E C O N O M I C S   B U L L E T I N

Exchange of intermediate goods and the agglomeration of firms

Jose Pedro Pontes
Technical University of Lisbon

Abstract

In a game where firms select locations,technological interactions through the exchange of intermediate goods bring about a multiplicity of locational equilibria and entail a pattern of agglomeration of the productive activity with the variation of the transport costs that is opposite to the one usually proposed in the literature, namely in VENABLES (1996). VENABLES, Anthony (1996), "Equilibrium locations of vertically linked industries", International Economic Review, 37 (2): 341−359.

The author wishes to thank Steve Brams, Pablo Garza, Joao Gata, Armando Pires , Vasco Santos for helpful comments and Masahisa Fujita for helpful suggestions. The usual disclaimer applies. This work had the support of the Research Unit of Complexity in Economics (UECE)
Citation: Pontes, Jose Pedro, (2002) "Exchange of intermediate goods and the agglomeration of firms." *Economics Bulletin*, Vol. 3, No. 27 pp. 1−7
Submitted: November 22, 2002. **Accepted:** November 22, 2002.
URL: http://www.economicsbulletin.com/2002/volume3/EB−02C70019A.pdf

1 Introduction

Technological interactions through the exchange of intermediate goods between firms that compete through the choice of locations have two different kinds of impact. The most conspicuous is the incentive that they provide to firms' spatial agglomeration. According to VENABLES (1996), this incentive follows a so-called "U-pattern": it is maximal for intermediate levels of the transport cost of inputs and negligible for extreme values. Assuming that there is some proportional relationship between the transport costs of intermediate goods and final goods, if these costs are very high, firms are dispersed in space in order to serve consumers. If transport costs are very low, no incentive for agglomeration exists.

A second feature of the exchange of intermediate goods is the multiplicity of locational equilibria, which was shown by FUJITA (1981) and that arises because a location is advantageous for a firm simply because it has in its neighborhood firms that are connected with it by input-output relations.

In this paper, the spatial effects of the trade of intermediate goods is dealt with in a game-theoretic framework that keeps the core result of multiplicity of locational equilibria and, at the same time, proposes a pattern of agglomeration that is opposite to VENABLES (1996).

2 Assumptions of the model

The economy obeys the following assumptions:

1. There are two countries that are labelled A and B. Country A is larger than country B but the asymmetry is bounded from above so that the relation between the populations n_a and n_b is given by $2n_b > n_a > n_b$.

2. There are two downstream firms, labelled 1 and 2, that produce a homogeneous consumer good, and an upstream firm, labelled firm 3, that produces and sells an intermediate good to the downstream firms.

3. There is no price competition, so that each firm charges a parametric mill price. Downstream firms charge mill price $\overline{p}$ and the upstream firm quotes the mill price p_w. The firms have constant unit production costs c and c_w that are strictly less than the respective mill prices.

4. Transport costs per unit of product per unit of distance are given by t for the consumer good and by t_w for the intermediate good. The distance function between two locations s_i, s_j is given by

$$d(s_i, s_j) = \begin{cases} 0 & if \quad s_i = s_j \\ 1 & if \quad s_i \neq s_j \end{cases}$$

5. Transport costs of the goods (both consumer and intermediate) are supported by the buyers.

6. Consumers have a 0-1 demand function with reservation price v. They purchase the consumption good from the downstream firm that charges the lower delivered price, provided that this does not exceed the reservation price. Otherwise they refrain from purchasing. If two firms charge the same delivered price, the consumers buys from each one with probability $\frac{1}{2}$.

7. Downstream firms use the intermediate good in fixed proportions with the final good: α units of input are required to produce one unit of output. The downstream firm produces and purchases the input if its profit is positive. Otherwise it exits the market.

8. The operating profit of a downstream firm i gross of the transport cost of the input is positive

$$\pi_i = \overline{p} - c - \alpha\overline{p}_w > 0 \tag{1}$$

3 The normal form of the game

The economy can be modelled by a static noncooperative game whose normal form is described in the following way. There are three players, downstream firms $1, 2$ and upstream firm 3. Each firm has a strategy set made by the following pure strategies: "locate in country A" (strategy A); "locate in country B" (strategy B); and "exit the market" (strategy E). The strategy of firm i $(i = 1, 2, 3)$ is expressed by s_i. The payoff function of the downstream firm i $(i = 1, 2)$ is

$$P_i(s_1, s_2, s_3) = \begin{cases} D_i(s_1, s_2)\left[\overline{p} - c - \alpha\overline{p}_w - \alpha t_w d(s_i, s_3)\right] & if\ s_i \text{ and } s_3 \epsilon \{A, B\} \\ 0 & otherwise \end{cases} \tag{2}$$

where $D_i(s_1, s_2)$ is the demand addressed to firm i. By definition, $D_i(s_1, s_2)$ is zero if firm i exits the market. The payoff function of the upstream firm is

$$P_3(s_1, s_2, s_3) = \begin{cases} \alpha(\overline{p}_w - c_w)\left[D_1(s_1, s_2) + D_2(s_1, s_2)\right] & if \quad s_3 \epsilon \{A, B\} \\ 0 & if \quad s_3 = E \end{cases} \tag{3}$$

The following Propositions describe the solution of the game.

Proposition 1 *If (s_1, s_2, s_3) is a Nash equilibrium, then either $s_1 = s_2 = s_3 = E$ or s_1, s_2 and $s_3 \epsilon \{A, B\}$.*

Proof. As firms do not compete in prices, a downstream firm that stays in the market has a positive demand in any location. Hence, if the upstream firm stays in the market, each downstream firm can achieve a positive profit by locating close to it in order to avoid the transport cost of the input (see 1). If a downstream firm stays in the market, the upstream firm can get a positive profit by entering the market (see 3). ■

Lemma 2 *$s_1 = s_2 = s_3 = E$ is a (weak) Nash equilibrium.*

Proof. The proof is obvious. ■

In order to describe the set of Nash equilibria when the three firms stay in the market, the following Lemma is a simplifying device.

Lemma 3 *If s_1, s_2 and $s_3 \epsilon \{A, B\}$, the locational decision of the upstream firm is independent of the strategy choices of the downstream firms.*

Proof. From 3, it is clear that if $s_3 \epsilon \{A, B\}$ the payoff of firm 3 is a constant function of s_3. ■

Lemma 3 simplifies the solution of the game: we first compute the best decision of the upstream firm and then solve the truncated 2-person game played by the downstream firms. The problem lies in the multiplicity of equilibria, namely the infinite multiplicity of best choices by firm 3. This multiplicity follows from the trade of intermediate goods in a spatial economy, as was outlined in the Introduction. Taking as a reference SCHELLING's (1960) concept of "focal point", equilibria where all firms stay in the market and the upstream firm locates in the large country A will be selected.[1]

We concentrate now on the classes of the truncated games played by the downstream firms as defined by parameters α, n_a, n_b and t, assuming that the input supplier locates in A. We make the assumption that the transport cost of the consumption good and the input change in proportion so that $t = t_w$. [2]. The remaining parameters are fixed in the following way:

$$\begin{aligned} \overline{p} &= \overline{p}_w = 1 \\ c &= 0 \\ v &= 2 \end{aligned} \tag{4}$$

With these specifications, constraint 1 becomes $0 < \alpha < 1$.

The first class of games corresponds to the situation where transport costs are high, so that each firm can only sell in the market where it is located: $\overline{p} + t > v$, or (according to 4) $t > 1$. As the game is symmetric, only the payoffs (given by 2) of firm 1 are included. The matrix is

$$\begin{array}{cccc} & & 2 & \\ & & A & B \\ 1 & A & (1-\alpha)\dfrac{n_a}{2} & (1-\alpha)\, n_a \\ & B & [1-\alpha(1+t)]\, n_b & [1-\alpha(1+t)]\dfrac{n_b}{2} \end{array} \tag{5}$$

[1] The selection of country A as the location of the input supplier is not necessary. The same fundamental results would be obtained if the upstream firm located in country B.

[2] This assumption is different from the one that is made in FUJITA and HAMAGUCHI (2001). However, it is realistic to think that the transport costs of final and intermediate goods have the same determinants.

Applying a best-reply structure preserving transformation, by subtracting a_{21} from the first column and a_{12} from the second column, we obtain the equivalent matrix

$$\begin{array}{ccc} & & 2 & \\ & & A & B \\ 1 & A & (1-\alpha)\left(\frac{n_a}{2}-n_b\right)+\alpha t n_b & 0 \\ & B & 0 & [1-\alpha(1+t)]\frac{n_b}{2}-(1-\alpha)n_a \end{array} \tag{6}$$

It is clear that $a_2 = [1-\alpha(1+t)]\frac{n_b}{2} - (1-\alpha)n_a < 0$. On the other hand, $a_1 = (1-\alpha)\left(\frac{n_a}{2}-n_b\right)+\alpha t n_b$ has a negative and a positive term. It is clear that a_1 increases strictly with α and t. Furthermore, a_1 has an infimum

$$\underline{a}_1 = (1-\alpha)\left(\frac{n_a}{2}-n_b\right)+\alpha n_b \tag{7}$$

$\underline{a}_1$ is positive if

$$\alpha > \frac{2n_b - n_a}{4n_b - n_a} \tag{8}$$

We can conclude that, in this class of games, if condition 8 is fulfilled, strategy A strictly dominates strategy B for both firms, so that an equilibrium in dominant strategies arises for any values of α and t. On the other hand, if condition 8 is not met, for low values of α and t, a_1 will be negative and we have two asymmetric Nash equilibria (A, B) and (B, A). For high values of α and t, a_1 will be positive so that we again have an equilibrium in dominant strategies with A as a strictly dominant strategy for both firms.

In the second class of games, transport costs are relatively low, so that each firm can sell not only in its local market but also in the distant country: $\overline{p}+t < v$, or, according to 4, $t < 1$. In this symmetric class of games, the payoff matrix of firm 1 is

$$\begin{array}{cccc} & & 2 & \\ & & A & B \\ 1 & A & (1-\alpha)\frac{n_a+n_b}{2} & (1-\alpha)n_a \\ & B & [1-\alpha(1+t)]n_b & [1-\alpha(1+t)]\frac{n_a+n_b}{2} \end{array} \tag{9}$$

It can be easily checked that, in the class of games represented by matrix 9, there is a unique equilibrium in dominant strategies for all values of the parameters, with A being a dominant strategy for both players.

It is possible to summarize the Nash equilibria in the (α, t) space in Figure 1.

[Insert here Figure 1]

4 Conclusions

If the intensity of use of the intermediate good is not too high (otherwise the location problem is trivial), the geography of production has the following pattern. If the transport cost is very high, the input cannot be imported and both firms locate in the large country. Consumers in the small country are not served. If the transport cost is intermediate, the firms disperse across the regions and supply local consumers. If transport costs are low, each firm locates in the central point of the spatial distribution of consumers (that is to say, the large country) and sells to consumers in both markets. This pattern is opposite to the one proposed by VENABLES (1996).

This opposite pattern follows from the different degree of divisibility of the upstream industry (a monopoly) and the downstream industry (a duopoly). If both industries had two firms, for high transport costs an upstream-downstream pair would locate in each country supplying the local customers, thus reproducing the standard argument. However, according to KOOPMANS and BECKMANN (1957) classical paper, the role of the exchange of intermediate goods in a spatial economy is non trivial precisely on account of differences of divisibility.

Further work in this field would include consideration of price competition in addition to the spatial competition that is exclusively analysed in this paper.

References

[1] FUJITA, Masahisa (1981), "Location of firms with input transactions", *Environment and Planning A*, 13: 301-320.

[2] FUJITA, Masahisa and Nobuaki HAMAGUCHI (2001), "Intermediate goods and the spatial structure of an economy", *Regional Science and Urban Economics,* 31: 79-109.

[3] KOOPMANS, Tjalling and Martin BECKMANN (1957), "Assignment problems and the location of economic activities", *Econometrica*, vol. 25, pp. 53-76.

[4] SCHELLING, Thomas (1960), *The Strategy of Conflict,* MA, Harvard University Press, 1960.

[5] VENABLES, Anthony (1996), "Equilibrium locations of vertically linked industries", *International Economic Review,* 37(2), May: 341-359.

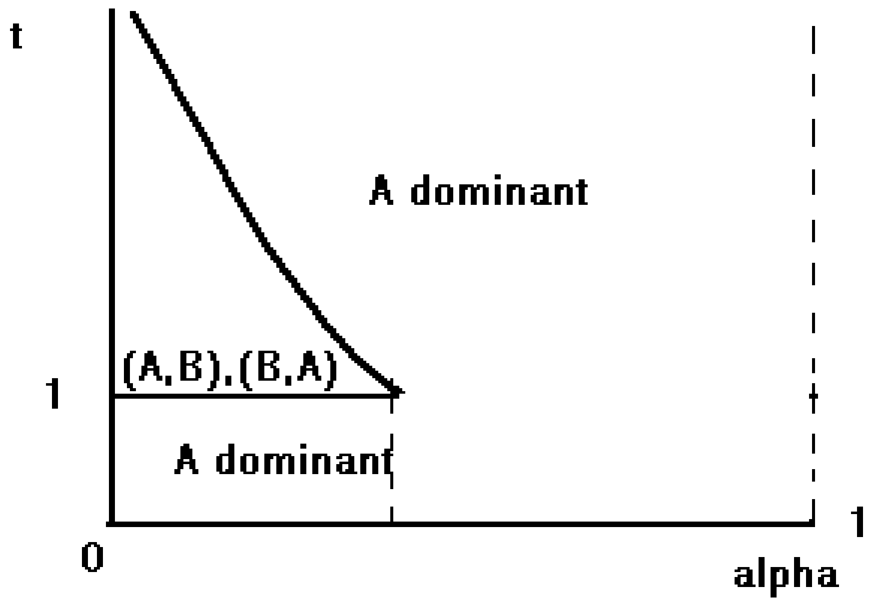

Figure 1: Nash equilibria in (α, t) space