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Capital Accumulation and Habit Formation

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Abstract

This paper investigates the impact of habits on savings and steady state capital intensity. Within the framework of an OLG economy with productive capital, a rise in the strength of habits increases savings if the steady state is asymptotically stable. Consequently, the steady state capital intensity as implied by an OLG model with habits is higher compared to the case with time−separable utility. If, however the initial steady state is unstable, a rise in the strength of habits lowers savings.

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1 Introduction

Psychologists offer significant evidence for habit persistence. The evidence indicates that a household's perceived well-being is much more related to recent changes than to absolute levels of consumption.[1] Modeling of this finding is possible when habit persistence is introduced into the utility function. In this case a consumer's utility depends not only on the level of her consumption but also on a reference level of past consumption (stock of habits).

This paper investigates the impact of (the strength of) habits on capital accumulation within a Diamond (1965)-style overlapping generations (OLG) framework with productive capital. The model resembles that of Decreuse and Thibault (2001), except that households only work in the first period of life. It shows that an increase in the strength of habits raises savings and capital intensity in a stable steady state. Compared to the case without habits (and with a time-separable utility function) habit persistence implies a steeper consumption path over time, because consumers need to offset the negative effects of an increasing stock of habits. Thus, young households raise savings which results in a higher steady state capital intensity.

In an OLG framework the impact of habits on savings has already been addressed by Lahiri and Puhakka (1998). They show for an OLG *exchange economy* that an increase in the strength of habits raises desired savings. This paper extends the analysis to a model with production and capital accumulation. One might presume that the result carries over from an exchange economy to one with production and capital accumulation. However, in an economy with productive capital two complications arise. First, in a model with capital accumulation higher savings imply a rise in the capital intensity, which in turn lowers the capital rental rate. A lower capital rental rate *lowers* desired savings. Second, a higher strength of habits implies a steeper consumption path over time. Ihori (1978) shows that along the "intertemporal consumption possibility curve" (see Section 3) there are segments along which a rise in savings (in the steady state capital intensity) can result in a flatter consumption path over time. For both reasons it is not obvious whether savings will be higher or lower in the new steady state compared to the initial steady state.

[1] Scitovsky (1992) lays out the state-of-the-art findings in psychological research on the theory of human satisfaction.

The paper demonstrates that under general assumptions the positive relationship between the strength of habits and savings (the steady state capital intensity) does indeed hold not only in an exchange economy but also in an economy with production for *stable* steady states. If, however, the steady state is unstable a rise in the strength of habits implies a lower steady state capital intensity.

2 An OLG Economy with Habits

We consider a fully competitive economy with productive capital that has two overlapping generations, one young household and one old household. Each household is alive for two periods and has perfect foresight. Economic activity is performed over infinite discrete time. Each young household is endowed with L_t units of labor, which are inelastically supplied to the labor market in the first period of life. Over time, the endowment of labor grows at an exogenous rate $n \geq -1$.

The young household receives a wage rate w_t per unit of labor. This is allocated to consumption, c_t^1, and savings, s_t. Superscript 1 denotes consumption in the first period of life, i.e., consumption of the young household: $c_t^1 + s_t = w_t$. Savings are equal to the purchased (depreciated) capital stock of the old household plus investment. Once the household becomes old and enters period 2, the only economic activity is consumption. Both savings and interest on savings are fully consumed: $c_{t+1}^2 = (1 + r_{t+1})s_t$. Superscript 2 refers to the older household and r denotes the real interest rate. Each young household maximizes utility subject to its intertemporal budget constraint.

$$c_t^1 + \frac{c_{t+1}^2}{1 + r_{t+1}} = w_t \tag{1}$$

A household's instantaneous utility depends not only on the level of its consumption but also on a reference level of past consumption (on a stock of habits). Thus, habit formation implies that households derive utility not only from the absolute level of consumption in the two periods but also from increasing consumption in the second period relative to the first period. The intertemporal utility function of a household born at time t becomes:

$$U_t = u\left(c_t^1\right) + \beta\, u\left(\hat{c}_{t+1}^2\right) \quad \text{where} \quad \hat{c}_{t+1}^2 = c_{t+1}^2 - \delta\, c_t^1. \tag{2}$$

The parameter β is the subjective discount factor. Following Lahiri and Puhakka (1998), instantaneous utility in the second period of life is derived from the difference of current consumption and a fraction of past consumption. Parameter $\delta \in [0,1]$ indexes the *strength of habits* or the *importance of past consumption* in the instantaneous utility function. The weight that is attached to past consumption increases in δ. If $\delta = 0$, the past consumption has no weight at all. Then (2) corresponds to the standard time separable model, and utility is fully determined by absolute consumption *levels* (and not by the changes in consumption).

ASSUMPTION 1 (Utility) *The felicity function $u : \mathbb{R}_+ \to \mathbb{R}$ is defined over non-negative consumption during the first and second periods of life. It is twice continuously differentiable, increasing in both arguments and strictly quasiconcave. In particular, it satisfies $u'(\cdot) > 0$ and $u''(\cdot) < 0$. As $\arg u \to 0 : u'(\cdot) \to \infty$ and as $\arg u \to \infty : u'(\cdot) \to 0$.*

The marginal rate of intertemporal substitution becomes:

$$\frac{d\,c^2_{t+1}}{d\,c^1_t} = -\frac{u'\left(c^1_t\right) - \delta\,\beta\,u'\left(\hat{c}^2_{t+1}\right)}{\beta\,u'\left(\hat{c}^2_{t+1}\right)}\,. \tag{3}$$

According to (3) a rise in the strength of habits, δ, lowers the (absolute value of the) marginal rate of intertemporal substitution. For any given reduction in first period consumption, stronger habits imply that less compensation is required to be equally well off in period 2. Therefore, at any given consumption path $(c_t,\, c_{t+1})$ the indifference curve becomes flatter.[2]

Production of a single commodity, Y_t, takes place according to a (linearly homogeneous) constant returns to scale technology (4) with two factors of production — capital, K_t, and labor, L_t. In (4), k denotes capital intensity and y stands for output per capita.

$$y_t = f(k_t) \quad \text{with } y_t \equiv \frac{Y_t}{L_t}\,, \;\; k_t \equiv \frac{K_t}{L_t} \tag{4}$$

ASSUMPTION 2 (Production) *(i) The production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ is invariant over time; specifically, there is no technological progress. It is*

[2]As Lahiri and Puhakka (1998) point out, the domain of δ is limited. In order for the indifference curves to be downward sloping, the parameters of the model must satisfy $0 \le \delta \le [u'(c^1_t)]/[\beta\,u'(\hat{c}^2_{t+1})]$.

twice continuously differentiable, increasing and strictly concave. Technology allows for positive and decreasing marginal product of k*. Particularly,* $f(0) = 0$*,* $f'(k) > 0$ *and* $f''(k) < 0$ *for all* k *strictly positive. (ii)* $\lim_{k\to 0} f'(k) = \infty$ *and* $\lim_{k\to\infty} f'(k) = 0$*. (iii)* $k\, f''(k) + f'(k) > 0$*.*

Output in period t is allocated to consumption and investment. Market clearing of the goods market requires (5).

$$y_t = c_t^1 + \frac{c_t^2}{1+n} + s_t \tag{5}$$

Firms are assumed to maximize profits. Moreover, the rate of depreciation of physical capital is set equal to one.[3] Since all agents are price takers, the factors of production are paid their respective marginal product.

$$w_t = f(k_t) - k_t f'(k_t) \tag{6}$$

$$1 + r_t = f'(k_t) \tag{7}$$

Considering the market clearing condition (5) along with (6) and (7), it follows that savings in period t constitute capital in period $t + 1$.

$$k_{t+1}(1+n) = s_t(w_t,\, 1 + r_{t+1}) \tag{8}$$

As households maximize utility (2) subject to the intertemporal budget constraint (1) they choose savings such that U_t is maximized given wages and correctly foreseen capital rental rates.

$$s_t(w_t,\, r_{t+1}) = \arg\max_s u(w_t - s_t) + \beta\, u\left((1 + r_{t+1})s_t - \delta(w_t - s_t)\right) \tag{9}$$

An *intertemporal equilibrium* in this OLG economy is an exogenously given endowment, k_0, and a sequence $\langle k_t \rangle_{t=1}^{\infty}$ following from optimization, (6), (7), (9), and market clearing, (8). A *steady state equilibrium* is a stationary capital intensity, k, such that

$$k = \frac{s\left(f(k) - k\, f'(k),\, f'(k)\right)}{1+n}\,. \tag{10}$$

From Assumption 2 it follows that there exists an upper bound to the attainable capital, $\tilde{k}$, such that $f(\tilde{k}) = (1+n)\,\tilde{k}$. Galor and Ryder (1991, p.387)

[3] Both assumptions — that the rate of depreciation is equal to one and that there is no technological progress — simplify the analysis without changing the results of the paper.

demonstrate that under the assumptions adopted in this paper there exists a steady state equilibrium. Moreover, all steady state equilibria lie in the interval $[0, \tilde{k})$. [4]

3 Capital Accumulation and Habits

This section analyzes the impact of habit formation on savings and capital accumulation. Considering (9), the first order condition (FOC) for the optimization problem becomes:

$$-u'\left(c_t^1\right) + \beta\left[f'(k_{t+1}) + \delta\right] u'\left(\hat{c}_{t+1}^2\right) = 0. \tag{11}$$

For an exchange economy Lahiri and Puhakka (1998) showed that habits raise desired savings. The following lemma shows that their result also holds for an economy with production in the short run.

LEMMA 1 *In the short run, for a given k_0, habit formation increases savings of the young household: $d\,s(k_0)/d\,\delta > 0$.*

Proof. Differentiating the FOC with respect to s_0 and δ and considering that k_0 is given by last period's savings, the following differential arises:

$$\frac{d\,s(k_0)}{d\,\delta} = \frac{-\beta\,u'\left(\hat{c}_1^2\right) - \left[f'(k_1) + \delta\right]\beta\,u''\left(\hat{c}_1^2\right)\left(-w_0 + s_0\right)}{u''\left(c_0^1\right) + \beta\,u''\left(\hat{c}_1^2\right)\left[f'(k_1) + \delta\right]^2}. \tag{12}$$

Since $(-w_t + s_t) \leq 0$, it follows that $d\,s_0/d\,\delta > 0$. □

The consumer is conscious of the fact that higher consumption now will lead to a higher future marginal utility in the future. Compared to the no-habits case with time separable utility, habit forming households choose a steeper consumption profile over time. Habits provide a reason for postponing consumption because households benefit not only from consumption levels but also from consumption *growth* (see Deaton 1992, p.34). A higher δ, thus, requires a steeper consumption profile over time.

[4] Let $\phi' \equiv d\,k_{t+1}/d\,k_t$. Assumptions 1 and 2 imply $\phi' \geq 0\,\forall\,k > 0$. If, moreover, the following conditions hold, there is a unique and stable steady state(see Galor and Ryder 1989): (i) $\lim_{k\to 0}\phi' > 1$, (ii) $\phi'' \leq 0\,\forall\,k > 0$, (iii) $d\,s/d\,q \geq 0 \Leftrightarrow u'(\hat{c}^2) \geq -(q+\delta)\,k\,(1+n)\,u''(\hat{c}^2)$.

PROPOSITION 1 *Consider an asymptotically stable steady state in an OLG economy with production. The greater the strength of habits, δ, the higher the steady state capital intensity: $d\,k/d\,\delta > 0$.*

Proof. Differentiating FOC (11) with respect to k and δ gives rise to the following differential (13):

$$\begin{aligned} \frac{d\,k}{d\,\delta} &= \frac{-\beta\,u'\,(\hat{c}^2) + [f'(k)+\delta]\,\beta\,u''\,(\hat{c}^2)\,c^1}{-u''(c^1)\gamma^1 + \beta\,u'\,(\hat{c}^2)\,f''(k) + \beta[f'+\delta]u''(\hat{c}^2)\gamma^2}. \\ \gamma^1 &\equiv -k\,f''(k) - (1+n) \\ \gamma^2 &\equiv [k\,f''(k) + f'(k)]\,(1+n) - \delta\,[-k\,f''(k) - (1+n)] \end{aligned} \tag{13}$$

Consider first the case when the capital intensity, k, is high enough that $-k\,f''(k) - (1+n) \leq 0$. Then $\gamma^1 \leq 0$ and $\gamma^2 > 0$. Since, by assumption, $k\,f''(k) + f'(k) > 0$, differential (13) is positive in this case: $d\,k/d\,\delta > 0$. This case refers to the "efficient zone" of *Ihori*'s (1978) intertemporal consumption possibility curve, i.e. $\partial c^2/\partial c^1 < 0$ (see Figure 1 below).
The differential $d\,k/d\,\delta$ is also positive if $-k\,f''(k) - (1+n) > 0$. Implicitly differentiate FOC (11) with respect to k_{t+1} as well as k_t. Then we can write $d\,k_{t+1}/d\,k_t$ as:

$$\begin{aligned} &\phi'(k_{t+1}, k_t) \equiv \frac{d\,k_{t+1}}{d\,k_t} = \\ &\frac{-u''(c_t^1)\,k_t\,f''(k_t) - \beta\,\delta\,[f'(k_{t+1}) + \delta]\,u''(\hat{c}_{t+1}^2)\,k_t\,f''(k_t)}{u''(c_t^1)\,(1+n) + u'(\hat{c}_{t+1}^2)\,\beta\,f''(k_{t+1}) + u''(\hat{c}_{t+1}^2)\,\beta\,[f'(k_{t+1}) + \delta]\,\gamma^3} \\ &\text{with } \gamma^3 \equiv [f'(k_{t+1}) + \delta + k_{t+1}\,f''(k_{t+1})](1+n)\,, \end{aligned} \tag{14}$$

where $\phi(k_{t+1}, k_t)$ represents the equation of motion of the (one-dimensional) dynamic system of the model. In order for a steady state to be stable $\phi'(k_{t+1}, k_t)$ must be smaller than unity in a neighborhood of the steady state. Both the numerator and the denominator are negative in (15). Therefore, the condition $\phi'(k) < 1$ implies $-u''(c^1)\gamma^1 + \beta\,u'\,(\hat{c}^2)\,f''(k) + \beta[f'+\delta]u''(\hat{c}^2)\gamma^2$ is negative. This expression, however, equals the denominator of (13) Thus, for a *stable* steady state the denominator of (13) is negative, and stability implies $d\,k/d\,\delta > 0$. □

A rise in the strength of habits lowers the marginal rate of intertemporal substitution, because households derive utility not only from total consumption in both periods but also from an increase in consumption in the second period. At a given interest rate households shift consumption from period one to period two, thus fostering savings. In the long run, at the new steady state, capital intensity, savings and the wage rate are higher, and the interest rate is lower than at the initial steady state. Moreover, the consumption path (c_t, c_{t+1}) is steeper compared to the initial steady state.

COROLLARY 1 *Consider an unstable steady state in an OLG economy with production. The greater the strength of the habits, δ, the lower the steady state capital intensity: $d\,k/d\,\delta < 0$.*

Proof. For an unstable steady state it holds that $\phi'(k_{t+1}, k_t)$ exceeds unity in the steady state. Because both the numerator and the denominator are negative in (15), the condition $\phi'(k) > 1$ requires $-u''(c^1)\gamma^1 + \beta\, u'\,(\hat{c}^2)\, f''(k) + \beta[f' + \delta]u''(\hat{c}^2)\gamma^2$ to be positive. Thus, for an *unstable* steady state the denominator of (13) is positive. Hence, instability of the steady state implies $d\,k/d\,\delta < 0$. □

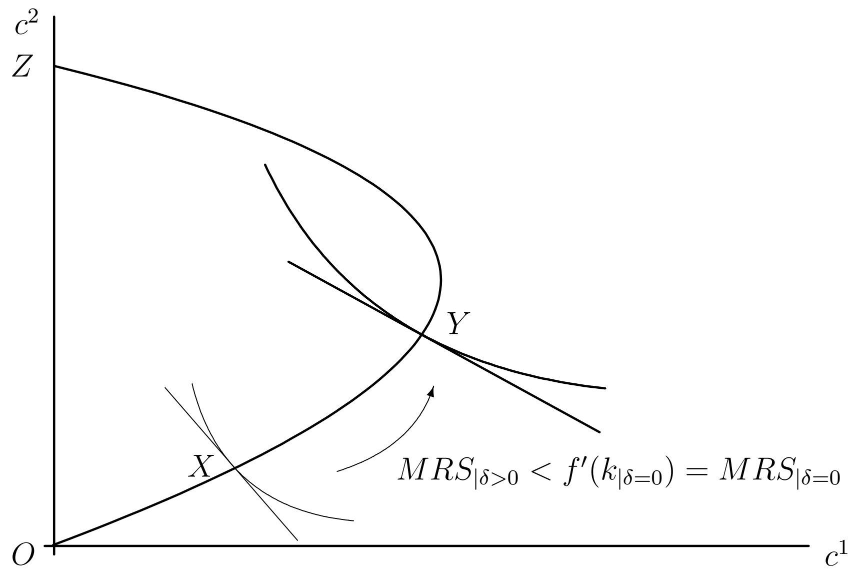

Figure 1. Habits and Optimum Consumption Plans

Figure 1 illustrates Proposition 1. The figure makes use of Ihori's (1978) consumption possibility curve ($OXYZ$). It represents the intersection of $\mathbb{R}^2_+$ with $(c^1(k), c^2(k))$, where $c1(k) = f(k) - k\,f'(k) - (1+n)\,k$ and $c^2 = (1+n)\,k\,f'(k)$. c^1 and c^2 describe the consumption possibilities — i.e., the household budget constraints are binding and the first order conditions for a profit maximum are satisfied — as functions of the steady state capital intensity. Each point along this curve is associated with a specific capital intensity; k is zero at the origin and steadily increases along $OXYZ$. As one moves upward, c^2 rises, the interest rate becomes lower and c^1 initially rises and then falls. Point X depicts an optimum for the case of δ equal to zero. However, if δ becomes larger, the marginal rate of substitution decreases and X is no longer an optimum. At point X, the slope of the indifference curve decreases and the marginal rate of substitution is less than the marginal product of capital. Thus, savings become larger and the economy moves toward point Y, which represents the steady state for $\delta > 0$.

Whenever $0 < \partial c^2 / \partial c^1 < 1 \Leftrightarrow -k\, f''(k) > (1+n)/(2+n)\, f'(k)$ along the intertemporal consumption possibility curve a rise in the steady state capital intensity implies a flatter consumption path over time and the optimum c^2/c^1-ratio declines. In this case the steady state is unstable and savings decline upon a rise in the strength of habits.

4 Conclusions

The present paper investigates the impact of habits (of a rise in the strength of habits) on savings and steady state capital intensity. It builds on Lahiri and Puhakka's (1998) analysis which shows that an increase in the strength of habits raises desired savings for an OLG *exchange economy*. The present paper considers an *economy with production* and capital accumulation. A rise in the strength of habits increases savings and steady state capital intensity if the steady state is asymptotically stable. Moreover, if the initial steady state is unstable, a rise in the strength of habits lowers savings.

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