E C O N O M I C S B U L L E T I N

Voting costs and voter welfare

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Abstract

This note considers the effect of the cost of voting on voters' welfare and their incentive to reduce voting costs in a simple model in which two single−voter teams participate to win an election from two alternatives. When each voter's benefit of winning is his private information but uncertain to his rival, reductions in the cost of voting have two opposing effects on each voter's welfare, namely, the direct effect and the indirect effect. And the voters may be worse off as their voting costs decrease. In this case, voters will face a prisoner's dilemma, as each voter has a unilateral incentive to reduce his cost of voting.

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1. Introduction

The literature on strategic voting (e.g., Ledyard, 1981, 1984; Palfrey and Rosenthal, 1983, 1985; Börgers, 2001) has largely focused on the effect of the cost of voting on election turnouts. The general result is that the cost of voting is the dominant factor for voters to participate or turn out. One important issue ignored in the literature is the effect of the cost of voting on voters' welfare and their incentive to reduce the cost of voting.

This note attempts to provide some insight to this issue. To make the point as straightforward as possible, I concentrate on a small electorate. Indeed, the electorate is so small such that there are only two voters, one on each of two competing teams. The voter on one team prefers one alternative, while the voter on the other team prefers the other alternative. If only one voter participates, he wins the election. If both voters participate or abstain, each wins the election with equal chance. Not winning yields zero benefit to a voter, while winning bestows positive benefit on the voter, and participation or voting is costly. Following the literature, I assume that each voter is risk neutral and his utility is given by the difference between the voter's expected benefit of winning and his cost of voting.

Given that there is only one voter on each team and the majority voting, coin toss tie-breaking rule, participation by a voter increases his chance of winning by 50%, no matter whether the other voter participates or not. Thus, a voter participates if and only if his benefit of winning is no less than twice of his cost of voting. Clearly, as the cost of voting decreases, each voter has a greater incentive to participate. One may naturally think that each voter's welfare will also increase as the cost of voting decreases. This indeed is the case when each voter's benefit of winning is public information, because his expected utility is simply half of the benefit of winning minus the cost of voting. However, the assumption that each voter's benefit of winning and his cost of voting are public information is quite strong. In reality, a voter usually has private information about his own benefit and/or cost, while other voters know about only the distributions of the voter's benefit and/or cost (Palfrey and Rosenthal, 1985). In the note, I assume that the cost of voting is the same to every voter, which is common knowledge, but each voter's benefit of winning is his private information. It is important to know whether the result for the competitive participation game of complete information extends to that of incomplete information.

Unfortunately, such an extension does not go through. As the cost of voting decreases, each voter may be worse off ex ante. The reason is as follows. Reductions in the cost of voting have two opposing effects on a voter's welfare. The first is the direct effect. Holding the probability that the other voter participates, the voter's expected utility, as one expects, increases as the cost of voting decreases. The direct effect on voters' welfare of reductions in the cost of voting is beneficial. The second is the indirect, strategic effect. A lower voting cost makes the other voter more likely to participate, reducing the chance that the voter wins the election and hence his expected utility. The indirect effect on voters' welfare of reductions in the cost of voting is detrimental. Depending on parameter values, either the direct effect or the indirect effect may dominate. In the latter case, as the cost of voting decreases, each voter is worse off ex ante.

The cost of voting may decrease exogenously. There may be financial reward to participate, for instance, a voter may be eligible for a drawing to win a lottery. Also, there may be team organizers who arrange to provide voters with door-to-door shuttle services from their homes to the poll station. Should there be the passage of a legislation allowing voters to use work rather than off-work time to go to the poll station, the cost of voting would decrease. More importantly, when each voter has private information about his benefit of winning, he may have a unilateral incentive to lower his cost of voting by incurring an additional expense, because the direct effect on the voter's welfare of reductions in the cost of voting is beneficial. For example, the voter can choose to mail his ballot rather than go to the poll station in person, and may choose to vote online if it is possible. The voter is willing to incur the additional expense so long as the gain of his expected utility through the direct effect outweighs the additional expense.

When the indirect effect dominates the direct effect, each voter faces a prisoner's dilemma. It is in his best interest to incur an additional expense to lower his cost of voting, but both voters would be better off if they could commit not to trying to lower their voting costs.

The reminder of the note is structured as follows. Section 2 describes the model and analyzes the effect of the cost of voting on voters' expected utility, and Section 3 concludes.

2. The Model

There are two teams, 1 and 2, each consisting N_1 and N_2 voters, and two alternatives to vote for. Voters on team 1 prefer alternative 1, while voters on team 2 prefer alternative 2. An election is held, and each voter on a team has two choices to make: participate to vote or abstain. If one alternative has more votes than the other, the alternative with the greater number of voters wins. In the event of a tie, a fair coin is tossed to decide the winner. The cost of voting is $c > 0$ to all voters. This is common knowledge. The benefit of not winning the election is normalized to 0. I consider two specifications of the benefit of winning. In the first, each team member's benefit is known. In the second, the benefit to each team member is that member's private information, but unknown to other members, either on his own team or on the other team.

In this note, I consider the simplest case of team structure in which there is only one voter on each team, $N_1 = N_2 = 1$. In this case, one can alternatively interpret the model as two candidates vote themselves in for an office or more generally, as two players participate to compete for a prize. If only one player participates, the player wins the prize. If both participate or abstain, they get the prize with equal chance.

Let us first analyze the benchmark case where each voter's benefit of winning is common knowledge. Assume further that the benefit of winning is the same to both voters and equal to b > 0. Given the majority voting, coin toss tie-breaking rule, participation increases a voter's chance of winning by 50% compared to not participation, regardless of the participation decision of the other voter. The expected benefit of participation is thus $b/2$, and the voter's expected utility is $b/2 - c$. The voter participates if and only if $b/2 - c > 0$. Suppose that $b > 2c$. Clearly, if the cost of voting c decreases, each voter's expected utility increases.

Sometimes, voters can 'invest' to reduce their voting costs. For example, they can take advantage of information technology by voting online. For simplicity, assume that the investment technology is such that, to reduce the cost of voting from c to $c - \varepsilon$, a voter has to incur an additional expense of $k\varepsilon^2/2$, with $k > 0$ and $\varepsilon > 0$.

With the availability of the investment technology, each voter's expected utility is $b/2 - (c - \varepsilon) - k\varepsilon^2/2$. It is easy to see that the voter will optimally choose $\varepsilon^{opt} = 1/k$, and the voter's expected utility is $b/2 - c + 1/(2k)$. We thus see that the access to the investment technology makes each voter better off.

The assumption that each voter has complete information about the other voter's benefit of winning and cost of voting is overly strong (Palfrey and Rosenthal, 1985). In the real-world situation, a voter usually has private information about his benefit of winning and/or cost of voting. In the following, I extend the analysis by assuming that each voter's benefit of winning is his private information, and that the other voter knows only about the distribution of his rival's benefit of winning. The two voters' benefits of winning are independently and identically distributed according to the cumulative function F(b), with the corresponding density function $f(b) = F'(b) > 0$, for $b \in [\underline{b}, \bar{b}]$, with $\bar{b} > \underline{b} \geq 0$. On the other hand, each voter's cost of voting is c, which is common knowledge. Clearly, the two voters are symmetric ex ante.

Assumption 1. $\underline{b} < 2c < \bar{b}$.

In the following, I focus on analyzing one voter, say, voter 1's ex ante probability to participate, p, and his expected utility, U, and use an asterisk * to denote the corresponding variable for the other voter, voter 2. As before, voter 1 participates if and only if his benefit of winning is no less than twice of his cost of voting. So, the ex ante probability that voter 1 participates is $p = 1 - F(2c)$. Similarly, the ex ante probability that voter 2 participates is $p^* = 1 - F(2c)$. Assumption 1 ensures that each voter participates with positive probability.

If voter 1 participates, then with probability $p^*/2 + 1 - p^* = 1 - p^*/2$ he wins the election. If he does not participate, then he wins with probability $(1 - p^*)/2$. So, voter 1's expected utility is

$$\begin{aligned} U(c, p^*(c)) &= p[(1 - p^*/2)\, \mathbf{E}b|_{b\geq 2c} - c] + (1 - p)[(1 - p^*)/2]\, \mathbf{E}b|_{b\leq 2c} \\ &= (1 - p^*/2)\int_{2c}^{\bar{b}} xf(x)dx + (1 - p^*)/2 \int_{\underline{b}}^{2c} xf(x)dx - [1 - F(2c)]c, \end{aligned} \tag{1}$$

where **E** is the expectation operator over b.

Differentiating U with respect to c, we have

$$dU/dc = \partial U/\partial c + \partial U/\partial p^*\, dp^*/dc. \tag{2}$$

The first term in (2) is the direct effect of the cost of voting on voter 1's expected utility, and the second term is the indirect effect.

Simple computation shows that $\partial U/\partial c = - p = F(2c) - 1 < 0$. The direct effect of reductions in the cost of voting on voter 1's expected utility is beneficial. As the cost of voting decreases, voter 1's expected utility increases. This implies that voter 1 has a *unilateral* incentive to reduce his voting cost.

Given $p^* = 1 - F(2c)$, $dp^*/dc = - 2f(2c) < 0$. It is easy to see that $\partial U/\partial p^* = - \mathbf{E}b/2 < 0$, So, $\partial U/\partial p^*\, dp^*/dc = f(2c)\mathbf{E}b > 0$. The indirect effect of reductions in the cost of voting on voter 1's expected utility is detrimental. A lower voting cost makes voter 2 more likely to vote, reducing the probability that voter 1 wins the election and resulting into lower expected utility to voter 1.

With substitution, we get

$$dU/dc = F(2c) + f(2c)\mathbf{E}b - 1. \tag{3}$$

If $dU/dc > 0$, then a lower cost of voting makes voter 1 worse off ex ante. The same is true for voter 2, as the two voters are symmetric ex ante. On the other hand, if $dU/dc < 0$, then each voter's expected utility increases as the cost of voting decreases.

Example 1. Assume that b is uniformly distributed over the unit interval [0, 1]. It is easy to see from (3) that, if $1/4 < c < 1/2$, $dU/dc > 0$, indicating that voters are worse off when their costs of voting decrease.

Recall that $\partial U/\partial c < 0$. This implies that each voter has a unilateral incentive to reduce his voting cost. However, when both voters respond to this incentive, both may be worse off. This happens when $dU/dc > 0$. In this case, each voter faces a prisoner's dilemma. It is in his best interest to lower his cost of voting, no matter whether the other voter lowers his voting cost or not. But when both try to lower their costs of voting, both are worse off compared to the case in which they could commit not to lowering their voting costs.

Example 1 (continued). Suppose now that there is an investment technology that enables voter 1 to lower his voting cost from c to $c - \varepsilon$ by incurring expense $k\varepsilon^2/2$, $k > 0$. The same technology is also available to voter 2. The voters play a two-stage game. In the first stage, they independently decide whether to lower their voting costs. In the second stage, they decide independently whether to participate.

Voter 1's choice in the second stage is easy. Recall that he participates if only if his benefit of winning is no less than twice of his cost of voting, $2(c - \varepsilon)$. So, $p = p(\varepsilon) = 1 - F(2c - 2\varepsilon)$. Similarly, $p^* = p^*(\varepsilon^*) = 1 - F(2c - 2\varepsilon^*)$.

In the first stage, voter 1 chooses ε to maximize his expected utility $V = U(c - \varepsilon, p^*(\varepsilon^*)) - k\varepsilon^2/2$, taking ε^* as given. The first-order condition is $1 - F(2c - 2\varepsilon) - k\varepsilon = 0$. The second-order condition requires that $k > 2$. The optimal ε is a function of k, $\varepsilon^{opt}(k) = (1 - 2c)/(k - 2)$.

One can easily compute from (1) that $U(c) = 1/4 - c/2 + c^2$. Suppose initially that $c = 1/3$. It is easy to show that, for $k > 6$, $c - \varepsilon^{opt}(k) > 1/4$, and hence $U(1/3) > U(1/3 - \varepsilon^{opt}(k))$. Indeed, by the convexity of U(.), one can further show that for $k > 4$, $U(1/3) > U(1/3 - \varepsilon^{opt}(k))$. Let $k = 8$. Then, $\varepsilon^{opt}(8) = 1/18$, and $c - \varepsilon^{opt}(8) = 1/3 - 1/18 = 5/18$. $U(1/3) = 63/324$, but $U(5/18) = 61/324$.

3. Conclusion

This note considers the effect of the cost of voting on voters' welfare. There are two teams, voting for two alternatives. The team that comes up with more voters wins the election, and when the two teams tie, a fair coin is tossed to decide the winning team. I analyze the simplest case in which each team is composed of a single voter. For simplicity, the two voters are assumed to be symmetric. I show that, when voters' benefits of winning are common knowledge, reductions in the cost of voting make each voter better off. And each voter has an incentive to reduce his cost of voting.

When each voter's benefit of winning is his private information but uncertain to his rival, the voters may be worse off as their voting costs decrease. Given the probability that the rival votes, each voter's expected utility increases as the voting cost decreases. This is the direct effect of voting costs on a voter's welfare. On the other hand, a lower voting cost makes the rival more likely to vote, making the voter less likely to win the election. This is the indirect effect of voting costs on a voter's welfare, which works against the direct effect. Depending on parameter values, either effect can dominate, implying that a lower cost of voting may make the voters either better off or worse off ex ante. Moreover, each voter has a unilateral incentive to reduce his cost of voting, as the direct effect on the voter's welfare of reductions in the cost of voting is always beneficial. When the indirect effect dominates the direct effect, each voter faces a prisoner's dilemma. It is in his best interest to reduce his cost of voting, no matter whether his rival reduces the cost of voting. But both would be better off if they could commit not to reducing their voting costs.

If the asymmetric status quo tie-breaking rule is used, under which a voter is designated as the winner if both teams come up the same number of participants, the analysis needs to be adjusted. But it can be shown that the qualitative result does not change. In other words, as the cost of voting decreases, both voters can be worse off ex ante in the majority voting, status quo tie-breaking rule.

The bare-bone model in which each team consists of only one voter abstracts many important issues. First, participation by each voter increases his chance of winning the prize by 50%, no matter whether his rival votes or not. If there is more than one voter on a team, this would not be the case. Each team member's net benefit of voting will depend on the voting decision of members on the competing team. Second, since there is only one voter on each team, there is no free-rider problem on each team. This is an important issue to consider when one analyzes voting behavior in a large electorate. It is interesting to study the effect of voting costs on each team member's voting behavior and his welfare, when each team consists of more than one voter. I leave it for future research.

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