

## Testing Growth Ratios via Pooled Error Correction Models

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### *Abstract*

The balanced growth and stochastic growth theory implies stable investment–output and consumption–output ratios. Our analysis supports cointegration between investment and output (IO–model) as well as consumption and output (CO–model) for single countries. Pooling the data we find cointegration and that output is weakly exogenous in the IO–specification. Weak exogeneity of output is not confirmed for the CO–specification on the pooled level. The growth ratio restrictions, however, are rejected. A small simulation study investigates the empirical properties of the employed panel tests in finite samples. It is found that the adopted bootstrap approach to determine critical values for common test statistics outperform the corresponding approximation.

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# 1 Introduction

The stochastic version of the balanced growth theory implies that the ratios between consumption and output (CO-ratio) as well as investment and output (IO-ratio) are stationary. For the Group of Seven (G7) Harvey, Leybourne and Newbold (1998) conduct unit root tests with fixed mean for each relation and cannot reject the unit root hypothesis in almost every case for both ratios. Similar results are obtained for the CO-ratio. In contrast, considering a smooth transition in mean they find for some countries (France, Italy, UK) evidence against a unit root in the CO-ratio.

As an alternative to discussing the empirical properties of growth ratios this study focuses on the dynamics of the underlying variables. Firstly, adopting a time series approach the stability of growth ratios implies cointegration between the involved variables and a particular cointegrating vector. These restrictions are tested within a cointegration framework. Secondly, the low power problem of cointegration tests is well known for small samples. To remedy this shortcoming the data is pooled to extend the information base. The adopted procedure, recently introduced by Herwartz and Neumann (2000) and applied by Herwartz and Reimers (2002) to test the purchasing power parity, allows for country specific transitory dynamics and to test growth ratio restrictions on the pooled level. It takes heteroskedasticity of error terms and cross sectional error correlation into account.

## 2 The theoretical background

Within the basic neoclassical growth theory of an one-sector economy due to Solow (1956), output ( $Y_t$ ) is generated from a constant returns to scale Cobb–Douglas production function

$$Y_t = \lambda_t K_t^{1-\gamma} L_t^\gamma, \tag{1}$$

where  $K_t$  and  $L_t$  denote the capital stock and labour input, respectively, and  $\lambda_t$  is the total factor productivity. The assumption of constant returns to scale allows to employ the production function in intensive form or in the corresponding per capita version. Considering a deterministic trend results in steady state growth if some assumptions concerning preferences, capital accumulation and resource constraints hold. To form the neoclassical growth model under uncertainty the total factor productivity is characterized as a random walk process with deterministic drift term (Brock and Mirman 1972, Donaldson and Mehra 1983). Under uncertainty the neoclassical model results in balanced growth implying a common growth rate. Since factor productivity has a random walk with drift representation the logarithms of output, consumption and investment are integrated of order one and share a common stochastic trend (King, Plosser, Stock and Watson 1991). The logarithmic growth ratios  $c_t - y_t$  and  $i_t - y_t$  follow stationary stochastic processes. The variables  $c_t$  and  $y_t$  ( $i_t$  and  $y_t$ ) are cointegrated. The growth ratios imply a linear restriction that can be tested within

a cointegration framework. If  $c_t + \beta_1 y_t$  and  $i_t + \beta_2 y_t$  are stationary the two hypotheses of interest are  $H_0 : \beta_1 = -1$  and  $H_0 : \beta_2 = -1$ , respectively.

### 3 Methodology

Assuming  $y_t$  to be weakly exogenous the conditional error correction model (ECM) for consumption reads as follows in the CO-model:

$$\Delta c_t = \nu_{11} + \alpha_{11}(c_{t-1} + \beta_1 y_{t-1}) + \gamma_{11} \Delta y_t + u_{1t}, t = 1, \dots, T. \quad (2)$$

Weak exogeneity of  $y_t$  implies that within the corresponding marginal process,

$$\Delta y_t = \nu_{12} + \alpha_{12}(c_{t-1} + \beta_1 y_{t-1}) + v_{1t}, \quad (3)$$

the error adjustment coefficient  $\alpha_{12}$  is zero.

The adopted single equation approach to infer on long-run equilibrium relations is asymptotically efficient if a set of assumptions can be made (Banerjee, Dolado, Galbraith and Hendry 1993, Chapter 6, Herwartz and Neumann 2000). Apart from weak exogeneity of  $y_t$  efficient inference within equations like (2) or (3) requires that both involved variables are nonstationary and integrated of order one, i.e. first differences  $\Delta y_t$  and  $\Delta c_t$  are stationary processes. In addition,  $c_t$  and  $y_t$  have to be cointegrated implying that there exists a linear combination of both variables which provides stationary residuals. Finally  $u_{1t}$  and  $v_{1t}$  are assumed to be serially uncorrelated error processes, and, moreover  $E[u_{1t}v_{1t}] = 0$ . Due to the latter assumptions it might be necessary to augment equations (2) and (3) conveniently with further stationary lagged variables. Presample values are assumed to be available throughout.

Analogously to the CO-model in (2) and (3) the IO-specification reads as follows:

$$\Delta i_t = \nu_{21} + \alpha_{21}(i_{t-1} + \beta_2 y_{t-1}) + \gamma_{21} \Delta y_t + u_{2t}, t = 1, \dots, T. \quad (4)$$

and

$$\Delta y_t = \nu_{22} + \alpha_{22}(i_{t-1} + \beta_2 y_{t-1}) + v_{2t}. \quad (5)$$

A test on significance of  $\alpha_{i1}$ ,  $i = 1, 2$ , is implicitly a test of the null hypothesis of cointegration. Under the alternative of no cointegration the  $t$ -ratio of  $\hat{\alpha}_{i1}$  ( $t_{\hat{\alpha}_{i1}}$ ) fails to be asymptotically normally distributed. For this case Kremers, Ericsson and Dolado (1992) show that the distribution of this statistic is somewhere between the standard normal distribution and the distribution of the Dickey-Fuller  $t$ -statistic. Similarly, a suitable hypothesis to test for weak exogeneity of  $y_t$  is  $H_0 : \alpha_{i2} = 0$ ,  $i = 1, 2$ .

In this study we concentrate on likelihood ratio tests (LR-tests). The LR-statistics derived from the single equation ECM are typically represented as  $T$  times the log ratio of the

sum of residual squared errors, estimated via OLS under a particular null hypothesis and its alternative. With  $RSS_0$  and  $RSS_1$  denoting these estimates the LR-statistic is

$$LR_n = T \ln \left( \frac{RSS_0}{RSS_1} \right). \quad (6)$$

For some testing problems  $LR_n$  is asymptotically  $\chi^2(q)$  distributed, with  $q$  denoting the number of excess parameters under the alternative hypothesis. To obtain an asymptotic  $\chi^2$ -distribution of the LR-test on weak exogeneity it is essential, however, to assume a homoskedastic error distribution. If this assumption is violated, the LR-statistic loses its pivotal property. In Herwartz and Neumann (2000) it is shown that in the case of heteroskedasticity the wild bootstrap, introduced by Wu (1986), is a convenient means to obtain critical values for LR-statistics.

Complementary to single equation analysis it is also of interest to test the economic model on the level of pooled economies. Increasing sample information is appealing to improve the empirical properties of common test procedures as e.g. LR-tests. To be more precise on the issue of pooling consider now a set of empirical models as in (2), i.e.

$$\Delta c_{nt} = \nu_{11n} + \alpha_{11n}(c_{nt-1} + \beta_{1n}y_{nt-1}) + \gamma_{11n}\Delta y_{nt} + u_{1nt}, \quad n = 1, \dots, N, \quad (7)$$

where  $N$  denotes the number of equations in the system. Assuming the error terms of the  $N$  equations to be contemporaneously uncorrelated a convenient generalization of the statistic given in (6) is

$$LR_N = \sum_{n=1}^N LR_n = T \sum_{n=1}^N \ln \left( \frac{RSS_{0n}}{RSS_{1n}} \right). \quad (8)$$

In (8) we implicitly assume that  $T$  observations are available for each equation. Note that  $LR_N$  is easily modified if this assumption is violated.

Herwartz and Neumann (2000) show that the wild bootstrap is convenient to obtain critical values for this statistic under typical stylized facts of empirical processes. Apart from heteroskedasticity the error terms are allowed to exhibit cross equation correlation. In the next section a brief investigation of the small sample properties of generating critical values for  $LR_N$  by means of the wild bootstrap will reveal that this approach reduces considerably size distortions involved when applying critical values from the  $\chi^2(qN)$ -distribution in small samples.

## 4 Empirical analysis

We test restrictions implied by the neoclassical growth model for the G7 economies, namely Canada (CA), Germany (GE), France (FR), Italy (IT), Japan (JA), United Kingdom (UK) and United States of America (US). For each country real gross domestic product

(Y), real consumption expenditure (C) and real gross investment (I) are investigated. Regression specific sample periods and available observations are given in Table 1. All series are transformed to logarithms of per capita quantities. We thank the Deutsche Bundesbank for providing the data. Except for German data all variables are seasonally adjusted.

Unit root tests are the starting point of the empirical analysis to check necessary conditions for cointegration.\* The unit root hypothesis cannot be rejected for the majority of the investigated time series measured in levels. First differences of almost all variables are found to be stationary. Before we turn to an investigation of the neoclassical growth model on the pooled level we discuss some results obtained for conditional single equation models as (2) and (4) augmented with marginal processes (3) and (5).

As mentioned efficient inference on long-run parameters  $\beta_{in}$ ,  $i = 1, 2$ , requires the marginal variables to be weakly exogenous. Estimation and inference results for  $\alpha_{i2n}$ ,  $i = 1, 2$ , are depicted in Table 1. Regarding estimated  $t$ -ratios of the adjustment coefficients we obtain for the CO-model (IO-model) evidence in favor of weak exogeneity of  $y_t$  in 5 (7) economies of the G7. The estimated adjustment coefficients in the conditional equation of the CO-model (2) applied to French and US data are not significant. Note that for both economies  $y_t$  was not found to be weakly exogenous. These results remain the same regarding suitable LR-statistics and generating critical values by means of a bootstrap procedure.

The hypotheses  $H_0 : \beta_{1n} = -1$  and  $H_0 : \beta_{2n} = -1$  are accepted for most conditional single equation models (Table 1 right hand side panel). Taking critical values from a  $\chi^2(1)$ -distribution the hypothesis  $H_0 : \beta_{1n} = -1$  ( $H_0 : \beta_{2n} = -1$ ) is rejected for JA and UK (CA, JA and UK) at the 5% significance level. Generating critical values via wild bootstrap the latter hypothesis is still rejected for JA (CA and UK). Throughout  $p$ -values obtained by the bootstrap procedure are larger compared to those obtained from the asymptotic approximation.

Before we turn to the issue of testing homogeneity and weak exogeneity on the pooled level we first try to characterize the empirical properties of the  $LR_N$ -statistic in (8) by means of a simulation study. A few properties of bootstrapping critical values for  $LR_N$  are discussed in Herwartz and Neumann (2000). A predominant feature of this approach is that its empirical properties depend on the dimension of the system ( $N$ ) and the error correction dynamics operating within the conditional models (2) or (4). The smaller  $\alpha_{11n}$  ( $\alpha_{21n}$ ) the weaker is the attractor keeping  $c_t$  and  $y_t$  ( $c_t$  and  $i_t$ ) in equilibrium. Thus e.g. empirical size properties of testing homogeneity or weak exogeneity suffer from weak error correction dynamics. As mentioned critical values for  $LR_N$  might also be taken from the  $\chi^2(qN)$ -distribution. Under cross equation error correlation, however, the asymptotic distribution of  $LR_N$  is no longer  $\chi^2$ -distributed. Thus superiority of the cross correlation consistent bootstrap procedure increases with the actual correlation pattern relating e.g. the error terms of (7).

To cope with data dependent empirical properties of the  $LR_N$ -statistic we generated

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\*Results from augmented Dickey Fuller tests (ADF-tests) applied to the series are available upon request.

2000 replications of the estimated CO- and IO-models, respectively. The simulations were performed under the assumptions of homogeneity ( $\beta_{in} = -1$ ,  $i = 1, 2$ ) and weak exogeneity of income ( $\alpha_{i2n} = 0$ ,  $i = 1, 2$ ). Error correction coefficients  $\alpha_{i1n}$  and coefficients governing contemporaneous impacts of  $\Delta y_t$  ( $\gamma_{11n}, \gamma_{21n}$ ) are chosen to be equal to the obtained parameter estimates and to satisfy stability conditions. Intercept terms and higher order lag coefficients are set to zero. Error terms  $u_{1nt}, v_{1nt}$  ( $u_{2nt}, v_{2nt}$ ) are drawn from a 7-dimensional multivariate normal distribution with covariance matrix estimated from their empirical counterparts. The relevant correlation patterns are displayed in Table 2.

Simulation results are given in Table 3. For the simulation design coming closest to the investigated empirical time series (upper panel of Table 3) we observe large empirical size distortions for both testing problems. Generating critical values by means of the wild bootstrap, however, is uniformly superior to the  $\chi^2$ -approximation. It turns out that for a given nominal significance level of 5% the corresponding empirical levels vary between 16.3% and 23.2% (18.3% and 26.1%) if critical values are taken from the bootstrap ( $\chi^2$ -distribution). Clearly the empirical size distortions can be addressed to the small sample size ( $T = 112$ ) and, more importantly, to the small error correction dynamics in the estimated CO- and IO-systems. The actually observed patterns of contemporaneous error correlation do not appear to contribute substantially to the empirical size properties. Size distortions decrease with the sample size but remain significant even for the case with  $T = 480$  observations. In any case the results obtained under contemporaneous error correlation and independence are quite similar. Imposing stronger error correction dynamics, however, the empirical size of the  $LR_N$ -test approaches the nominal level. If the simulated processes are characterized by error correction coefficients which are five times larger than those estimated for the CO- and IO-system the empirical size is quite close to the nominal one for sample size  $T = 240$  (or larger). The empirical size properties of the tests are mirrored when looking at the nominal test level necessary to achieve an empirical size of 5%. In Table 3 these quantities are also given ( $\alpha^*$ ). In small samples the bootstrap procedure is again slightly superior to the  $\chi^2$ -approximation. Under the empirical covariance scenario with  $T = 112$  it turns out that for this procedure nominal test levels of 0.5% to 0.9% deliver the target size of 5% empirically.

Test results using pooled data are depicted in Table 4. The hypotheses of joint insignificance of estimated error correction coefficients in the marginal and conditional equation of the CO-models for all G7 economies is rejected. On the one hand this result supports the hypothesis of cointegration on the pooled level, on the other hand, weak exogeneity of output is rejected. When testing on weak exogeneity of output in the CO-model we can confirm its rejection by means of the simulation results discussed before. The null hypothesis is rejected at the 5% level since the  $LR_N$ -statistic is larger than the implied size adjusted critical values ( $\alpha^* = 0.6\%$  or  $0.8\%$  applying the  $\chi^2$ -approximation or the wild bootstrap, respectively). For the second system, the IO-model, we again find evidence in favor of cointegration on the pooled level. For the marginal process the LR-statistic testing  $H_0 : \alpha_{22n} = 0, n = 1, \dots, 7$ ,

is 8.07 and, thus, not significant. Therefore we conclude output to be weakly exogenous for investment on the pooled level.

Testing linear restrictions for the cointegrating vector a value of the corresponding LR-statistic for the hypothesis  $H_0 : \beta_1 = -1$  ( $\beta_2 = -1$ ) of  $LR_N = 24.7$  ( $LR_N = 36.2$ ) is obtained from a pooled set of equations (2) (equations (4)). Applying the wild bootstrap to generate appropriate critical values it turns out that both hypotheses are rejected at any reasonable nominal level. The obtained bootstrap p-values 0.004 ( $H_0 : \beta_1 = -1$ ) and 0.001 ( $H_0 : \beta_2 = -1$ ) are both smaller than nominal test levels providing an empirical size of 5% ( $\alpha^* = 0.7\%$  and 0.5%, respectively). Thus the homogeneity hypothesis is not supported by the data. Looking at the single equation results in Table 1 it appears that both pooled statistics are dominated by single country results. The large statistics for the G7 are due to highly significant rejections of  $H_0 : \beta_1 = -1$  ( $H_0 : \beta_2 = -1$ ) obtained for the Japanese (Canadian) single equation ECM.

## 5 Conclusion

The pooled data results are not consistent with the balanced growth theory for the G7 economies. Following Temple (1999) or Mankiw, Romer and Weil (1992) the rejection of growth ratio hypotheses may be addressed to the applied definition of investment, which does not take human capital into account. Since our sample is composed of highly industrialized countries we conjecture that specifying a Cobb-Douglas technology including human capital will not change our main conclusion. In our view, the rejection of the growth ratio hypotheses supports growth models implying more general CO- and IO-relations.

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Table 1: Estimates and test results of long-run elasticities and error adjustment coefficients

Eq	Estimates			$H_0 : \alpha_{ijn} = 0$			$H_0 : \beta_{in} = -1$		
	$T$	$\hat{\alpha}$	$\hat{\beta}$	LR <sub>n</sub>	p-asy	p-wild	LR <sub>n</sub>	p-asy	p-wild
Canada, 1970:1 - 1998:2									
(2)	114	-.154 (2.92)	-0.947 (22.1)	8.727	.003	.010	2.050	.152	.217
(3)	112	.102 (1.79)	-1.052 (15.1)	3.358	.067	.060			
(4)	110	-.206 (4.32)	-1.664 (17.5)	18.83	.000	.000	13.43	.000	.001
(5)	112	-.025 (1.79)	-1.210 (3.28)	3.437	.064	.093			
France, 1970:1 - 1998:2									
(2)	114	-.056 (0.95)	-0.852 (3.07)	.991	.320	.316	1.244	.265	.278
(3)	112	.174 (3.52)	-1.138 (39.4)	12.71	.000	.010			
(4)	109	-.064 (2.79)	-0.528 (3.11)	7.915	.005	.003	2.955	.086	.064
(5)	112	.001 (0.12)	-10.42 (0.12)	0.015	.902	.912			
Germany, 1970:1 - 1998:4									
(2)	111	-.097 (2.42)	-1.102 (12.6)	6.380	.012	.012	1.032	.310	.300
(3)	111	.022 (0.38)	-1.231 (2.17)	0.165	.684	.761			
(4)	111	-.109 (3.83)	-1.346 (7.28)	15.19	.000	.003	3.511	.061	.088
(5)	111	-.005 (0.30)	2.053 (0.19)	0.104	.747	.771			
Italy, 1970:1 - 1997:4									
(2)	106	-.030 (1.61)	-1.085 (13.7)	2.829	.093	.160	0.509	.476	.506
(3)	106	.104 (1.85)	-1.222 (29.1)	3.725	.054	.032			
(4)	110	-.052 (2.46)	-0.670 (3.51)	6.220	.013	.011	1.343	.246	.245
(5)	108	-.020 (1.74)	0.305 (0.68)	3.259	.071	.049			
Japan, 1970:1 - 1998:2									
(2)	111	-.262 (4.40)	-0.942 (77.0)	18.79	.000	.000	12.12	.000	.006
(3)	112	-.024 (0.32)	-0.662 (0.73)	0.113	.737	.752			
(4)	110	-.046 (2.43)	-1.429 (8.14)	6.197	.013	.025	6.944	.008	.058
(5)	108	-.005 (0.43)	0.335 (0.09)	0.195	.658	.754			
United Kingdom, 1970:1 - 1997:4									
(2)	105	-.093 (2.48)	-1.250 (21.2)	6.405	.011	.064	5.043	.025	.064
(3)	107	.063 (1.22)	-1.263 (10.4)	1.575	.209	.351			
(4)	110	-.148 (2.70)	-1.360 (10.0)	7.536	.006	.005	5.512	.019	.004
(5)	110	-.005 (0.23)	-0.931 (0.47)	0.057	.812	.767			
United States, 1970:1 - 1997:4									
(2)	108	.069 (1.22)	-1.173 (15.2)	1.578	.209	.213	2.741	.098	.119
(3)	106	.426 (6.82)	-1.118 (85.8)	40.44	.000	.000			
(4)	107	-.085 (3.06)	-1.192 (9.20)	9.661	.002	.002	2.473	.116	.181
(5)	110	-.016 (0.96)	-.656 (1.14)	1.003	.316	.326			

Each country panel contain four rows. First two lines provide results for the CO-model, third and fourth lines give results for the IO-model. Columns two and three include estimates and  $t$ -ratios parantheses. In each panel the information period is given.  $T$  denotes the number of available observations. The equations for Germany contain seasonal dummies. p-asy and p-wild denote the p-values obtained from the asymptotic approximation and wild bootstrap inference, respectively.

Table 2: Correlation estimates for conditional ECMs and marginal processes

	CA	FR	GE	IT	JA	UK	US
CO-Model							
CA	1.00	0.13	0.03	0.00	0.08	0.06	<b>0.26</b>
FR	0.01	1.00	<b>0.22</b>	0.14	-0.02	0.01	0.19
GE	0.17	<b>0.42</b>	1.00	-0.01	0.12	-0.08	0.05
IT	0.00	<b>0.28</b>	0.17	1.00	-0.01	-0.09	-0.05
JA	0.14	0.07	0.09	0.06	1.00	0.09	0.05
UK	0.09	<b>-0.19</b>	-0.13	-0.14	0.17	1.00	0.11
US	<b>0.23</b>	0.17	0.07	-0.11	0.11	-0.05	1.00
IO-model							
CA	1.00	0.01	0.11	0.12	0.10	0.02	0.05
FR	-0.01	1.00	0.15	-0.01	0.04	0.03	-0.13
GE	0.17	<b>0.36</b>	1.00	<b>0.24</b>	0.02	-0.14	-0.07
IT	-0.04	0.17	<b>0.22</b>	1.00	0.07	-0.19	-0.03
JA	0.13	0.10	-0.03	<b>-0.25</b>	1.00	-0.03	-0.10
UK	<b>0.27</b>	0.16	0.31	-0.05	<b>0.25</b>	1.00	0.00
US	<b>0.19</b>	-0.12	0.05	<b>-0.27</b>	0.13	0.01	1.00

Upper blocks provide correlation estimates for the conditional equations (2) and (4). Similarly lower blocks show correlation patterns estimated for the marginal processes (3) and (5). Bold entries indicate significance at the 5%-level. The relevant critical value is  $|2/\sqrt{112}| = 0.189$ .

Table 3: Empirical properties of  $LR_N$  for the estimated CO- and IO-systems.

			Empirical covariance				Identity			
			CO		IO		CO		IO	
			$\hat{\alpha}$	$\alpha^*$	$\hat{\alpha}$	$\alpha^*$	$\hat{\alpha}$	$\alpha^*$	$\hat{\alpha}$	$\alpha^*$
$T$	$H_0$		$\alpha_{i1n} = \hat{\alpha}_{i1n}, \alpha_{i2n} = 0$							
112	$\beta_{in} = -1$	asy	<b>.240</b>	.004	<b>.261</b>	.004	<b>.244</b>	.003	<b>.246</b>	.004
		wild	<b>.204</b>	.007	<b>.232</b>	.005	<b>.209</b>	.005	<b>.220</b>	.006
	$\alpha_{i2n} = 0$	asy	<b>.195</b>	.006	<b>.183</b>	.008	<b>.183</b>	.007	<b>.189</b>	.008
		wild	<b>.187</b>	.008	<b>.166</b>	.009	<b>.163</b>	.009	<b>.179</b>	.009
240	$\beta_{in} = -1$	asy	<b>.141</b>	.014	<b>.153</b>	.009	<b>.149</b>	.012	<b>.150</b>	.010
		wild	<b>.124</b>	.016	<b>.140</b>	.013	<b>.136</b>	.012	<b>.139</b>	.013
	$\alpha_{i2n} = 0$	asy	<b>.134</b>	.014	<b>.124</b>	.017	<b>.142</b>	.016	<b>.119</b>	.015
		wild	<b>.127</b>	.015	<b>.117</b>	.019	<b>.133</b>	.016	<b>.113</b>	.018
480	$\beta_{in} = -1$	asy	<b>.088</b>	.025	<b>.119</b>	.021	<b>.103</b>	.021	<b>.103</b>	.025
		wild	<b>.084</b>	.027	<b>.114</b>	.022	<b>.104</b>	.020	<b>.099</b>	.025
	$\alpha_{i2n} = 0$	asy	<b>.100</b>	.024	<b>.098</b>	.020	<b>.096</b>	.025	<b>.094</b>	.019
		wild	<b>.096</b>	.027	<b>.099</b>	.021	<b>.094</b>	.026	<b>.095</b>	.020
			$\alpha_{i1n} = 2\hat{\alpha}_{i1n}, \alpha_{i2n} = 0$							
480	$\beta_{in} = -1$	asy	<b>.066</b>	.037	<b>.088</b>	.031	<b>.070</b>	.030	<b>.078</b>	.031
		wild	<b>.064</b>	.038	<b>.085</b>	.032	<b>.071</b>	.032	<b>.074</b>	.033
	$\alpha_{i2n} = 0$	asy	<b>.070</b>	.038	<b>.077</b>	.030	<b>.070</b>	.037	<b>.079</b>	.030
		wild	<b>.067</b>	.038	<b>.074</b>	.031	<b>.068</b>	.038	<b>.077</b>	.030
			$\alpha_{i1n} = 5\hat{\alpha}_{i1n}, \alpha_{i2n} = 0$							
240	$\beta_{in} = -1$	asy	<b>.073</b>	.036	<b>.073</b>	.034	<b>.075</b>	.031	<b>.069</b>	.034
		wild	.061	.041	<b>.064</b>	.038	<b>.069</b>	.036	<b>.063</b>	.039
	$\alpha_{i2n} = 0$	asy	.062	.038	<b>.063</b>	.037	<b>.067</b>	.036	.054	.042
		wild	.061	.038	.057	.039	.062	.039	.052	.044
480	$\beta_{in} = -1$	asy	.056	.044	<b>.066</b>	.039	.054	.042	.058	.040
		wild	.053	.047	<b>.064</b>	.041	.055	.042	.059	.042
	$\alpha_{i2n} = 0$	asy	.054	.045	.057	.041	.056	.047	.061	.038
		wild	.057	.045	.055	.042	.054	.045	.060	.036

Size estimates ( $\hat{\alpha}$ ) for tests on homogeneity ( $H_0 : \beta_{in} = -1$ ) and weak exogeneity of income ( $H_0 : \alpha_{i2n} = 0$ ). Critical values are from the  $\chi^2(14)$ -distribution (asy) or from wild bootstrap (wild). Error terms are drawn from the empirical covariance matrix or from identity covariance matrix. The nominal significance level ( $\alpha$ ) is 5%.  $\alpha^*$  is the nominal test level providing an empirical size of 5%. Bold entries indicate empirical size distortions ( $|\hat{\alpha} - \alpha|$ ), which are significant at the 1% level. Since each model is simulated 2000 times the relevant 99% confidence interval is [0.0373, 0.0626].

Table 4: Results of pooled tests

eq	$H_0 : \alpha_{ijn} = 0$			$H_0 : \beta_{in} = -1$		
	LR <sub>N</sub>	p-asy	p-wild	LR <sub>N</sub>	p-asy	p-wild
	CO-process					
$\Delta c_t$	45.7	1.00e-07	0.000	24.7	0.001	0.004
$\Delta y_t$	62.1	5.79e-11	0.000	55.7	1.08e-09	0.000
	IO-process					
$\Delta i_t$	71.5	7.20e-13	0.000	36.2	6.75e-06	0.001
$\Delta y_t$	8.07	0.367	0.359	17.8	0.013	0.032