E C O N O M I C S B U L L E T I N

Some Evidence of Decreasing Volatility of the US Coincident Economic Indicator

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Abstract

The paper treats the issue of the decreasing volatility of the U.S. economy which has been observed since the mid−1980s. As a measure of volatility the residual variance of a composite economic indicator is used. This indicator is constructed as a common dynamic factor with Markov switching and hence it incorporates both the comovements of different macroeconomic variables and the asymmetry between the contractions and expansions. Two additional regimes are included capturing the secular shift in the volatility. Furthermore, the mixed frequency is allowed for, permitting the use both of monthly and quarterly component series. The low mean regime probabilities comply to the NBER business cycle dating, while the low variance regime probabilities indicate the beginning of 1984 as a possible date of the structural break in volatility.

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1 Introduction

This paper examines the secular volatility decline in the US economy which presumably took place in the mid-1980s. The state of the US economy is characterized here by an unobserved coincident economic indicator. Therefore the structural shift in volatility, if any, will be captured by the one-time change in the residual variance of this indicator.

The tool used to carry out the study is the dynamic factor analysis. A standard common factor model with regime switching was modified, first, to incorporate the additional two regimes reflecting the structural break in volatility and, second, to make use of the mixed-frequency component series.

The advantages of this approach are several. Firstly, considering a few macroeconomic time series at once rather than only one allows capturing one of the key business cycle characteristics — the common fluctuations of these variables.

Secondly, the probabilistic approach to the volatility shift adopted in this paper does not impose any predefined breakpoint rather estimating the moment when the structural break occurred which is reflected in the conditional regime probabilities.

Thirdly, the use of mixed-frequency data permits achieving higher efficiency of the estimates since relevant additional information is utilized which is measured at lower frequency and normally is left aside as having too many "missing observations".

The remainder of the paper is organized as follows. Section 2 sets up a basic model with Markov switching and discusses its possible extensions. In section 3 the empirical analysis using the US Post-World War II monthly and quarterly macroeconomic time series is conducted. Section 4 summarizes the main findings of the paper. All the illustrative material — tables and graphs — is placed into appendix which follows the list of references.

2 Model

We consider a set of the coincident time series which are supposed to evolve at the same pace as a current state-of-affairs indicator (e.g. GDP). The common dynamics of the coincident time series are explained by a latent common factor. The idiosyncratic dynamics of each time series are captured by one specific factor per each observed time series. Formally:

$$\Delta y_t = \Gamma \Delta c_t + u_t \tag{1}$$

where Δy_t is the $n \times 1$ vector of the observed time series in the first differences; Δc_t is the scalar representing the latent common factor in the first differences; u_t is the $n \times 1$ vector of the latent specific factors; Γ is the $n \times 1$ factor loadings vector linking the observed series with the common factors; t is the time subscript ($t = 1, 2, ..., T$).

This model may be extended to the mixed-frequency case, i.e., a case where the component time series[1] are observed at different frequencies, e.g., monthly and quarterly. The methodology of the mixed-frequency common factor model with linear and regime-switching dynamics is described in Mariano and Murasawa (2000) and Kholodilin (2001), respectively.

Assume that we have $n = n_1 + n_2$ observable component series. The first n_1 component series, y_1, are observed at lower frequency (each $f > 1$ periods), while the remaining n_2 series, y_2, are measured at a higher frequency which we may normalize to 1. Thus, if we have quarterly and monthly data, $f = 3$ and we observe $y_1 = \{y_{13}, y_{16}..., y_{1.T-3}, y_{1.T}\}$ and $y_2 = \{y_{21}, y_{22}..., y_{2.T-1}, y_{2.T}\}$. Denote by y_{1t}^* the values of the first n_1 component series that we might have observed if these series were measured at the same frequency as y_2, that is, $y_1^* = \{y_{11}^*, y_{12}^*, ..., y_{1.T-1}^*, y_{1.T}^*\}$. The observed lower-frequency series can be expressed as an arithmetic average of these unobserved values:

$$y_{1t} = \frac{1}{f} \sum_{i=0}^{f-1} L^i y_{1t}^* \tag{2}$$

where the l.h.s. variable is measured at periods $f, 2f, ..., T$ and the r.h.s. variable is observed each period, i.e. $1, 2, ..., T$.

After taking the first differences of the observed lower-frequency series[2], the growth rates of these series would be:

$$(1 - L^f) y_{1t} = \frac{1}{f} (\sum_{i=0}^{f-1} L^i)^2 (1 - L) y_{1t}^* \tag{3}$$

where L is the lag operator and $(\sum_{i=0}^{f-1} L^i)^2 = \sum_{i=0}^{2f-1} (f + 1 - |i - f|) L^i$.

To estimate the model at the higher frequency, the unobserved values of the lower-frequency time series are treated as missing. As Mariano and Murasawa (2000) showed, they can be replaced by any random variable as long as it is independent of the parameters of the model. In particular, these missing observations may be substituted by zeros. Thus, the growth rates of the first n_1 variables expressed at the higher frequency can be constructed as:

[1] Here we consider only the case of the flow variables.

[2] Let, for example, y_{1t} be the quarterly series. Thence their first difference is the quarterly growth rate. But since our model is expressed in terms of the higher (monthly) frequency, to designate this first-order difference we need operator $\Delta^3 = 1 - L^3$.

$$(1-L)\tilde{y}_{1t}^{*} = \begin{cases} (1-L^{f})y_{1t}, & \text{if } t = f, 2f, ..., T \\ 0 & \text{otherwise} \end{cases}$$

Therefore the vector of the growth rates of all the n observed series, Δy_t, measured at the higher frequency may be decomposed as:

$$\begin{pmatrix} (1-L)\tilde{y}_{1t}^{*} \\ (1-L)y_{2t} \end{pmatrix} = \Gamma \begin{pmatrix} \frac{1}{f}(\sum_{i=0}^{f-1} L^{i})^{2} I_{t} \\ 1 \end{pmatrix} (1-L)c_{t} + \begin{pmatrix} \frac{1}{f}(\sum_{i=0}^{f-1} L^{i})^{2} I_{t} \\ 1 \end{pmatrix} u_{t} \quad (4)$$

where I_t is the indicator function: $I_t = 1$ when $t = f, 2f, ..., T$ and $I_t = 0$ otherwise; Γ is the $n \times 2$ factor loadings matrix defined somewhat differently than that from (1):

$$\Gamma = \begin{pmatrix} \Gamma_1 & O_{n_1} \\ O_{n_2} & \Gamma_2 \end{pmatrix}$$

where Γ_1 and Γ_2 are the vectors of factor loadings for the lower- and high-frequency series, respectively; O_n is $n \times 1$ the vector of zeros.

The dynamics of the unobserved common factor is described in terms of a nonlinear autoregressive (AR) model:

$$\Delta c_t = \mu(s_t) + \phi(L)\Delta c_{t-1} + \varepsilon_t \quad (5)$$

where $\mu(s_t)$ is the state-dependent intercept of the common coincident factor which takes different values depending on the regime (low in recessions and high in expansions); s_t is the unobserved regime variable; $\phi(L)$ is the common factor AR lag polynomial of order p; ε_t is the serially uncorrelated common factor disturbance term with state-dependent variance:

$$\varepsilon_t \sim NID\left(0, \sigma^2(s_t)\right)$$

The common factor grows faster during the upswings and slower (or even decreases) during the downswings of the economy. Here we introduce an additional dimension of the problem by allowing the common factor residual variance, $\sigma^2(s_t)$, to have its own regimes: regime of low and high volatility. We assume that the mean state variable, s_t^{μ}, is independent of the residual variance state variable, s_t^{σ}. This kind of model was used by McConnell and Perez Quiros (2000) to examine the Post-World War II evolution of the US quarterly real GDP.

The changes in the regimes follow a first-order Markov chain process, which is summarized by the transition probabilities matrix with a characteristic element $p_{ij} = prob(s_t = j | s_{t-1} = i)$.

Since we have two parameters — mean and variance — each of which passes through its own low and high regimes, the whole process should be cast in a four regimes framework as it is done in McConnell and Perez Quiros (2000). Namely:

$s_t = 0$	$s_t = 1$	$s_t = 2$	$s_t = 3$
$s_t^\mu = 0$	$s_t^\mu = 1$	$s_t^\mu = 0$	$s_t^\mu = 1$
$s_t^\sigma = 0$	$s_t^\sigma = 0$	$s_t^\sigma = 1$	$s_t^\sigma = 1$

where s_t^μ and s_t^σ are the unobserved state variables for common factor mean (intercept) and common factor residual variance, respectively. Each state variable has its own 2×2 transition probabilities matrix:

		s_t^j	
		High	Low
s_{t-1}^j	High	p_{11}^j	$1 - p_{11}^j$
	Low	$1 - p_{22}^j$	p_{22}^j

where $j = \{\mu, \sigma\}$.

Given that the state variables s_t^μ and s_t^σ staying behind the evolution of the common factor intercept and residual variance are independent, the 4×4 transition probabilities matrix, π, governing the behavior of the "composite" state variable, s_t, would look as:

$$\begin{pmatrix} p_{11}^\mu p_{11}^\sigma & (1-p_{11}^\mu)p_{11}^\sigma & p_{11}^\mu(1-p_{11}^\sigma) & (1-p_{11}^\mu)(1-p_{11}^\sigma) \\ (1-p_{22}^\mu)p_{11}^\sigma & p_{22}^\mu p_{11}^\sigma & (1-p_{22}^\mu)(1-p_{11}^\sigma) & p_{22}^\mu(1-p_{11}^\sigma) \\ p_{11}^\mu(1-p_{22}^\sigma) & (1-p_{11}^\mu)(1-p_{22}^\sigma) & p_{11}^\mu p_{22}^\sigma & (1-p_{11}^\mu)p_{22}^\sigma \\ (1-p_{22}^\mu)(1-p_{22}^\sigma) & p_{22}^\mu(1-p_{22}^\sigma) & (1-p_{22}^\mu)p_{22}^\sigma & p_{22}^\mu p_{22}^\sigma \end{pmatrix}$$

In fact, $\pi = \pi^\mu \otimes \pi^\sigma$, where π^μ and π^σ are the transition probabilities matrices for the state variables s_t^μ and s_t^σ.

Thus, in our four-regime model we have four state-dependent means, μ_{ij}, where $i = \{s_t^\mu = 0, s_t^\mu = 1\}$ and $j = \{s_t^\sigma = 0, s_t^\sigma = 1\}$, and two state-dependent residual variances, σ_j^2, where $j = \{high, low\}$.

A constrained version of the above model was also considered following the model proposed by Kim and Nelson (1999a) for the US real GDP. They treat the low volatility regime as an absorbing state. This means that whenever the system attains this state, it remains there forever. This assumption translates into the following constraint imposed on the transition probabilities matrix π^σ:

$$\pi^\sigma = \begin{pmatrix} p_{11}^\sigma & 1 - p_{11}^\sigma \\ 0 & 1 \end{pmatrix}$$

The quantity $\frac{1}{1-p_{11}^{\sigma}}$ measures the expected duration of the high volatility regime and hence indirectly indicates the approximate location of the breakpoint. The two models — unrestricted and restricted — can be compared using the standard likelihood ratio (LR) test.

The idiosyncratic factors are mutually independent and are modelled as AR processes:

$$u_t = \Psi(L)u_{t-1} + \eta_t \tag{6}$$

where $\Psi(L)$ is the sequence of q ($q = \max\{q_{1,\dots}, q_n\}$, where q_i is the order of the AR polynomial of the $i-th$ idiosyncratic factor) $n \times n$ diagonal lag polynomial matrices and η_t is the $n \times 1$ vector of the mutually and serially uncorrelated normally distributed shocks:

$$\eta_t \sim NID\left(\begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_n^2 \end{pmatrix}\right)$$

The model is expressed in the state-space form and estimated via maximum likelihood using the Kalman filter and smoother as in Kim and Nelson (1999b).

3 Real example

The previous research (e.g. McConnell and Perez Quiros (2000)) implies that a structural break in the US economy had presumably taken place in early 1984. For the sake of illustration, without trying to detect the breakpoints for the major US macroeconomic series, we analyze their behavior before and after that date. Figures 1-2 show the evolution of the growth rates of the US real GDP (quarterly data) and the four monthly component series of the US composite economic indicator as estimated by Stock and Watson (1988): employees on nonagricultural payrolls (EMP); personal income less transfer payments (INC); index of industrial production (IIP); and manufacturing and trade series (SLS). Furthermore, their means and the two-standard-deviations band for two subperiods — 1959:1-1983:12 for monthly data (1959:2-1983:4 for GDP) and 1984:1-1998:12 for monthly data (1984:1-1998:4 for GDP) — are plotted. The means have not apparently changed much, while the variances, especially those of GDP, EMP, and IIP have undergone an important decline.

The quantitative characterization of these changes is found in Tables 1a-1b of Appendix which contain means, standard deviations (St.dev.), coefficients of variation (CV), minima (Min), and maxima (Max) of the time series in question before and after the beginning of 1984. As comparison of Tables 1a and 1b shows, the means have decreased, although not significantly. The variances have decreased, especially those of EMP and IIP, which in the second subsample have experienced almost double reduction. The coefficients of variation fell down too, safe for the case of IIP whose variance diminished faster than the mean. It seems also that the growth rates have changed asymmetrically: if in two cases out of five (INC and SLS) the lowest growth rates where attained in the second subperiod, only in one case out of five (SLS) the highest growth rate had been achieved after January 1984.

The formal test[3] of the differences between the means and variances in the two subsamples is contained in Table 2. The columns two and four represent the test statistics values — Z distributed as a normal and F following $F(n_1, n_2)$ distribution — for means and variances, respectively. n_1 and n_2 stand for the sizes of each of two subsamples. The p-values (see columns three and five of Table 2) of these test statistics allow testing the null hypothesis of no difference between the moments of the two subperiods. Only for the mean of the variable INC the null hypothesis may be rejected at significance level of 10%, while the rest of the means seem not to change. The variances of all the time series appear to have changed significantly. The largest decrease in the growth rate variance hit EMP, GDP, and IIP.

Given the fact that the GDP has experienced large volatility structural break, as our own calculations show and as was discovered by McConnell and Perez Quiros (2000), we decided to estimate both single-frequency model based only on the monthly time series stretching from January 1959 to December 1998 (namely: EMP, INC, IIP, and SLS) and a mixed-frequency model using, in addition to the monthly series, the quarterly GDP. The time series in levels were logged, then their first differences were taken and multiplied by 100. Finally, all the component series were demeaned and normalized.

Four models have been estimated: (1) unrestricted single-frequency model; (2) unrestricted mixed-frequency model; (3) restricted single-frequency model, and finally (4) restricted mixed-frequency model.

All the models were estimated under the identifying assumption of the first factor loading being equal 1. The parameter estimates, together with their standard errors, of the unrestricted and restricted single-frequency models can be found in Table 3, while those of the unrestricted and constrained mixed-frequency models in Table 4. In the four models the common factors follow AR(1), while the specific factors follow AR(2).

[3] For details on inferences based on two samples see Devore (1987).

The LR test of the restriction imposed on the transition probabilities matrix of the variance state variable shows that the null hypothesis of $p_{22}^{\sigma} = 1$ can be rejected at 5% significance level: the test value for the single-frequency model is equal 16.6 and for the mixed-frequency model it is 13.8 (see the first rows of Tables 3 and 4) vs. $\chi^2_{0.95}(1) = 3.84$.

The low common factor residual variance is more than nine times smaller than the high common factor residual variance. This difference is somewhat surprising given that the pre-1984 and post-1984 ratio of variances of the individual component time series does not exceed 4. The state-dependent means corresponding to the high variance regime (μ_{11} and μ_{21}) are much greater in the absolute value than their counterparts in the low-volatility regime. This implies that the shift in the volatility was accompanied by a "stabilization" of the growth rates. Both the recessions and expansions became milder.

The expected duration of the high volatility state computed using the estimate of the transition probability, p_{11}^{σ}, in both cases (single- and mixed-frequency models) is equal 333 months. If we assume that the high variance state commences at the very beginning of the sample (February 1959), the 333rd period will correspond to October 1986. The date is somewhat late compared to the beginning of 1984 proposed as the date of the start of volatility decline.

Figure 3 displays the estimate of the common coincident factor obtained using the single-frequency model and mixed-frequency model plotted against the National Bureau of Economic Research (NBER) business cycle dates. Five major recessions can be observed on the picture.

On Figures 4-5 the conditional (smoothed) probabilities of the low mean regime (sum of the conditional probabilities corresponding to the regimes 1 and 3) and low mean — high variance regime (conditional probabilities corresponding to the regime 1) for both the single- and mixed-frequency models are depicted. Both conditional probabilities are contrasted against the NBER chronology. However, only the conditional probabilities of the low mean — high variance regime display fairly high degree of conformity to this dating — see Figure 5. The low regime probabilities detect quite a bit of false signals: 2 false recessions in 1960s, 1 in 1980s, and 1 in 1990s. The picture is the same for the two models, although the mixed-frequency model attenuates slightly the false alarms compared to the single-frequency model.

Figure 6 displays the smoothed probabilities of the low variance regime (sum of the conditional probabilities corresponding to the regimes 0 and 1) for both models. It seems that, regardless of model, this regime becomes much more probable since February-March of 1984. From that period on the conditional probabilities of the coincident economic indicator having low residual variance are almost always — safe for two short interruptions — exceeding 0.7. This evidence is in accordance with the finding of McConnell and Perez Quiros (2000) who using the quarterly GDP data signal the first quarter of 1984 as the beginning of low volatility "era".

The low mean and low variance regime probabilities corresponding to the restricted models — with single and mixed frequency — are displayed on Figures 7 and 8, respectively. The recession (low mean) probabilities obtained from the constrained model do not differ from those resulting from the unconstrained estimation. The restricted model low volatility regime probabilities are much smoother thanks to the restriction imposed on the transition probability, p_{22}^{σ}. The smoothed probabilities signal the arrival of the low volatility regime earlier than the filtered conditional probabilities do. One can clearly see the frontier between the high and low variance regimes which passes through the middle of 1984.

4 Concluding remarks

The paper has examined 4 single-factor models with Markov switching: two unrestricted models with a single (monthly) frequency and two restricted models with two observation frequencies (monthly and quarterly). The common factor's intercept and residual variance experience independently shifts in the regimes. This leads to considering a four-regime model where for each of the two common factor residual variance regimes (low and high volatility) there is a pair of common factor mean regimes (low and high mean).

In the restricted models a constraint is imposed on the transition probabilities matrix of the variance state variable forcing the low variance regime to be an absorbing state. The restriction results in smoother conditional regime probabilities but finds no support in the real data — the difference between the log-likelihood function values of the restricted and unrestricted models being statistically insignificant.

The models were estimated using the US monthly and quarterly macroeconomic data covering the period 1959-1998. The shift in the composite economic indicator appears to have happened in the early 1984. This conforms to the previous findings, e.g. McConnell and Perez Quiros (2000), Chauvet and Potter (2001). A tight link between our model's recession probabilities and the NBER chronology is evident.

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5 Appendix

Table 1a. The component series statistics in first subperiod

	Mean	St.dev.	CV	Min	Max
Monthly data 1959:1-1983:12					
EMP	0.189	0.283	1.50	-0.86	1.23
INC	0.277	0.447	1.61	-1.21	1.68
IIP	0.296	1.050	3.54	-4.25	6.00
SLS	0.286	1.120	3.94	-3.10	3.10
Quarterly data 1959:1-1983:4					
GDP	0.833	1.100	1.32	-2.43	3.73

Table 1b. The component series statistics in second subperiod

	Mean	St.dev.	CV	Min	Max
Monthly data 1984:1-1998:12					
EMP	0.179	0.137	0.76	-0.24	0.56
INC	0.225	0.382	1.70	-1.27	1.22
IIP	0.257	0.525	2.05	-1.34	2.06
SLS	0.286	0.923	3.23	-3.27	3.55
Quarterly data 1984:1-1998:4					
GDP	0.735	0.534	0.727	-1.03	2.29

Table 2. Testing significance of differences between the means and variances of two subsamples

	Mean Z	Mean p-value	Variance F	Variance p-value
EMP	0.518	0.302	4.27	0.0
INC	1.350	0.088	1.37	0.011
IIP	0.540	0.295	4.00	0.0
SLS	0	0.500	1.47	0.002
GDP	0.752	0.226	4.24	0.0

Table 3. Estimated parameters of unrestricted and restricted models with single-frequency data

Parameter	Unrestricted: -1072.8		Restricted: -1081.1	
	Estimate	St. error	Estimate	St. error
p_{11}^{μ}	0.946	0.023	0.975	0.010
p_{22}^{μ}	0.927	0.028	0.879	0.049
p_{11}^{σ}	0.923	0.041	0.997	0.003
p_{22}^{σ}	0.956	0.020	1	-
μ_{1high}	0.102	0.035	0.060	0.018
μ_{2high}	-0.167	0.050	-0.180	0.035
μ_{1low}	0.036	0.012	0.004	0.006
μ_{2low}	-0.023	0.015	-0.163	0.036
γ_{INC}	1.52	0.116	1.49	0.112
γ_{IIP}	4.17	0.268	3.94	0.259
γ_{SLS}	3.25	0.234	3.13	0.231
ϕ	0.306	0.113	0.332	0.092
$\psi_{EMP.1}$	0.122	0.049	0.101	0.048
$\psi_{EMP.2}$	0.453	0.053	0.486	0.054
$\psi_{INC.1}$	-0.026	0.059	-0.039	0.052
$\psi_{INC.2}$	0.042	0.059	0.033	0.054
$\psi_{IIP.1}$	-0.134	0.077	-0.079	0.065
$\psi_{IIP.2}$	-0.097	0.069	-0.069	0.060
$\psi_{SLS.1}$	-0.430	0.051	-0.413	0.051
$\psi_{SLS.2}$	-0.212	0.049	-0.203	0.049
σ_{high}^2	0.038	0.007	0.027	0.003
σ_{low}^2	0.002	0.001	0.001	0.001
σ_{EMP}^2	0.016	0.002	0.014	0.002
σ_{INC}^2	0.107	0.008	0.106	0.008
σ_{IIP}^2	0.249	0.027	0.276	0.027
σ_{SLS}^2	0.627	0.046	0.639	0.047

Table 4. Estimated parameters of unrestricted and restricted models with mixed-frequency data

Parameter	Unrestricted: -1490.9		Restricted: -1497.8	
	Estimate	St. error	Estimate	St. error
p_{11}^{μ}	0.950	0.020	0.976	0.010
p_{22}^{μ}	0.923	0.029	0.883	0.067
p_{11}^{σ}	0.945	0.029	0.997	0.003
p_{22}^{σ}	0.958	0.019	1	-
μ_{1high}	0.208	0.055	0.115	0.044
μ_{2high}	-0.331	0.073	-0.330	0.069
μ_{1low}	0.052	0.020	0.004	0.011
μ_{2low}	-0.051	0.024	-0.341	0.072
γ_{EMP}	0.514	0.039	0.514	0.041
γ_{INC}	0.813	0.064	0.793	0.066
γ_{IIP}	2.10	0.134	2.06	0.136
γ_{SLS}	1.71	0.117	1.68	0.121
ϕ	0.256	0.100	0.321	0.110
$\psi_{GDP.1}$	-0.825	0.103	-0.872	0.088
$\psi_{GDP.2}$	0.034	0.120	-0.014	0.097
$\psi_{EMP.1}$	0.125	0.048	0.117	0.048
$\psi_{EMP.2}$	0.451	0.053	0.492	0.053
$\psi_{INC.1}$	-0.056	0.054	-0.048	0.054
$\psi_{INC.2}$	0.026	0.045	0.031	0.063
$\psi_{IIP.1}$	-0.062	0.064	-0.053	0.065
$\psi_{IIP.2}$	-0.056	0.058	-0.058	0.058
$\psi_{SLS.1}$	-0.449	0.051	-0.425	0.050
$\psi_{SLS.2}$	-0.226	0.049	-0.209	0.049

Table 4. continuation

Parameter	Unrestricted		Restricted	
	Estimate	St. error	Estimate	St. error
σ_{high}^2	0.121	0.023	0.100	0.018
σ_{low}^2	0.005	0.004	0.005	0.003
σ_{GDP}^2	0.229	0.051	0.242	0.048
σ_{EMP}^2	0.016	0.002	0.015	0.002
σ_{INC}^2	0.103	0.007	0.105	0.007
σ_{IIP}^2	0.274	0.025	0.281	0.026
σ_{SLS}^2	0.609	0.045	0.625	0.045

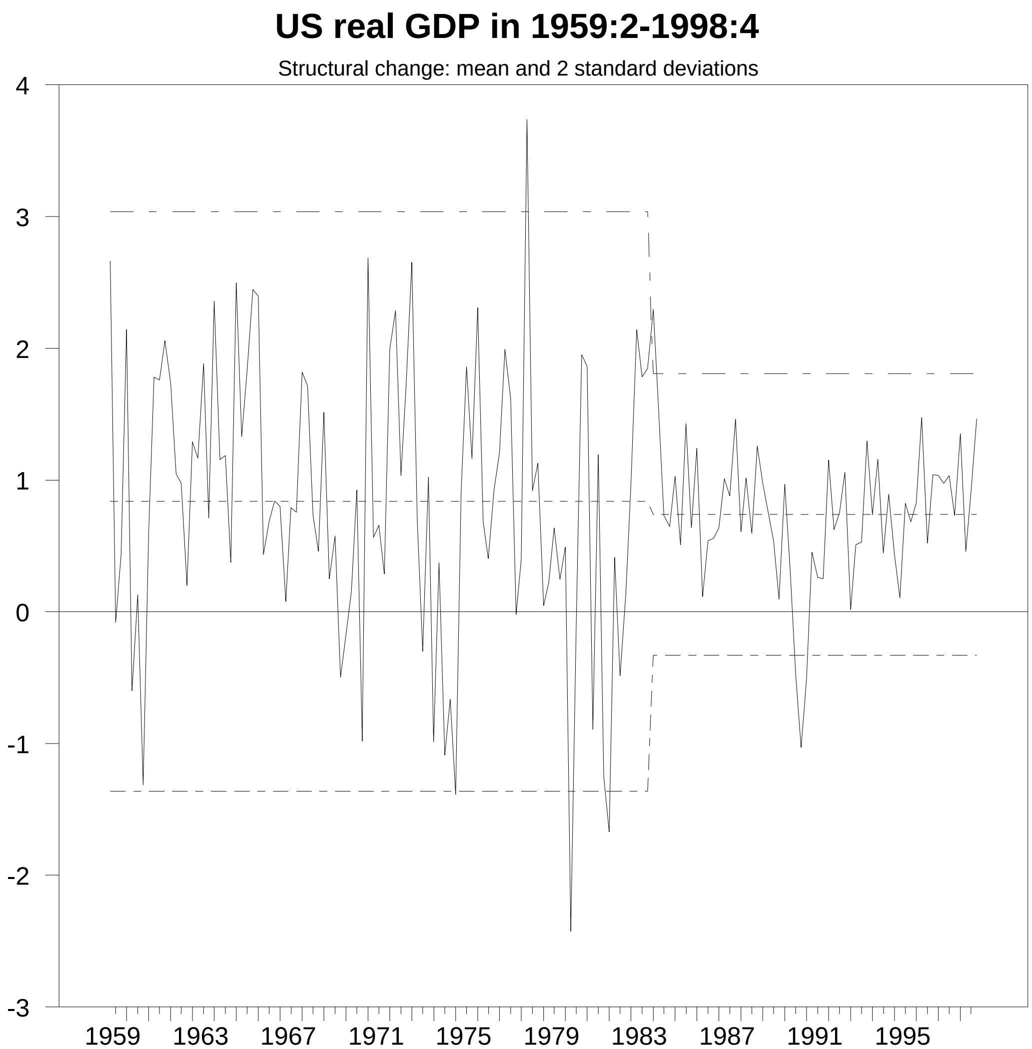

US real GDP in 1959:2-1998:4
Structural change: mean and 2 standard deviations
4
3
2
1
0
-1
-2
-3
1959
1963
1967
1971
1975
1979
1983
1987
1991
1995

Figure 1:

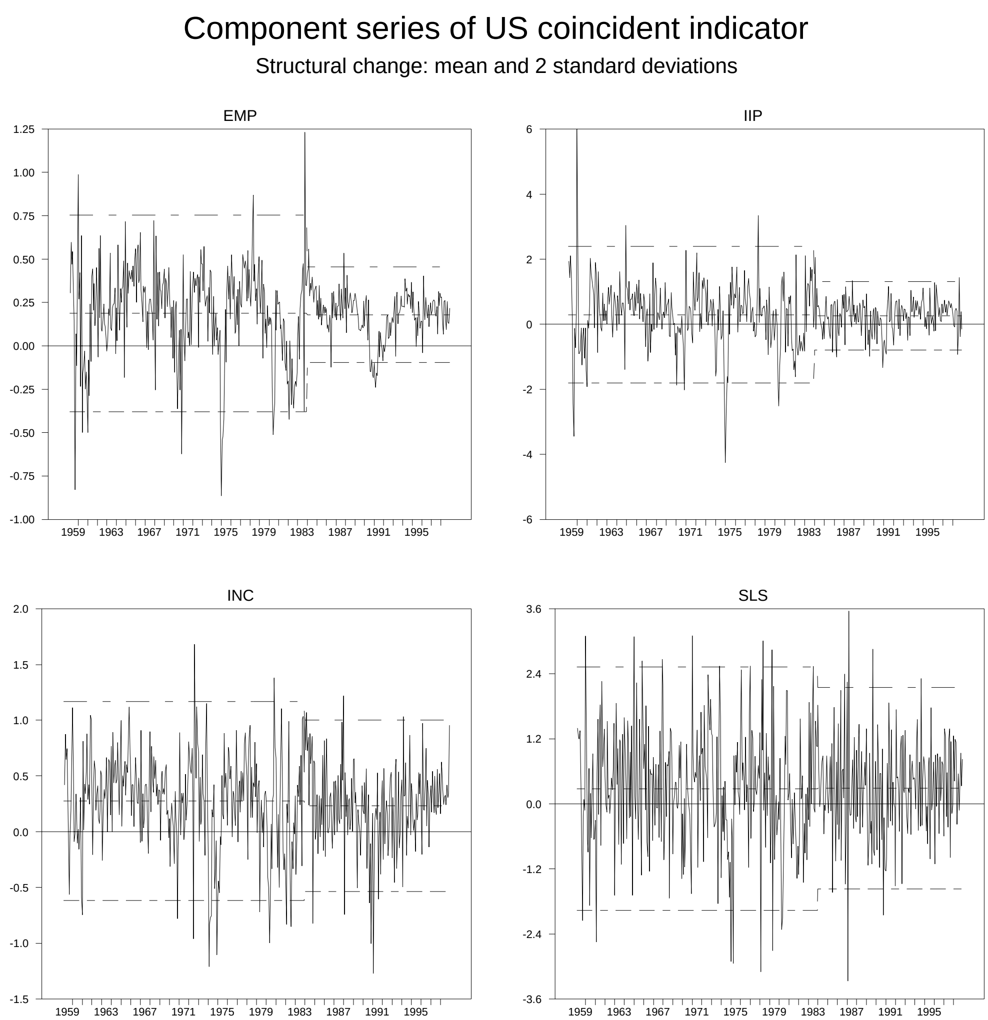

Component series of US coincident indicator
Structural change: mean and 2 standard deviations
EMP
IIP
INC
SLS

Figure 2:

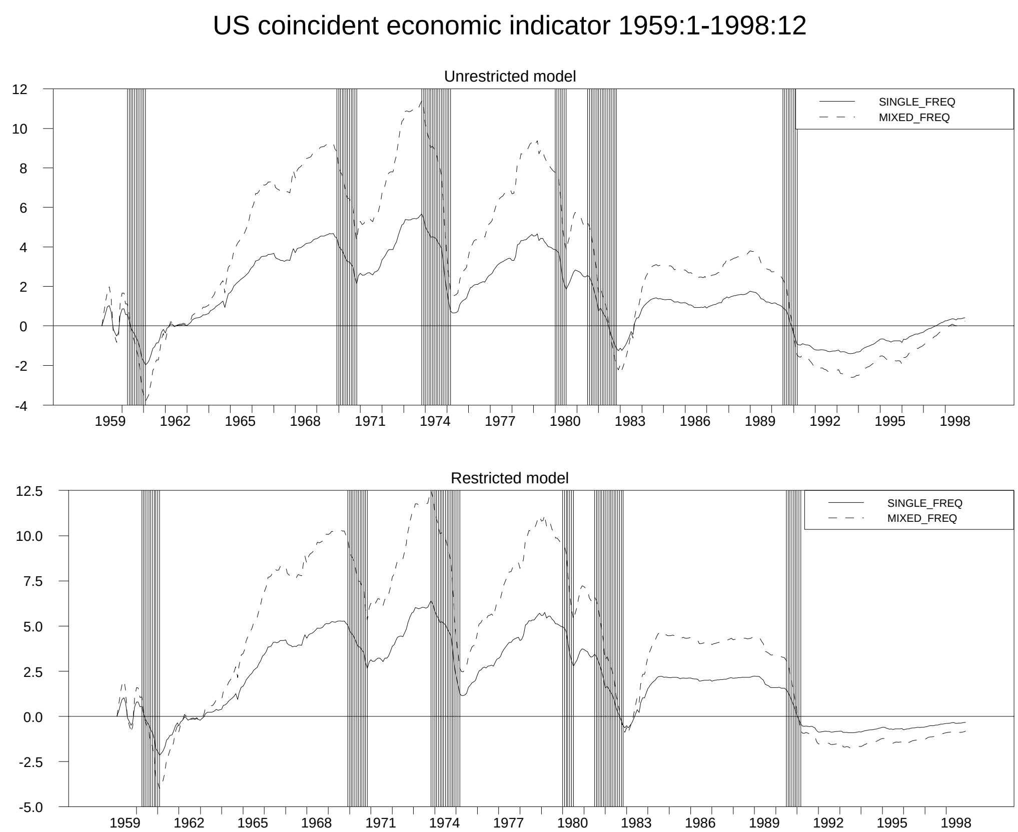

US coincident economic indicator 1959:1-1998:12
Unrestricted model
SINGLE_FREQ
MIXED_FREQ
12
10
8
6
4
2
0
-2
-4
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Restricted model
SINGLE_FREQ
MIXED_FREQ
12.5
10.0
7.5
5.0
2.5
0.0
-2.5
-5.0
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998

Figure 3:

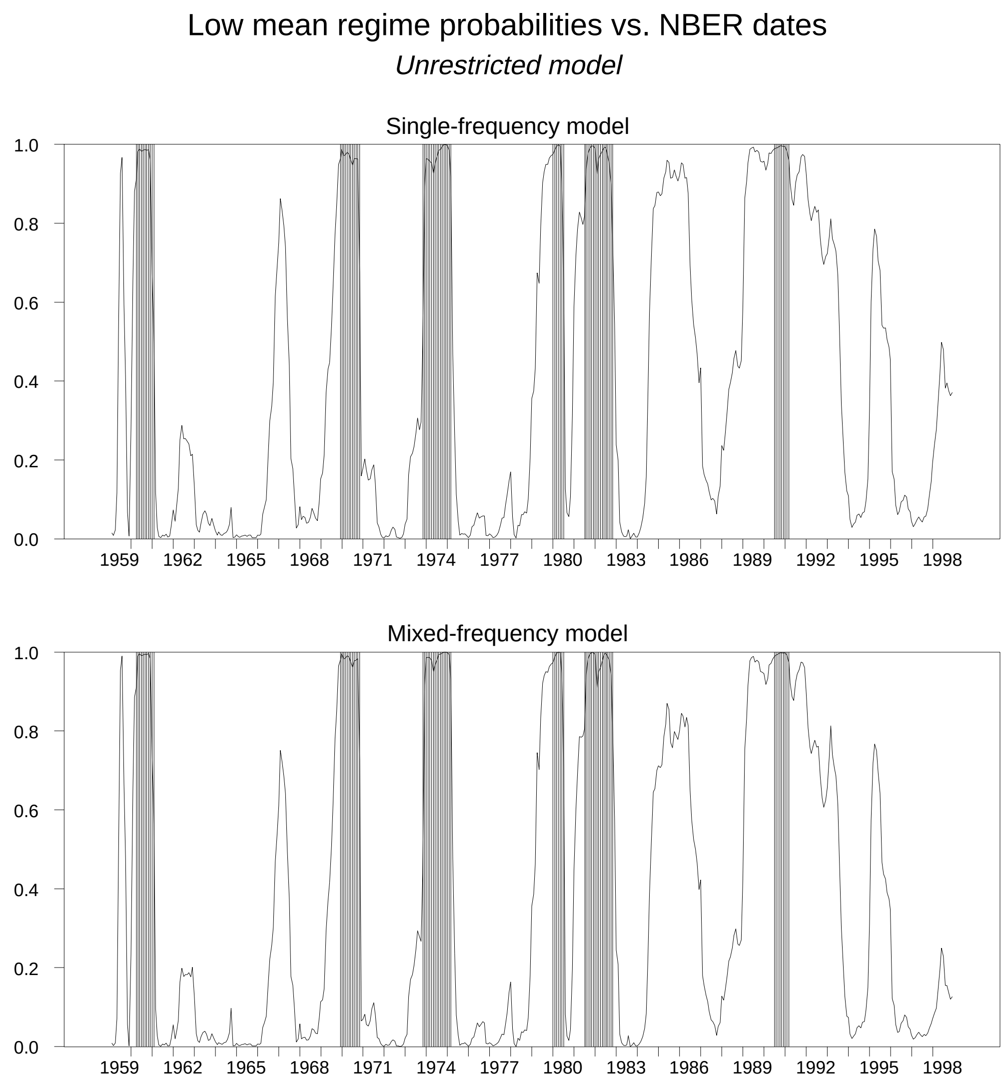
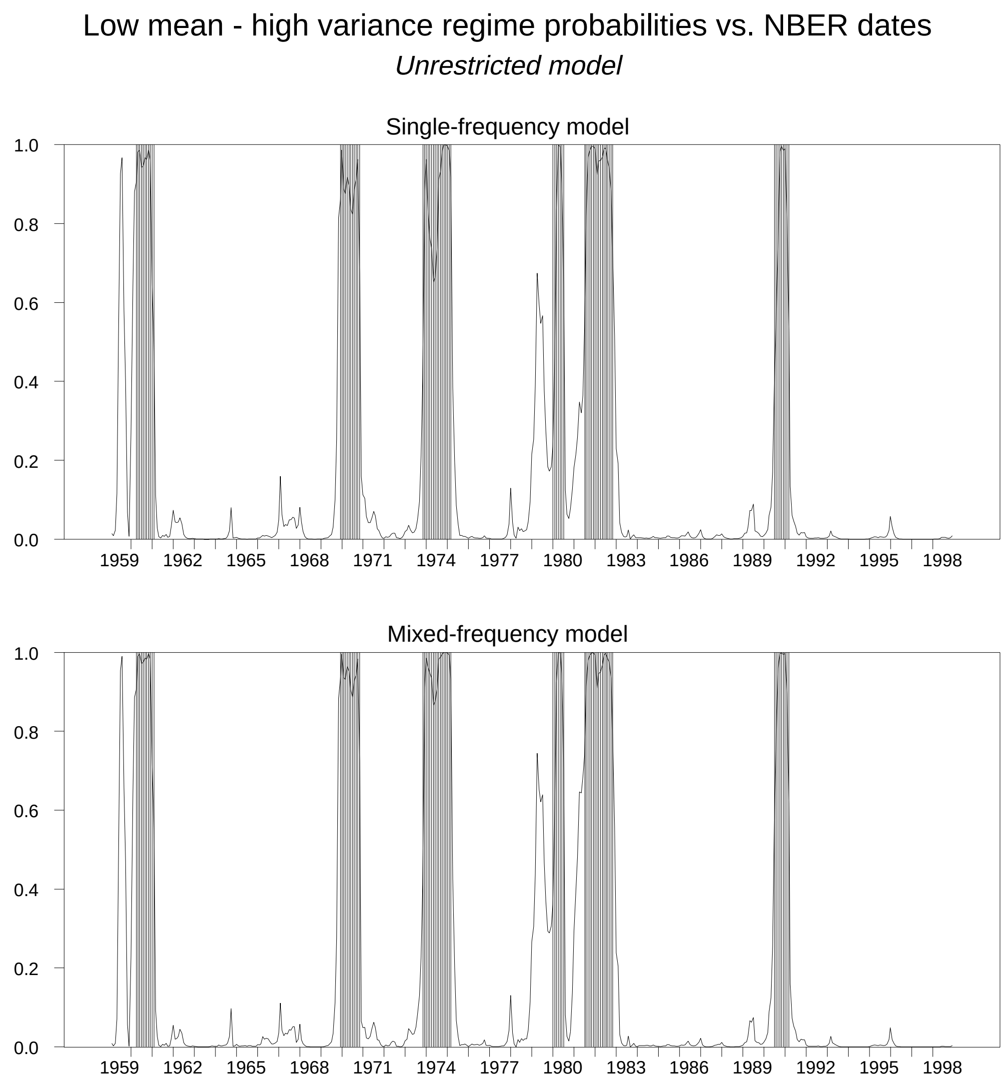
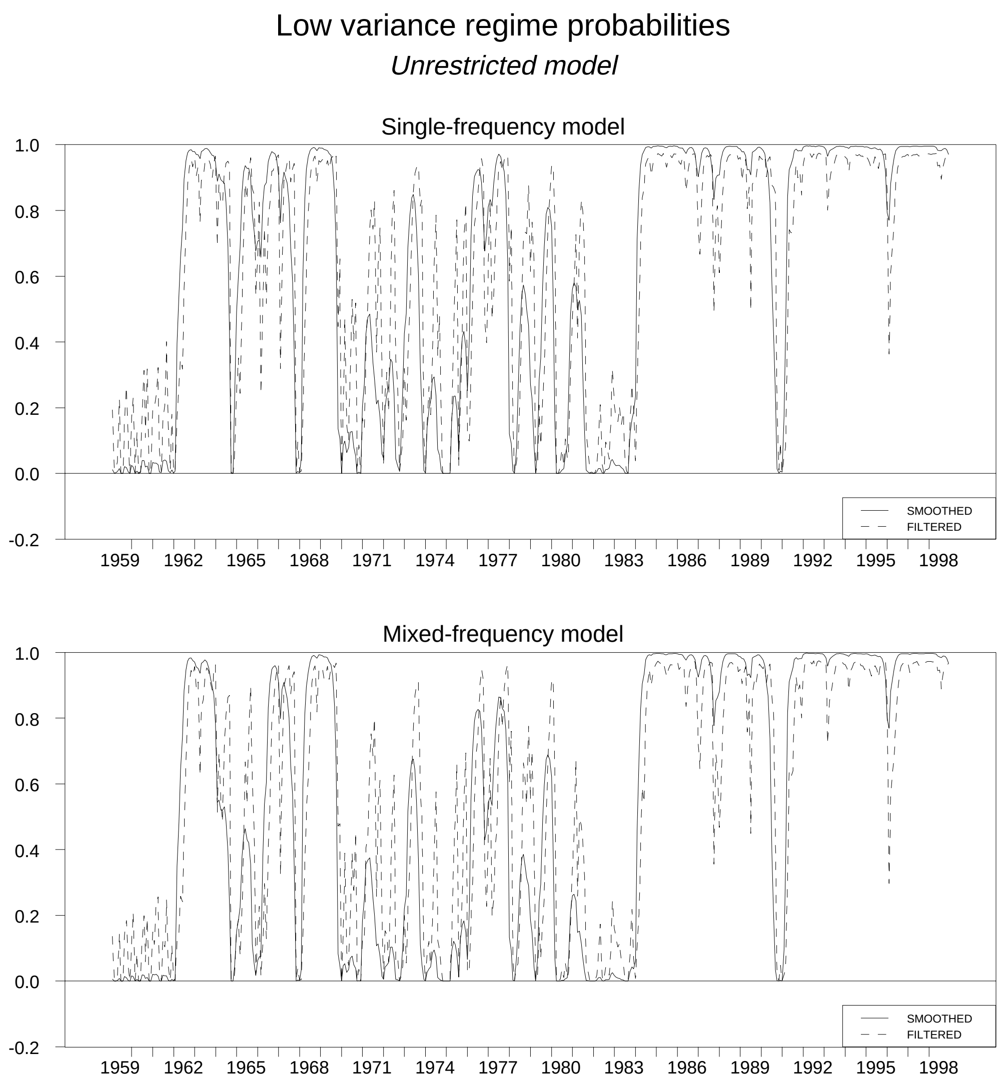

Low mean regime probabilities vs. NBER dates
Unrestricted model
Single-frequency model
1.0
0.8
0.6
0.4
0.2
0.0
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Mixed-frequency model
1.0
0.8
0.6
0.4
0.2
0.0
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998

Figure 4:



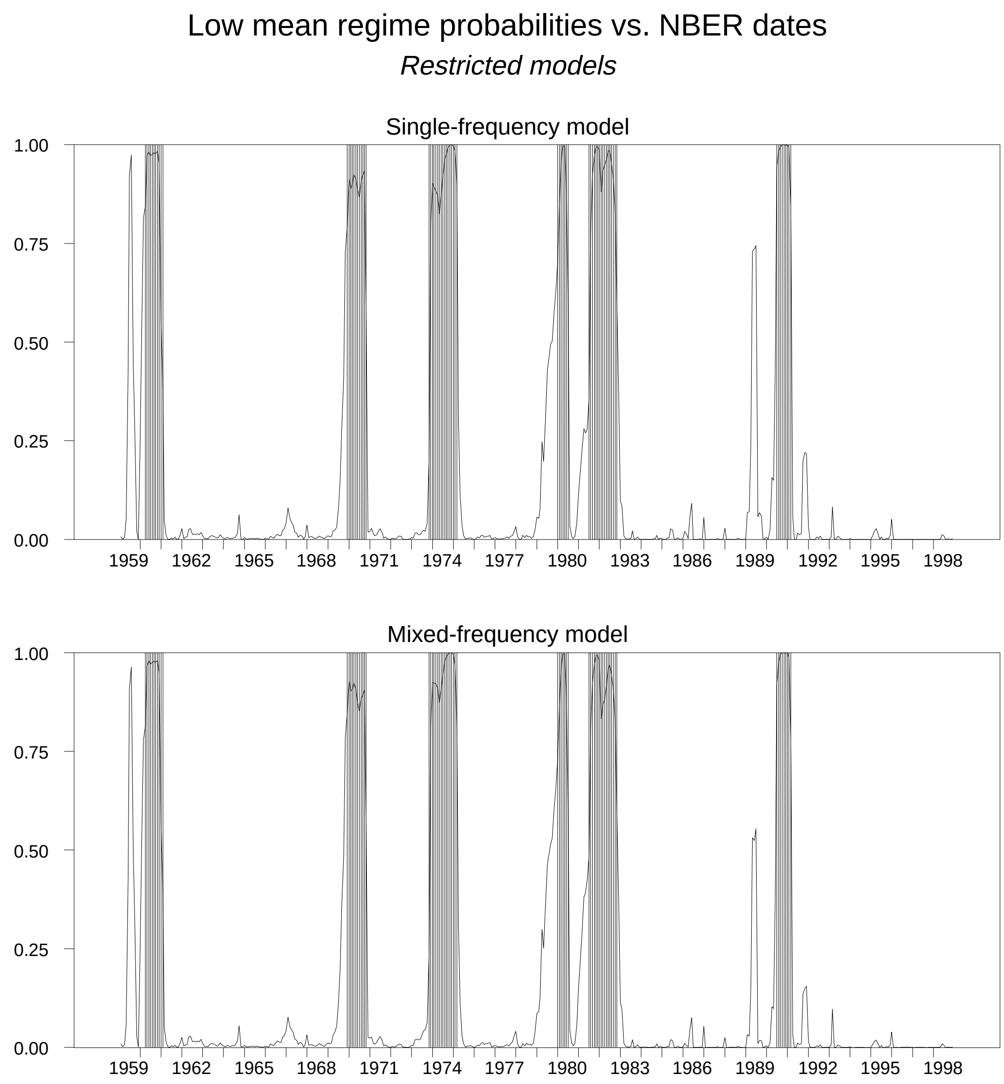
Low mean - high variance regime probabilities vs. NBER dates
Unrestricted model
Single-frequency model
1.0
0.8
0.6
0.4
0.2
0.0
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Mixed-frequency model

Figure 5:



Low variance regime probabilities
Unrestricted model
Single-frequency model
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
SMOOTHED
FILTERED
1959 1962 1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995 1998
Mixed-frequency model
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
SMOOTHED
FILTERED
1959 1962 1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995 1998

Figure 6:



Low mean regime probabilities vs. NBER dates
Restricted models
Single-frequency model
1.00
0.75
0.50
0.25
0.00
1959 1962 1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995 1998
Mixed-frequency model
1.00
0.75
0.50
0.25
0.00
1959 1962 1965 1968 1971 1974 1977 1980 1983 1986 1989 1992 1995 1998

Figure 7:

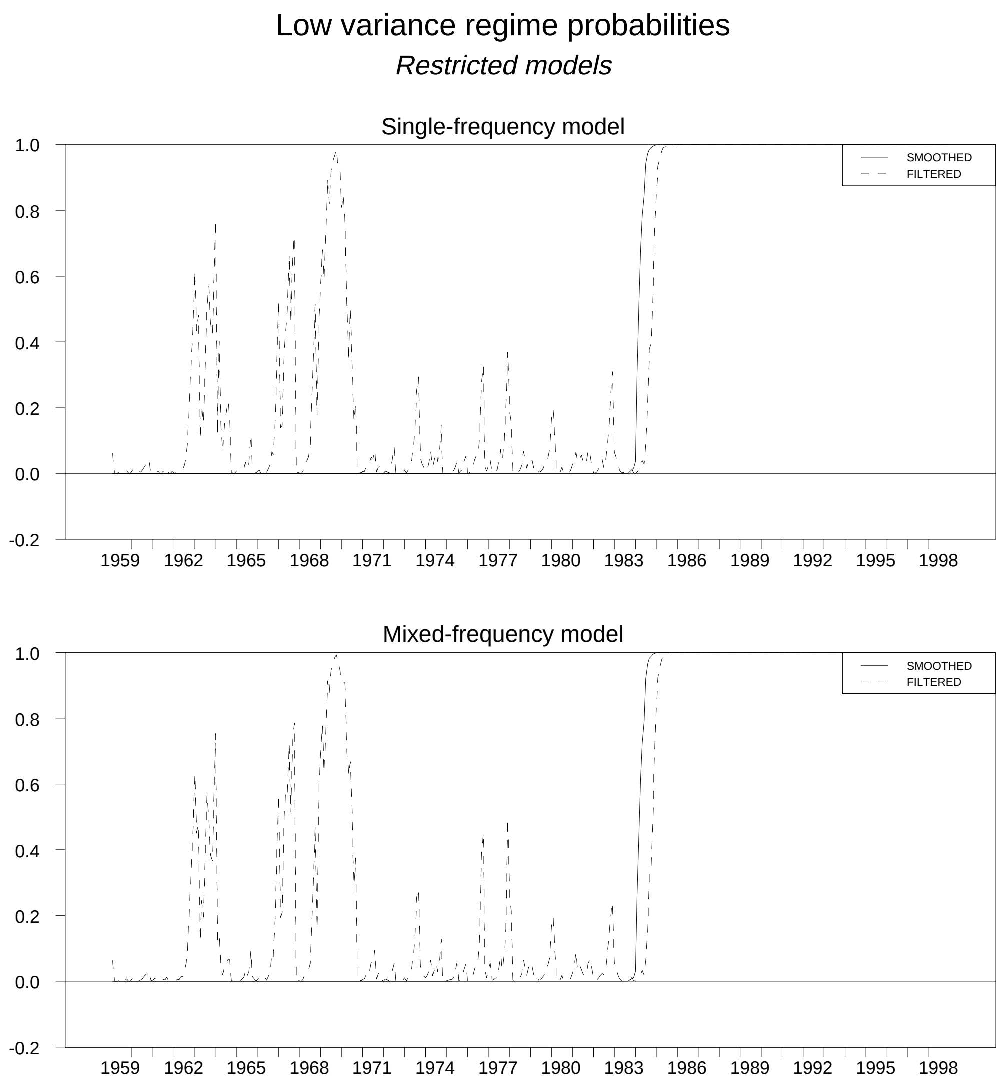

Low variance regime probabilities
Restricted models
Single-frequency model
SMOOTHED
FILTERED
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998
Mixed-frequency model
SMOOTHED
FILTERED
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
1959
1962
1965
1968
1971
1974
1977
1980
1983
1986
1989
1992
1995
1998

Figure 8: