theta_i > 0$.[1]

Moreover, suppose that cleaning up activities, defined by the function $R_i(a_i(t))$, become less efficient as abatement expenditures increase, so that $R^{'}(a_i(t)) > 0$, $R_i^{''}(a_i(t)) < 0, \forall a_i(t) \geq 0$ with $R_i(0) = 0$. Additionally, following Selden and Song, assume that $\lim_{a_i \to 0} R_i^{'}(a_i(t)) = \gamma < +\infty$. This assumption incorporates corner solutions to Forster's model.

Assume also that net emissions, $P_i(t)$, are additively separable into gross emissions, $g_i(t)$, minus the reduction in pollution that depends on the amount of abatement expenditures, $a_i(t)$, and ignore (without loss of generality) the fact that the natural environment absorbs certain amount of pollution. That is, $P_i(k_i(t), a_i(t)) = g_i(t) - R_i(a_i(t))$.

In most cases pollution is a global or transboundary problem involving interactions between countries. In the acid rain case, for instance, winds transport a percentage, σ_i, of pollutants emitted by each country, P_i, to its neighbor. Thus, the flow of emissions in country i will be $\psi_i[g_i - R_i(a_i)] + \sigma_i[g_j - R_j(a_j)]$, where $\psi_i = 1 - \sigma_i$.[2] Hence, the marginal abatement cost for each country depends on its own abatement level, while each country's marginal abatement benefit depends on world-wide abatement.

Finally suppose that preferences, technologies and natural environments are exactly the same in both countries.

[1] This assumption is the unique change regarding Selden and Song's (1995) model and it does not affect the results concerning the J curve for abatement. In their paper Selden and Song assume that $G_i^{''}(k_i(t)) \geq 0$.

[2] Subscripts referring time have been omitted to simplify notation.

3 The J curve for abatement

Consider that countries do not cooperate and each government chooses time paths for abatement and capital in order to maximize the present discounted value of welfare for an infinite time horizon, taken the time path of the other country net emissions, $P_j = g_j - R(a_j)$, as given.[3]

Alternatively, the optimization problem can be decomposed into two states. The first one is a static problem that establishes the optimal allocation rule to consumption or abatement at each period. The second one is a dynamic problem that determines the optimal level of current expenditure, $e_i = c_i + a_i$.

In order to choose the optimal expenditures on abatement, each country solves the following problem:

$$\max_{e_i \geq a_i \geq 0} u(a_i) = B_i(e_i - a_i) - D_i[\psi_i(g_i - R_i(a_i)) + \sigma_i P_j]$$

Lemma 1 *An interior solution, $a_i^* = a_i(e_i, g_i, P_j)$, to the abatement problem of country $i = 1, 2$ has the following properties: $a_{ie_i} = \frac{-B_i^{''}}{\Delta} > 0$, $a_{ig_i} = \frac{\psi_i^2 R_i^{'} D_i^{''}}{\Delta} > 0$, $a_{iP_j} = \frac{\sigma_i \psi_i R_i^{'} D_i^{''}}{\Delta} > 0$, $da_i/da_j = -R_j^{'} da_i/dP_j < 0$, where $\Delta = \psi_i \left\{ \psi_i \left(R_i^{'} \right)^2 D_i^{''} - R_i^{''} D_i^{'} \right\} - B_i^{''} > 0$ and subscripts denote partial derivatives. Moreover, the marginal willingness to pay for abating pollution can be defined as: $m_i(e_i, g_i) = \frac{1}{R_i^{'}[a_i(e_i, g_i, P_j)]} > 0$.*

The necessary and sufficient conditions that the amount of abatement expenditures must satisfy to maximize welfare are:

$$B_i^{'}(e_i - a_i) \geq \psi_i D_i^{'}[\psi_i(g_i - R_i(a_i)) + \sigma_i P_j] R_i^{'}(a_i), a_i \geq 0, \forall i = 1, 2. \quad (1)$$

This condition implicitly defines the optimal value of expenditures on abatement. For $a_i > 0$, condition (1) holds with equality and, by applying the theorem of the implicit function, it is easy to show that each country's

[3] The most commonly employed control variables in differential games are open-loop and feedback strategies. The former are optimal for the original game as specified by its initial condition, while, the latter are optimal for every subgame evolving from it. Nevertheless, since feedback strategies are more difficult to characterize, this note focuses on open-loop strategies.

optimal level of abatement, a_i, increases with its current expenditure, e_i, gross emissions, g_i, and the other country's net emissions, P_j, while it decreases with the other country's level of abatement, a_j; that is, each country has incentives to free ride on the abatement efforts of its neighbor.

The indirect utility function, defined as $u_i(a_i^*) = v_i(e_i, g_i)$, has the following properties: $v_{ie_i} = B_i' > 0$, $v_{ig_i} = -\psi_i D_i' < 0$. Then, the marginal willingness to pay for abating pollution can be defined as $de_i/dg_i = -v_{g_i}/v_{e_i} = 1/R_i'[a_i]$.

Proposition 1 *It is optimal to spend no resources on abating emissions for small levels of current expenditure. However, there is a critical value, $e_i = E_i(g_i, P_j)$, from which welfare improves by allocating resources to abatement. Then, the marginal willingness to pay for a reduction in damage is an increasing function of current expenditure and gross emissions.*

Define $e_i = E_i(g_i)$ by $B_i'(e_i) = \psi_i D_i'(\psi_i g_i + \sigma_i P_j) R_i'(0)$. This condition specifies the boundary of a range of values of gross emissions and current expenditure for which the solution to the above optimization problem is $a_i^* = a_i(e_i, g_i) = 0$. As far as marginal damage is an increasing function in gross emissions and marginal benefits decrease as consumption increases, $E_i(g_i)$ is a strictly decreasing function in gross emissions or, equivalently, in capital per capita. In the region below and on this function abatement expenditures are equal to zero, otherwise, $a_i(e_i, g_i) > 0$.

For $e_i > E_i(g_i)$ we have that $m_{ie_i} = -(1/R_i') R_i'' a_{ie_i} > 0$ and $m_{ig_i} = -(1/R_i') R_i'' a_{ig_i} > 0$.

Proposition 2 *Current expenditure and capital increase until reaching their steady state values.*

Once the optimal level of abatement has been chosen, the control variables of the accumulation problem reduce to one, so that each government has to solve the following problem:

$$\max_{e_i > 0} \int_0^\infty e^{-\rho_i t} v_i(e_i, \theta_i F(k_i))\, dt,$$

$$s.t. \quad \dot{k} = F(k_i) - \delta_i k_i - e_i, \; k_i(0) = k_0 \geq 0,$$

where ρ_i denotes the discount rate and δ_i the depreciation rate of capital per capita.

The solution to the accumulation problem must satisfy the following conditions:

$$v_{ie_i} = B_i' [e_i - a_i (e_i, g_i, P_j)] = \mu_i, \quad (2)$$

$$F_i' (k_i) [1 - \theta_i m_i (e_i, g_i, P_j)] = \rho_i + \delta_i - \frac{\dot{\mu}_i}{\mu_i}, \quad (3)$$

$$\dot{k}_i = F_i (k_i) - \delta_i k_i - e_i. \quad (4)$$

Thus, the steady state levels of current expenditure and capital are given by the following expressions:[4]

$$F_i' (k_i^*) [1 - \theta_i m_i (e_i^*, g_i^*, P_j)] = \rho_i + \delta_i \quad (5)$$

$$e_i^* = F_i (k_i^*) - \delta_i k_i^*, \quad (6)$$

Regarding the transition to the steady state, the solution to the system (3)-(4) defines the time paths for the costate and state variables. Alternatively, the system can be defined in terms of the state and control variables. Differentiating (2) with respect to time and taking into account (3) we have:

$$\dot{e}_i = -\frac{v_{ie_i}}{v_{ie_ie_i}} \left\{ F_i' \left[\theta_i \frac{v_{ie_ig_i}}{v_{ie_i}} (F_i (k_i) - \delta_i k_i - e_i) + 1 - \theta_i m_i \right] - \rho_i - \delta_i \right\} \quad (7)$$

The local stability of the steady state can be inferred from the Jacobian matrix of the non-linear system evaluated at the steady state:

$$|J| = \frac{v_{e_i}}{v_{e_ie_i}} \left\{ \theta_i F_i' (k_i^*) [F_i' (k_i^*) - \delta] m_{ie_i} - \left[F_i'' (k_i^*) (1 - \theta_i m_i) - (\theta_i F_i' (k_i^*))^2 m_{ig_i} \right] \right\}$$

where $v_{e_ie_i} = \left(1 - a_{ie_i}^*\right) B_i'' (e_i^* - a_i (e_i^*, g_i^*)) < 0$ and, according to (5), $1 - \theta_i m_i > 0$ and $F_i' (k_i^*) - \delta_i > 0$. Then, $|J| < 0$ and the steady state is locally stable. It is easy to see from (4) and (7) that the stable path has a positive slope. (See Fig. 1).

An example of the acid rain problem in Europe is the case of Finland and the former Soviet Union where, as Kaitala *et al. (1992)* show, there

[4] The steady state of this exogenous growth model is characterized by a constant level of capital per capita so that $\dot{k} = 0$ and $\dot{\mu} = 0$.

are important asymmetries between the two affected areas concerning the coefficients of transportation, damage and abatement functions, etc. In this note the effect of differences in the discount rate and the technology of production on the steady state value of capital per capita, current expenditure and abatement is analyzed.

Proposition 3 *The country with the smallest temporal rate of discount and/or the less polluting technology will be characterized by the highest steady state levels of capital per capita, current expenditure and abatement.*

Changes in the rate of discount, ρ_i, and/or the technology used to produce final goods and capital, defined by the parameter θ_i, will affect the steady state levels of capital and current expenditure.

By differentiating (5) we get $dk_i^*/d\rho_i = 1/\Pi$ and $dk_i^*/d\theta_i = (1/\Pi)\, F_i^{'} m_i$, where $\Pi = F_i^{''}\,[1-\theta_i m_i] - \theta_i F_i^{'}\left[m_{ie_i}\,(de_i/dk_i) + m_{ig_i}\,(dg_i/dk_i) + m_{iP_j}\,(dP_j/dk_i)\right]$, $m_{ie_i} = -\left(1/\left(R_i^{'}\right)^2\right) R_i^{''} a_{ie_i} > 0$, $m_{ig_i} = -\left(1/\left(R_i^{'}\right)^2\right) R_i^{''} a_{ig_i} > 0$, $m_{iP_j} = -\left(\sigma_i/\psi_i\left(R_i^{'}\right)^2\right) a_{ig_i} R_i^{''} > 0$. Moreover, $de_i/dk_i = F_i^{'} - \delta_i > 0$, $dg_i/dk_i = \theta_i F_i^{'} > 0$ and $dP_j/dk_i = -\left(\sigma_j/\psi_j\right) a_{jg_j} R_j^{'}\left[1 - a_{ig_i} R_i^{'}\right] dg_i/dk_i < 0$.[5]

The first term of Π is clearly negative. Regarding the second term, on the one hand, an increase in the stock of capital increases the marginal willingness to pay for abating pollution by increasing current expenditure and gross emissions but, on the other hand, it reduces the marginal willingness to pay for abating pollution by reducing pollution in the other country. That is, as the reaction functions for abatement expenditures have a negative slope, a higher steady state stock of capital for country i will increase the level of abatement expenditures of country j and, as a result, net pollution P_j will decrease. Notice, however that, $m_{iP_j} = (\sigma_i/\psi_i)\, m_{ig_i} < m_{ig_i}$ (if we make the reasonable assumption that $\psi_i > \sigma_i$) and $|dP_j/dk_i| < |dg_i/dk_i|$ because $0 < \sigma_j/\psi_j < 1$, $0 < a_{jg_j} R_j^{'} < 1$ and $0 < 1 - a_{ig_i} R_i^{'} < 1$. Thus, $\Pi < 0$ and $dk_i^*/d\rho_i < 0$, $dk_i^*/d\theta_i < 0$. Graphically, the locus (6) does not depend on parameters ρ_i and θ_i while the locus (5) shifts to the right as either ρ_i or θ_i decreases.

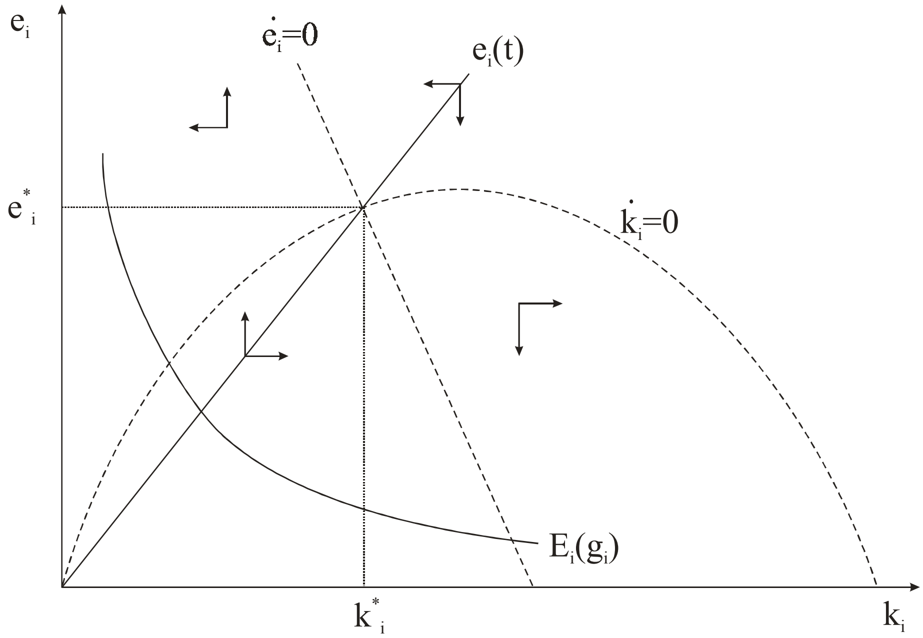
Countries that care about future generations have a low rate of discount so that, as they are patient, they will reach a steady state characterized by

[5] Notice that $0 < 1 - R_i^{'}\,(.)\, a_{ig_i} < 1$ for $i = 1, 2$.

a higher level of capital and current expenditure. Thus, since expenditures on abatement are positively related to capital and current expenditure, the steady state level of abatement will also be higher. Similarly, the optimal steady state stock of capital is higher for countries that use cleaner technologies. Since the amount of gross emissions generated from each unit of capital is smaller, it is possible to reach the optimal level of pollution with a higher stock of capital per capita. Furthermore, a higher stock of capital per capita will be related with higher steady state levels of current expenditure and abatement.

4 Conclusions

It has been shown that for low levels of current expenditure and capital it is optimal to care exclusively about economic growth and postpone cleaning activities. However, once a certain level of income is reached, welfare increases by reallocating resources to pollution control. Expenditures on abatement have been proved to be an increasing function of gross emissions and current expenditure. Moreover, current expenditure and capital increase along the time path to the steady state and, consequently, abatement expenditures also increase with income. Therefore, a J curve for abatement has been replicated. A static comparative analysis has confirmed that the smaller the rate of discount and/or the less polluting a technology is, the higher the steady state stock of capital, current expenditure and abatement will be.



Steady state and stable path.

References

[1] Forster, B.A. (1973). "Optimal Capital Accumulation in a Polluted Environment", *Southern Economic Journal*, **39**: 544-547.

[2] Kaitala, V., Pohjola, M. and Tahvonen, O. (1992). "An Economic Analysis of Transboundary Air Pollution between Finland and the Former Soviet Union, *Scandinavian Journal of Economics* **94(3)**: 409-424.

[3] Selden, T.M. and Song D. (1995). "Neoclassical Growth, the J Curve for Abatement, and the Inverted U Curve for Pollution", *Journal of Environmental Economics and Management*, **29**: 162-168.